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$(gsp)^*$-CLOSED SETS IN BITOPOLOGICAL SPACES

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ABSTRACT

In this paper we have introduced a new class of sets called (i, j)$(gsp)^*$-closed sets in bitopological spaces which is properly placed in between the class of closed sets and gsp-closed sets. As an application, we introduce two new spaces namely, T^*_{gsp}-space, gT^*_{gsp}-space.

***Keywords**: (i, j)$(gsp)^*$-closed set, T^*_{gsp}, gT^*_{gsp}-spaces.*

1. INTRODUCTION

Levine [10] introduced the class of g-closed sets in 1970. Maki.et.al [12] defined αg-closed sets in 1994. Arya and Tour [3] defined gs-closed sets in 1990. Dontchev [8], Gnanambal [9] Palaniappan and Rao[17] introduced gsp-closed sets, gpr-closed sets and rg-closed sets respectively. Veerakumar [18] introduced g^*-closed sets in 1991. J.Dontchev [8] introduced gsp-closed sets in 1995. Levine [10] Devi. *et al.* [5,6] introduced $T_{1/2}$- spaces, T_b spaces and ${}_\alpha T_b$ spaces respectively. Veerakumar [18] introduced $T_{1/2}$*-spaces. The purpose of this paper is to introduce the concepts of (i,j) $(gsp)^*$-closed sets, T^*_{gsp}-space, gT^*_{gsp}-space are introduced and investigated.

2. PRELIMINARIES

Throughout this paper (X,τ), (Y,σ) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ_1,τ_2) cl(A) and int(A) denote the closure and the interior of A respectively. The class of all closed subsets of a space (X,τ_1,τ_2) is denoted by (X,τ_1,τ_2) The smallest semi-closed (resp.pre-closed and α-closed) set containing a subset A of (X,τ_1,τ_2) is called the semi-closure (resp.pre-closure and α-closure) of A and is denoted by scl(A)(resp.pcl(A) and αcl(A))

Definition 2.1: A subset A of topological space (X,τ_1,τ_2) is called
(1) a pre-open set[14] if $A \subseteq \text{int}(cl(A))$ and a pre-closed set if cl(int(A)) $\subseteq A$
(2) a semi-open set [11] if $A \subseteq cl(\text{int}(A))$ and a semi-closed set if $\text{int}(cl(A)) \subseteq A$
(3) a semi-preopen set[1] if $A \subseteq cl(\text{int}(cl(A))$ and a semi-preclosed set[1] if $\text{int}(cl(\text{int}(A))) \subseteq A$
(4) an α-open set [15] if $A \subseteq \text{int}(cl(\text{int}(A)))$ and an α-closed set [15] if $cl(\text{int}(cl(A))) \subseteq A$
(5) a regular-open set[14] if intcl(A)=A and an regular-closed set [14] if A= intcl(A)

Definition 2.2: A subset A of topological space (X,τ_1,τ_2) is called
(1) a generalized closed set (briefly g-closed) [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ_1,τ_2)
(2) generalized semi-closed set(briefly) gs-closed [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ_1,τ_2)
(3) an α- generalized closed set (briefly αg-closed) [12] if $\alpha\, cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ_1,τ_2)

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(4) a generalized semi pre-closed set (briefly gsp-closed) [8] if sp $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ_1,τ_2)
(5) a regular generalized closed set (briefly rg-closed) [17] if sp $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X,τ_1,τ_2)
(6) a generalized pre-closed set (briefly gp-closed) [13] if p $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ_1,τ_2)
(7) a generalized pre regular-closed set (briefly gpr-closed) [9] if p $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ_1,τ_2)
(8) a wg-closed set [16] if $cl(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X,τ_1,τ_2)
(9) a gsp-closed set [8] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is gsp-open in (X,τ_1,τ_2)

Definition: 2.3: A topological space (X,τ) is said to be
(1) a $T_{1/2}$ space [10] if every g-closed set in it is closed.
(2) a T_b space [6] if every gs-closed set in it is closed.
(3) a αT_b space [5] if every αg-closed set in it is closed.

3. BASIC PROPERTIES OF (i, j)$(gsp)^*$ - CLOSED SETS

We introduce the following definitions

Definition 3.1: A subset A of bitopological space (X,τ_1,τ_2) is called a (i,j) $(gsp)^*$-closed set if τ_j-cl(A)$\subseteq U$ whenever $A \subseteq U$ and U is τ_i-gsp open.

