



SATISFACTORY ROOMMATES PROBLEM

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ABSTRACT

The present paper describes an algorithm, which will determine satisfactory matching of classical roommate's problem based on preference value, using assignment method.

Key words: Optimal matching, Preference Value, Satisfactory value matrix, Satisfactory Level.

1. INTRODUCTION

The stable roommate's problem is the extension of stable marriage problem [1]. It is a well known problem of matching $2n$ people into disjoint pairs to achieve a certain type of stability. The input to the problem is a set of $2n$ preference lists, one for each person i , where person i 's list is a rank ordering of the $2n-1$ people other than i . A roommate assignment A is a pairing of $2n$ people into n disjoint pairs. Assignment A is said to be unstable if there are two people who are not paired together in A , but who each prefer the other to their respective mates in A . Such a pair is said to be block Assignment A . An Assignment which is not unstable is called stable. An instance of the stable roommate's problem is called solvable if there is at least one stable assignment [3]. It is known that there are unsolvable instances of the stable roommate's problem [1]; the problem of finding an efficient algorithm to determine if an instance is solvable was proposed by Knuth and only recently solved by R. Irving [2]. In the next section, we introduced classical satisfactory roommate's problem and related definitions, to describe an algorithm, to find solution.

2. CLASSICAL SATISFACTORY ROOMMATE'S PROBLEM

The classical satisfactory roommate's problem (SFRP). is closely related to the stable roommate's problem. In the satisfactory roommate's problem each person in the set of even cardinality n ranks the $n-1$ others in order of preference. The object is to find satisfactory matching of roommate's problem. This is the partition of the set into $n/2$ pairs of roommates based on the individual satisfactory level. It is known that some of the instances of the satisfactory roommate's problem are unsolvable. In this paper, we define preference value for the preference list and by making use of this, an algorithm has described for finding matching. In this concept, in order to obtain optimal (satisfactory) matching, we have applied assignment method.

3. RELATED DEFINITIONS

In order to describe the new algorithm to find the matching using assignment technique, we need to introduce the following definitions.

Definition: 3.1 Satisfactory Value Matrix (SVM)

Matrix Representation of preference values [4] form a $n \times n$ matrix such that row and column represents members of group and it is denoted $SVM=[v_{ij}]$, where $[v_{ij}] = -$ (not possible to define) if $i=j$ and if $i \neq j$ then $[v_{ij}]$ is equal to the sum of preference value of j^{th} member with respect to i^{th} member and preference value of i^{th} member with respect to j^{th} member and it is equal to

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$$\frac{(n - (j - 1))}{n} + \frac{(n - (i - 1))}{n}$$

Definition: 3.2 Satisfactory level

The product of preference value of matching and 50 is the satisfactory level of that matching.

Definition: 3.3 Optimal (Satisfactory) matching

The matching which satisfies both members of a pair to maximum possible extend (Satisfactory level) is defined as optimal matching.

In other words, optimal matching is a matching in which change of any pair will lead to increase of satisfactory level of one person and decrease of other. But no way, satisfactory level of any one will increase without affecting satisfactory level of the other.

4. ABOUT ASSIGNMENT MODEL

The assignment Model is one of the fundamental combinatorial optimization models in the branch of operations research in mathematics. In its most general form, if there are a number of agents and a number of tasks that are to be assigned to agents. Any agent can be assigned to perform any task, incurring some cost that may vary depending on the agent-task assignment. The objective of this model is to assign all tasks on one to one basis to all agents in such a way that the total cost of the assignment should be minimized/maximized. To resolve this type of assignment problem Hungarian method is used.

4.1 Hungarian algorithm

1. Convert the maximization problem into minimization problem, by subtracting all the elements from the largest element of the matrix or by multiplying the matrix elements by -1.

2. (a) Row operation:

In matrix obtained from step 1, subtract the minimum element of each row from all the elements of that row. The reduced matrix will have at least one zero in each row.

(b) Column operation:

In matrix obtained from step 2(a), subtract the minimum element of each column from all the elements of that column. The reduced matrix will have at least one zero in each column.

3. Draw the minimum number of lines (horizontal and/or vertical) that are needed to cover all the zeros in the matrix obtained from step 2(b). If the minimum numbers of lines are equal to the order of the matrix then go to step 5. else go to the next step.

4. Find the smallest nonzero element (k) from the uncovered area, in the matrix obtained from step 3, and subtracts it from each element of uncovered area and adds it with the elements lie in the position of intersection of the lines and retain other elements as it and go to step 3.

5. Starting with first row of the matrix obtained from step 3, examine the rows/columns one by one until a row/column containing exactly one zero. If it is found, mark that zero by () and cross mark all the zeros lies in the column/row in which () marking is made. This procedure should be repeated until all zeros are marked or crossed.

6. Assigned zero marked by () in step 5 will give the optimal assignment. The sum of values in the cell marked by (), in the given assignment matrix will give optimal assignment value.

