International Journal of Mathematical Archive-9(3), 2018, 25-26
Available online through www.ijma.info ISSN 2229 – 5046

ON THE K-METRO DOMINATION NUMBER OF CARTESIAN PRODUCT OF P_2XP_n

BASAVARAJU G C*[1], VISHUKUMAR M[2] AND RAGHUNATH P[3]

[1,2]Department of Mathematics, REVA University, Bang.alor-64, INDIA.

[3]Principal, BEST PU College, Chikkaballapur Karnataka, INDIA.

(Received On: 21-01-18; Revised & Accepted On: 10-02-18)

ABSTRACT

A dominating set D of a graph $G = G(V,E)$ is called metro dominating set G if for every pair of vertices u, v there exists a vertex w in D such that $d(u,w) \neq d(v,w)$. The k- metro domination number of Cartesian product of P_2XP_n $\left(\gamma_{\beta_k}(P_2XP_n)\right)$, is the order of smallest k- dominating set of P_2XP_n which resolves as a metric set. In this paper we calculate the k- metro domination number of Cartesian product of P_2XP_n.

Keywords: *Metric Dimension, Landmark, k- dominating set, Metro dominating set.*

INTRODUCTION

Let $G(V,E)$ be a graph. A subset of vertices $D \subseteq V$ is called a dominating set $(\gamma - set)$ if every vertex $V - D$ adjacent to at least one vertex in D. The minimum cardinality of dominating set is called domination number of the graph G and it is denoted by $\gamma(G)$.

The metric dimension of a graph G is denoted by $\beta(G)$, is defined as the cardinality of a minimal subset $S \subseteq V$ having the property that for each pair of vertices u, v in G there exists a vertex in w in S such that $d(u,w) \neq d(v,w)$, the coordinate of each vertex v of $V(G)$ with respect of each landmark u_i belongs to S is defined as usual with i^{th} component of v as $d(u, v_i)$ for each i and is of dimension $\beta(G)$.

Metro domination number introduced by B.Sooryanaraya and Raghunath.P [8]. Fink and Je-cobson [11] in 1985 introduced the concept of multiple domination. A subset D of $V(G)$ is k- dominating in G if every vertex of $V - D$ has at least k neighbours in D. The cardinality of minimum k-dominating set is called k-domination number of G and is denoted by $\gamma_k(G)$.

A dominating set D of a graph $G(V,E)$ is called metro dominating set of G if for each pair of vertices u, v there exists a vertex w *in* D such that $d(u,w) \neq d(v,w)$. *For example:* The set of darkened vertices of the graph G, *of figure*1, is 2-metro dominating set and hence $\gamma_{\beta_2}(G) = 3$.

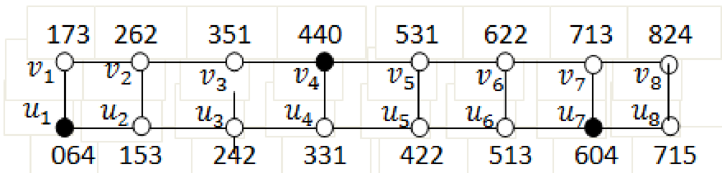

Figure − 1: $\gamma_{\beta_2}(P_2XP_8) = 3$.

Theorem: 1.1(3). For all $m \geq 2$ *and* $n \geq 3$ we have that $\beta(P_mXP_n)$ is 2 if n is odd, and 3 if n is even.

Theorem: 1.2(2). Let $K \geq 1$ then $\gamma_k(P_2XP_n) = \begin{cases} \frac{n}{2k} + 1 & if\ n \equiv 0(mod 2k) \\ \left\lceil \frac{n}{2k} \right\rceil & Otherwise \end{cases}$

Corresponding Author: Basavaraju G C[1],*
[1]Department of Mathematics, REVA University, Bang.alor-64, INDIA.

2. OUR RESULTS

Theorem 2.1: For all $m, n, \gamma_{\beta_2}(P_2XP_n) = \left\lceil \frac{n+1}{3} \right\rceil, n \geq 7$.