Remark 3.2: By setting $\tau_1 = \tau_2$ in definition (1.1) a (i,j) $(gsp)^*$-closed set is a $(gsp)^*$-closed set.

Proposition 3.3: Every τ_j-closed set A is (i, j) $(gsp)^*$-closed.

Proof: Let A be τ_j-closed. Let $A \subseteq U$ and U is τ_i-gsp open.τ_j- cl(A) = A$\subseteq U$ whenever $A \subseteq U$ and U is τ_i-gsp open.
$\therefore$ A is (i, j) $(gsp)^*$-closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.4: X= {a, b, c},τ_1= {φ,X,{c},{a, c}},τ_2= {φ,X,{a}}
Then A= {b} is (i, j) $(gsp)^*$-closed but it is not τ_j-closed.
$\therefore$ Every (i, j)$(gsp)^*$-closed set need not be τ_j-closed.

Proposition 3 .5: Every (i, j) $(gsp)^*$-closed set is (i, j) g-closed.

Proof: Let A be (i, j) $(gsp)^*$-closed. Then τ_j-cl(A)$\subseteq$ U whenever A$\subseteq$ U and U is τ_i- gsp open.
Let us prove that A is (i, j) g-closed. Let $A \subseteq U$ where U be τ_i-open. Then $A \subseteq U$ where U is τ_i gsp-open.
Then τ_j-cl(A)$\subseteq U$, since A is (i, j) $(gsp)^*$-closed.
$\therefore$ A is (i, j) g-closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.6: X= {a, b, c},τ_1= {φ,X,{c},{a, c}},τ_2= {φ,X,{b},{a, b}}
Then A= {a} is (i, j) g-closed but it is not (i, j) $(gsp)^*$-closed.
$\therefore$ Every (i, j) g-closed set need not be (i, j) $(gsp)^*$-closed.

Proposition 3.7: Every (i, j) $(gsp)^*$-closed set is (i, j) gs-closed.

Proof: Let A be (i, j) $(gsp)^*$-closed.Then τ_j-cl(A)$\subseteq U$ whenever $A \subseteq U$ and U is τ_i gsp-open.
Let us prove that A is (i, j) gs-closed. Let $A \subseteq U$ and U is τ_i- open. Then $A \subseteq U$ and U is τ_i gsp-open.
$\therefore$ τ_j-cl(A)$\subseteq U$, since A is (i,j) $(gsp)^*$-closed.τ_j-scl(A)$\subseteq cl(A) \subseteq U$,
$\therefore$ τ_j-cl(A)$\subseteq U$, whenever $A \subseteq U$ and U is τ_i –open.
$\therefore$ A is (i,j) gs-closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.8: X= {a, b, c},τ_1= {φ,X,{c},{a,c}},τ_2= {φ,X,{b},{a,b}}
Then A= {a} is (i, j) gs-closed but it is not (i, j) $(gsp)^*$-closed.
$\therefore$ Every (i, j) gs-closed set need not be (i, j) $(gsp)^*$-closed.

Proposition 3.9: Every (i,j))$(gsp)^*$-closed set is (i,j) αg-closed.

Proof: Let A be (i, j) $(gsp)^*$-closed.Then τ_j-cl(A)$\subseteq U$ whenever $A \subseteq U$ and U is τ_i gsp-open.
Let us prove that A is (i, j) αg-closed.Let $A \subseteq U$ and U is τ_i- open.
Then $A \subseteq U$ where U is τ_i gsp-open.τ_j-cl(A)$\subseteq U$, since A is (i, j) $(gsp)^*$-closed.
τ_j-αcl(A)$\subseteq cl(A) \subseteq U$ $\therefore \tau_j$-αcl(A)$\subseteq U$ whenever $A \subseteq U$ and U is τ_i –open.
Hence A is (i, j) αg-closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.10: X= {a, b, c},τ_1= {φ,X,{c},{a,c}}, τ_2= {φ,X,{b},{a,b}}
Then A= {a} is (i, j) αg-closed but it is not (i, j) $(gsp)^*$-closed.
$\therefore$ Every (i, j) αg-closed set need not be (i, j) $(gsp)^*$-closed.