5. SATISFACTORY MATCHING ALGORITHM FOR ROOMMATES (SMAR)

1. Get the preference lists of all members
2. Form a Satisfactory Value Matrix (SVM)
3. Apply Hungarian method to find optimal (satisfactory) matching for all members such that the total assignment value should be maximized. That indicates optimal satisfactory level.

Example: 1 Consider the problem instance of size 4 based on order of preference.

1	2 3 4
2	1 3 4
3	1 2 4
4	1 2 3

The Satisfactory value Matrix is

$$\text{SVM} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} - & 2 & \frac{5}{3} & \frac{4}{3} \\ 2 & - & \frac{4}{3} & 1 \\ \frac{5}{3} & \frac{4}{3} & - & \frac{2}{3} \\ \frac{4}{3} & 1 & \frac{2}{3} & - \end{pmatrix} \end{matrix}$$

The resultant matching for the above instance is (1, 2) and (3, 4). The above result shows that the satisfactory level of matching (1, 2) is 100% and for (3, 4) is 33.33 %.

Example: 2 Consider the problem instance of size 6 based on order of preference.

1	2	6	4	3	5
2	3	5	1	6	4
3	1	6	2	5	4
4	5	2	3	6	1
5	6	1	3	4	2
6	4	2	5	1	3

The above instance will not give any solution.

Example: 3 Consider the problem instance of size 8 based on order of preference.

1	2	5	4	6	7	8	3
2	3	6	1	7	8	5	4
3	4	7	2	8	5	6	1
4	1	8	3	5	6	7	2
5	6	1	8	2	3	4	7
6	7	2	5	3	4	1	8
7	8	3	6	4	1	2	5
8	5	4	7	1	2	3	6

The Satisfactory value Matrix is

$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix} & \begin{pmatrix} - & \frac{12}{7} & \frac{2}{7} & \frac{12}{7} & \frac{12}{7} & \frac{6}{7} & \frac{6}{7} & \frac{6}{7} \\ \frac{12}{7} & - & \frac{12}{7} & \frac{4}{7} & \frac{6}{7} & \frac{12}{7} & \frac{6}{7} & \frac{6}{7} \\ \frac{2}{7} & \frac{12}{7} & - & \frac{12}{7} & \frac{6}{7} & \frac{6}{7} & \frac{12}{7} & \frac{6}{7} \\ \frac{12}{7} & \frac{2}{7} & \frac{12}{7} & - & \frac{6}{7} & \frac{6}{7} & \frac{6}{7} & \frac{12}{7} \\ \frac{12}{7} & \frac{6}{7} & \frac{6}{7} & \frac{6}{7} & - & \frac{12}{7} & \frac{2}{7} & \frac{12}{7} \\ \frac{6}{7} & \frac{12}{7} & \frac{6}{7} & \frac{6}{7} & \frac{12}{7} & - & \frac{12}{7} & \frac{2}{7} \\ \frac{6}{7} & \frac{6}{7} & \frac{12}{7} & \frac{6}{7} & \frac{2}{7} & \frac{12}{7} & - & \frac{12}{7} \\ \frac{6}{7} & \frac{6}{7} & \frac{6}{7} & \frac{12}{7} & \frac{12}{7} & \frac{2}{7} & \frac{12}{7} & - \end{pmatrix} \end{matrix}$$

The matching by SMAR, for the above instance, is

{(1, 2), (3, 4), (5, 6),(7, 8)},
{(1, 5), (2, 6), (3, 7),(4, 8)},
{(1, 4) (2, 3), (5, 6) (7, 8)},
{(1, 2) (3, 4), (5, 8) (6, 7)},
{(1, 4) (2, 6), (3, 7) (5, 8)},
{(1, 5) (2, 6), (3, 4) (7, 8)},
{(1, 5) (2, 3), (4, 8) (6, 7)} and
{(1, 2) (3, 7), (4, 8) (5, 6)}

The satisfactory level of all the above pair is 85.71 each.

For the above instance, by using SMAR, we got 8 set of optimal (satisfactory) matching and the satisfactory level is 85.71 for all pairs in all set of matching.

CONCLUSION

The Satisfactory Roommates problem is essentially a version of Stable Roommates problem involving just one set. In this paper, an algorithm (SMAR) is described, by defining preference value for the preference list of Roommates instance and satisfactory value matrix. This algorithm results a matching in which each pair attains maximum satisfactory level i.e. optimum. So SMAR results satisfactory matching of roommates.

REFERENCES

- [1] D. Gale and L.S. Shapley. College admissions and the stability of marriage. American Mathematical Monthly, 69(1962), pp 9-15.
- [2] Robert W. Irving An Efficient Algorithm for the Stable Roommates Problem, Journal of Algorithms 6(1985), 577-595.
- [3] D. Gusfield The structure of the stable roommate problem-efficient representation and enumeration of all stable assignments. SIAM Journal of computing, 17(4):742-769, 1988.
- [4] T. Ramachandran, K. Velusamy and T. Selvakumar Best optimal stable matching. AMS, Vol. 5, 2011, no. 75, 3743 – 3751.