Theorem 2.2: For all $m, n,\ \gamma_{\beta_3}(P_2XP_n) = \left\lceil \frac{n+1}{4} \right\rceil$, n ≥ 9.

Theorem 2.3: For all $m, n,\ \gamma_{\beta_4}(P_2XP_n) = \left\lceil \frac{n+1}{5} \right\rceil, n \geq 11$.

Theorem 2.4: For all $m, n,\ \gamma_{\beta_k}(P_2XP_n) = \left\lceil \frac{n+1}{k+1} \right\rceil, n \geq 2k + 3$.

Proof: Consider P_2XP_n as two canonical copies of P_n with vertices labelled $u_1, u_2, \ldots\ldots u_n$ and $v_1, v_2, \ldots\ldots v_n$ with for each i, u_iv_i the only edges between the two paths. By using the theorem 1.1 and theorem 1.2, since a metro dominating set *D* is also a dominating set then we show that $\gamma_{\beta_k}(P_2XP_n) \geq \left\lceil \frac{n+1}{k+1} \right\rceil$. (1)

To prove reverse inequality we find a metro dominating set of cardinality $\left\lceil \frac{n+1}{k+1} \right\rceil$.

$D_1 = \{u_{2(l-k)+1} \colon l \geq 1\},\ n \equiv 0(mod2(k+1))$

$D_2 = \{v_{2l-k+2} \colon l \geq 1\},\ n \equiv k + 2(mod2(k+1))$

Let us choose the vertices dominates at least $2k + 1$, hence minimum number of vertices required to dominate the vertices of P_2XP_n is $\left\lceil \frac{n+1}{k+1} \right\rceil$. By using the theorem 1.1 we note that the dominating set which satisfies the above condition also serves as metric basis. Thus $\gamma_{\beta_k}(P_2XP_n) \leq \left\lceil \frac{n+1}{k+1} \right\rceil$. (2)

Therefore from (1) and (2), $\gamma_{\beta_k}(P_2XP_n) = \left\lceil \frac{n+1}{k+1} \right\rceil$.

REFERENCES

1. Buckley and Harary, *Distance in Graph*, Addison-Wesley, 1990.
2. Antoaneta Klobucar, *On the K-dominating number of cartesian product of two paths*, Math. Slovaca, 55(2005).No.2, 141-154.
3. Jose Caceres, Marya L, Puertas, Carmen Hernando, Mercue Mora, Ignacio M.Pelayo, Carlos Seara and David R.Wood, *On the metric dimension of cartesian product of graph.* http://www.Arxiv.org/math/0507527,march 2005, 1-23.
4. Basavaraju.G.C, Vishukumar.M and Raghunath.P, *Metro domination of square path*, Annals of Pure and applied mathematics, Vol.14, No.3, 2017, 539-545.
5. M.alishahi and Shalmaee, *Domination number of square of cartesian product of cycles*, open Journal of Discrete mathematics, 5(2015)88-94.
6. Lakshminarayana.S and Vishukumar.M, *On the K-Metro domination number of paths*, annals of Pure and applied mathematics, Vol.14, No.3, 2017, 593-600.
7. S.Khuller, B.Raghavachari and A.Rosenfeld, *Landmarks in graph*, Discrete Appl.Math, 70(3): 217- 229, 1996.
8. Raghunath P and Sooryanarana B, *Metro domination number of a graph*, Twetieth annual Conference of Rmanujan Mathematics Society, july 25-30(2005) university of Calicut, Calicut.
9. EL-ZAHAR.M-PAREEK.C.M, *The Domination number of products of graphs*, Ars Combin.31 (1991), 223-227.
10. B.Sooryanarayana, K.N.Geetha, *On the k-metric dimension of graphs*, j.MathComput.Sci.4 (2014), No.5, 861-878.
11. JACOBSON.M.S.-KINCH.L.F, *On the domination number of products of graphs I,*Ars Combin. 18(1983), 33-44.
12. Sandi Klavzar and Norbert Seifter, *Dominating Cartesian product of cycles*, Disc.applied Mathematics 59(1995) 129-136.