Proposition 3.11: Every (i, j))$(gsp)^*$-closed set is (i, j) gsp-closed.

Proof: Let A be (i,j) $(gsp)^*$-closed.Then τ_j-cl(A)$\subseteq U$ whenever $A \subseteq U$ and U is τ_i gsp-open.
Let us prove that A is (i, j) gsp-closed. Let $A \subseteq U$ where U is τ_i- open. Then $A \subseteq U$ where U is τ_i gsp-open.Then τ_j-cl(A)$\subseteq U$, since A is (i, j) $(gsp)^*$-closed.τ_j-spcl(A)$\subseteq$ τ_j-cl(A)$\subseteq U$
$\therefore \tau_j$-spcl(A)$\subseteq$U whenever $A \subseteq U$ and U is τ_i –open.
$\therefore$ A is (i, j) gsp-closed.

Example 3.12: X= {a, b, c},τ_1= {φ,X,{c},{a,c}},τ_2= {φ,X,{a}}
Then A= {c} is(i, j) gsp-closed but it is not (i, j) $(gsp)^*$-closed.
$\therefore$ Every (i, j) gsp-closed set need not be (i, j) $(gsp)^*$-closed.

Proposition 3.13: Every (i, j))$(gsp)^*$-closed set is (i, j) rg-closed.

Proof: Let A be (i, j) $(gsp)^*$-closed.Then τ_j-cl(A)$\subseteq U$ whenever $A \subseteq U$ and U is τ_i gsp-open.
Let us prove that A is (i, j) rg-closed. Let $A \subseteq U$ where U be τ_i – gsp-open.
Then $A \subseteq U$ where U is τ_i regular open.
$\therefore$ τ_j-cl(A)$\subseteq U$,since A is (i , j) $(gsp)^*$-closed.
$\therefore$ A is (i, j) rg-closed.

The converse of the above proposition is not true which is proved in the following example.

Example 3.14: X= {a, b, c},τ_1= {φ, X,{c},{a, c}},τ_2= {φ, X,{a}}
A= {c} is (i,j) rg-closed butit is not (i, j) $(gsp)^*$-closed.
$\therefore$ Every (i, j) rg-closed set need not be (i, j) $(gsp)^*$-closed.

Proposition 3.15: Every (i, j))$(gsp)^*$-closed set is (i,j) gp-closed but not conversely.

Proof: Let A be (i,j) $(gsp)^*$-closed.Then τ_j-cl(A)$\subseteq U$ whenever $A \subseteq U$ and U is τ_i gsp-open.
Let us prove that A is (i, j) gp-closed. Let $A \subseteq U$ and U is τ_i - open. Then $A \subseteq U$ and U is τ_i gsp-open.
Therefore τ_j-cl(A)$\subseteq U$,since A is (i, j) $(gsp)^*$-closed.τ_j-pcl(A)$\subseteq \tau_j cl(A) \subseteq U$.
$\therefore$ τ_j-pcl(A)$\subseteq U$ .Therefore A is (i, j) gp-closed.

The converse of the above proposition need not be true in general as seen in the following example

Example 3.16: X= {a, b, c},τ_1= {φ, X,{c},{a,c}},τ_2= {φ, X,{a}}

Then A= {c} is (i, j) gp-closed but it is not (i, j) $(gsp)^*$-closed.
∴ Every (i, j) gp-closed set need not be (i,j) $(gsp)^*$-closed.

Proposition 3.17: Every (i, j))$(gsp)^*$-closed set is (i, j) $g$pr-closed.

Proof: Let A be (i, j) $(gsp)^*$-closed.Then τ_j-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is τ_i gsp-open.
Let us prove that A is (i,j) gpr-closed. Let $A \subseteq U$ where U is τ_i- regular open.
Then $A \subseteq U$ where U is τ_i gsp-open.
$\Rightarrow$ τ_j-cl(A) $\subseteq U$, since A is (i, j) $(gsp)^*$-closed.
∴ τ_j-pcl(A) $\subseteq$ $\tau_i$cl(A) $\subseteq$ U.
∴ τ_j-pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i regular open.
∴ A is (i, j) gpr-closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.18: X= {a, b, c},τ_1= {φ,X,{c},{a,c}},τ_2= {φ,X,{a}}
Every subset A is gpr-closed. Then A= {a} is (i, j) gpr-closed but it is not (i, j) $(gsp)^*$-closed

Every (i, j) gpr-closed set need not be (i, j)$(gsp)^*$-closed.

