International Journal of Mathematical Archive-9(2), 2018, 265-273
Available online through www.ijma.info ISSN 2229 – 5046

TIME DELAY SEIRS E-EPIDEMIC MODEL FOR COMPUTER NETWORK

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(Received On: 01-01-18; Revised & Accepted On: 03-02-18)

ABSTRACT

In this paper, an attempt has been made to formulate a delayed SEIRS (Susceptible – Exposed – Infectious – Recovered – Susceptible) e – epidemic model with anti-malicious software and death due to malicious objects in computer network. It is proved that the malicious objects-free periodic solution is globally attractive if the rate of use of anti-malicious software is large enough and the solution is uniformly persistent if the rate is less than some critical value by using the comparison theorem. Our results indicate that a long latent period of the malicious objects or a proper use of anti-malicious software rate will lead to extinction of the malicious objects. Numerical methods are used to solve and simulate the system of equations developed and analyze the attacking behavior of malicious objects in the computer network.

***Key Words**: Malicious Objects; Threshold; Global Attractive; Simple Mass Action Incidence.*

***2000 Classification:** 92D30.*

INTRODUCTION

Now a days computer networks have become an essential tool for everyday life. Revolution has come in the field of education, information and defense through computer networks. The Internet has significantly reshaped the way people communicate with each other in recent years. People are sending messages, such as emails, playing games and sharing news, ideas and entertainment with their friends on the Internet. It has made the life easier and world accessible with the touch of button. But everything is not well in cyber world, it is facing several challenges in the form of malicious objects. These malicious objects are worms, virus and Trojan horse. Malicious objects have tremendous influence on computer network. Currently, e-mail has become one of the main factors for the transmission of malicious objects. Transmission of malicious objects in computer network is epidemic in nature and is analogous to biological epidemic diseases. Controlling the malicious objects in computer network have been an increasingly complex issue in recent years. In order to curb the malicious object, we propose an SEIRS model with time delay in infectious class. In recent years, much attention has been given for the persistence and global stability of the epidemic model with time delays. Mishra and Saini [1] introduced SEIRS model with latent and temporary immune period which is helpful to study about the malicious objects propagation with time delay in computer network. Mishra *et.al* have introduced different mathematical models on the fixed period of temporary immunity after the use of anti-malicious software, effect of quarantine, using fuzzy mathematics [2-5]. In epidemiology, the concept of distributed time delay and pulse vaccination, given by the authors [1,13] has developed the interesting changes in mathematical models and revealed effective strategies for the eradication of malicious objects. The basic concept on classical epidemic models introduced by Kermack and Mckendrick [13, 15, 16], gives the temporal evolutions of worms depending on the parameters in computer network. Mena-Lorca and Hethcote [17] studied five SIRS epidemic models for populations of varying size. Thieme [9] studied an SIRS epidemic model with population size dependent on contact rate and exponential demographics. The epidemic models of SEIR or SEIRS type without delays are widely studied [6-8, 10, 14]. Cooke and Van den Driessche [12] introduced and studied the disease transmission model of SEIRS type with exponential demographic structure.

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In the SEIRS model, once the malicious objects enter into the computer network, the nodes become susceptible and in the later phase they get exposed to the attack. Once the node is exposed to malicious object, then after a certain time delay the nodes become infected. After its get infected, anti malicious software is run which helps the node to recover from the attack and provide temporary immunity to the node in the network. There is no permanent immunity for the nodes which are in the cyber space.

The SEIRS e-epidemic model with time delay: We assume a total number of nodes N is partitioned into four compartments such as Susceptible, Exposed, Infectious, and Recovered. S(t), E(t), I(t) and R(t) denote the number of susceptible, exposed, Infectious, recovered nodes in computer network.

We have the system of differential equations

$$\frac{dS}{dt} = \mu(1 - S(t)) - \beta S(t)I(t) + \delta R(t)$$
$$\frac{dE}{dt} = \beta S(t)I(t) - \beta e^{-\mu\omega}S(t-\omega)I(t-\omega) - \mu E(t)$$
$$\frac{dI}{dt} = \beta e^{-\mu\omega}S(t-\omega)I(t-\omega) - (r+\mu+\alpha)I(t)$$
$$\frac{dR}{dt} = rI(t) - \mu R(t) - \delta R(t)$$
$$\frac{dN}{dt} = \mu - \mu N(t) - \alpha N(t) \qquad (1)$$
$$S(t^+) = (1-\theta)S(t)$$
$$E(t^+) = E(t); \quad t = K\tau$$