Proposition 3.19: Every (i, j))$(gsp)^*$-closed set is (i,j) wg-closed but not conversely

Proof: Let A be (i,j) $(gsp)^*$-closed.Then τ_j-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is τ_i gsp-open.
Let us prove that A is (i,j) wg-closed. Let $A \subseteq U$ where U is τ_i - open. Then $A \subseteq U$ where U is τ_i gsp-open.
$\Rightarrow \tau_j$-cl(A) $\subseteq U$, since A is (i, j) $(gsp)^*$-closed.τ_2-cl(int(A)) $\subseteq \tau_2 cl(A) \subseteq U$
∴ τ_2-cl(int(A $\subseteq U)$ whenever τ_i - open.
∴ A is (i, j) wg-closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.20: X= {a, b, c},τ_1= {φ,X,{b},{a,b}},τ_2= {φ,X,{a}}
Then A= {b} is (i, j) wg -closed but not (i, j) $(gsp)^*$-closed.
∴ Every (i, j) wg-closed set need not be (i, j) $(gsp)^*$-closed.

4. APPLICATION OF (i, j) $(gsp)^*$-CLOSED SET

Definition 4 .1: A space (X,τ) is called a T^*_{gsp}- space if every $(i,j)(gsp)^*$-closed set is $\tau_j$closed.

Definition 4.2: A space (X,τ) is called a gT^*_{gsp}- space if every g-closed set is $(i,j)(gsp)^*$ closed.

Definition 4.3: A space (X,τ) is called a gT^*_{gsp}- space if every g-closed set is $(gsp)^*$ closed.

Theorem 4.4: Every $T_{1/2}$-space is T^*_{gsp}- space

Proof: Let (X, τ) be a $T_{1/2}$- space Let us prove that (X,τ) is a T^*_{gsp}- spaceLet A be a $(i,j)(gsp)^*$-closed set.
Since every $(i,j)(gsp)^*$-closed set is g-closed, A is τ_jg-closed.

Since (X,τ) is a $T_{1/2}$-space, A is $\tau_j$closed.
∴ (X,τ) is a T^*_{gsp}- space

The converse of the above theorem need not be true in general as seen in the following example.

Example 4.5: Let X = {a, b,c} and τ= {φ,X,{a},{a,b}}
Then in example3.5 we have proved that the g-closed sets are φ, X,{b},{a,b},{b,c} and the $(i,j)(gsp)^*$-closed sets are φ, X, {b},{a,b}, {b,c} Since all the $(i,j)(gsp)^*$-closed sets are closed, (X,τ) is a T^*_{gsp}- space.
∴ A= {b, c} is τ_jg-closed but it is not closed,and hence it is not a $T_{1/2}$- space. Hence a T^*_{gsp}- space need not be a $T_{1/2}$-space.

Theorem 4.6: Every αT_b-space is T^*_{gsp}- space but not conversely.

Proof: Let (X, τ) be a αT_b- space Let us prove that (X,τ) is a T^*_{gsp}- space. Let A be a (i, j), $(gsp)^*$-closed set. Every $(i, j)(gsp)^*$-closed set is $\tau_j\alpha$g-closed and hence A is $\tau_j\alpha$g-closed.

Since (X,τ) is a αT_b-space, A is $\tau_j$closed.
$\therefore$ (X,τ) is a T^*_{gsp}- space

The converse of the above theorem need not be true in general as seen in the following example.

Example 4.7: Let X = {a, b, c} and τ= {φ, X, {a}, {a, b}}
Then in example 3.10 we have proved that the αg-closed sets are φ, X, {a},{b},{c},{a, b}, {a, c},{b, c} and the $(i, j)(gsp)^*$-closed sets are φ, X, {b},{a, b}, {b, c}.

Since the $(i, j)(gsp)^*$-closed sets are closed, (X,τ) is a T^*_{gsp}- space.
$\therefore$ A= {b} is $\tau_j\alpha$g-closed, but it is not closed and hence it is not a αT_b- space.
Hence a T^*_{gsp}- space need not be a αT_b- space.

Theorem 4.8: Every T_b-space is T^*_{gsp}- space but not conversely.

Proof: Let (X, τ) be a T_b- space. Let us prove that (X, τ) is a T^*_{gsp}- space. Let A be a $(i, j)(gsp)^*$-closed set**.** Every $(i, j)(gsp)^*$-closed set is $\tau_j$gs-closed and hence A is $\tau_j$gs-closed.

Since (X,τ) is a T_b-space, A is $\tau_j$closed.
$\therefore$ (X,τ) is a T^*_{gsp}- space.

The converse of the above theorem need not be true in general as seen in the following example.

Example 4.9: Let X = {a, b, c} and τ= {φ, X, {a},{a,b}}
Then in example 3.8 we have proved that the gs-closed sets are φ, X, {a} {b},{c},{a, b}, {a, c},{b, c} and the $(i, j)(gsp)^*$-closed sets are φ, X, {b},{a, b}, {b, c}.

Since the $(gsp)^*$-closed sets are closed, (X,τ) is a T^*_{gsp}- space.
$\therefore$ A= {b} is gs-closed, but it is not closed, and hence it is not a αT_b- space.
Hence a T^*_{gsp}- space need not be a T_b- space.

REFERENCES

1. D. Andrijevic, semi- preopen sets, Mat. Vesnik, 38(1) (1986), 24-32.
2. I. Arokiarani, K. Balachandran and J. Dontchev, some characterizations of gp-irresolute and gp-continuous maps between topological spaces, Mem. Fac. Sci. Kochi. Univ. Ser. A. Math., 20 (1999), 93-104.
3. S.P. Arya and T. Nour, Characterization of s-normal spaces, Indian J.Pure.Appl.Math., 21 (1990), 717-719.
4. K. Balachandran, P. Sundaram and H. Maki, On generalized continuous maps in topological spaces, Mem. Fac. Kochi. Univ. Ser.A.Math., 12 (1991), 5-13.
5. R. Devi, K. Balachandran and H. Maki, generalized α-closed maps and α-generalized closed maps, Indian J.Pure.Appl.Math., 29(1)(1998),37-49.
6. R. Devi, H. Maki and K. Balachandran, Semi-generalized closed maps and generalized closed maps, Mem. Fac. Sci.Kochi.Univ.Ser.A.Math.,14(1993), 41-54.
7. R. Devi, H. Maki and K. Balachandran, Semi-generalized homeomorphisms and generalized semi-homeomorphisms in topological spaces, Indian J.Pure.Appl.Math., 26(3) (1995), 271-284.
8. J. Dontchev, on generalizing semi-preopen sets, Mem. Fac. Sci. Kochi. Ser. A, Math., 16 (1995), 35-48.
9. Y. Gnanambal, on generalized preregular closed sets in topological spaces, Indian J. Pure. Appl. Math., 28 (3) (1997), 351-360.
10. N. Levine, generalized closed sets in topology, Rend. Circ. Math. Palermo, 19 (2) (1970), 89-96.
11. N. Levine, semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70 (1963), 36-41.
12. H. Maki, R. Devi and K. Balachandran, Associated topologies of generalized α-closed sets and α-generalized closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 15 (1994), 51-63.
13. H. Maki, J. Umehara and T. Noiri, Every topological spaces is pre-$T_{1/2}$, Mem. Fac. Sci. Kochi. Univ. Ser. A, Math., 17 (1996), 33-42.
14. A.S. Mashhour, M.E. Abd EI-Monsef and S.N.EI-Deeb, on pre-continuous and weak pre-continuous mappings proc.Math. and Phys.Soc.Egypt, 53(1982),47-53.

15. O. Njastad, on some classes of nearly open sets, pacific J.Math., 15(1965),961-970.
16. N. Nagaveni, studies on generalizations of homeomorphisms in topological spaces, Ph.D, thesis, Bharathiar University, Coimbatore, 1999.
17. N. Palaniappan and K.C. Rao, Regular generalized closed sets, Kyungpook Math.J.,33(2)(1993),211-219.
18. M.K.R.S. Veerakumar, Between closed sets and $g-closed$ sets, Mem. Fac. Sci. Kochi. Univ. Ser.A, Math., 17 (1996), 33-42.

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