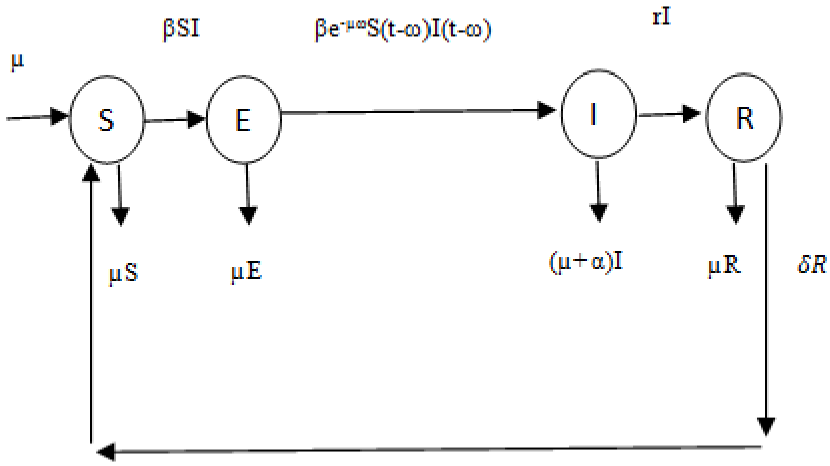

Figure-1: Schematic diagram for flow of malicious objects in computer network with time delay.

$$I(t^+) = I(t)$$
$$R(t^+) = R(t) + \theta S(t)$$

Where all coefficients are positive constants. The model derived as follows

(a) The susceptible nodes become infectious at the rate βI, where β is the constant rate.
(b) The birth rate is equal to the death rate and is denoted by μ.
(c) The time delay ω is the latent period of the malicious objects.
(d) We assume θ ($0 < \theta < 1$) is fraction of susceptible to whom the anti malicious software is inoculated at $t = k\tau$, $k \epsilon N$, where τ denotes period of run of anti malicious software.
(e) The run of anti malicious does not provide permanent immunity, i.e. R(t) contains vaccinated as well as recovered individuals.

Model (1) is subject to the restriction S(t) + E(t) + I(t) + R(t) = N(t). Note that the variable E does not appear in the first, third and fourth equations of system (1). We consider two reduced systems separately.

$$\frac{dS}{dt} = \beta S(t)I(t) + \mu(1 - S(t)) + \delta R(t)$$
$$\frac{dI}{dt} = \beta e^{-\mu\omega}S(t-\omega)I(t-\omega) - (r+\mu+\alpha)I(t)$$
$$\frac{dR}{dt} = rI(t) - bR(t) - \delta R(t) \qquad (2)$$
$$\frac{dE}{dt} = \beta S(t)I(t) - \beta e^{-\mu\omega}S(t-\omega)I(t-\omega) - \mu E(t)$$
$$E(t^+) = E(t)$$

The initial condition of system (2) is given as

$$S(\theta) = \varphi_1(\theta),\ I(\theta) = \varphi_2(\theta),\ R(\theta) = \varphi_3(\theta),\ N(\theta) = \varphi_4(\theta);\ -\omega \le \theta \le 0 \tag{3}$$

where $\varphi = (\varphi_1, \varphi_2, \varphi_3, \varphi_4)T \ \varepsilon\ C_+ = C([-\omega, 0], R_+^4)$, $\varphi_i(0)>0$, i=1,2,3,4.

$G = \{(S, E, I, R)\in R^4 / S \ge 0; E \ge 0; I \ge 0; R \ge 0; S + E + I + R \le N\}$. It is easy to show that G is positively invariant with respect to system (2).

To prove our main results we need the following lemma

Lemma 1: Let us consider the following impulsive differential equations

$$\frac{du}{dt} = a - bu(t);\quad t \ne K\tau \tag{4}$$
$$u(t^+) = (1-\theta)u(t);\quad t = K\tau$$

Where a > 0, b > 0, 0$< \theta < 1$. Then, there exist a unique positive periodic solution of system (3).

$$\bar{u}(t) = \frac{a}{b} + \left(u^* - \frac{a}{b}\right)e^{-b(t-K\tau)},\ \text{where } u^* = \frac{a(1-\theta)(1-e^{-b\tau})}{b(1-(1-\theta)e^{-b\tau})},\ \text{which is globally asymptotically stable.}$$

Proof: Integrating and solving the first equation of system (4) between pulses, we get

$$u(t) = \frac{a}{b} + \left(u(K\tau) - \frac{a}{b}\right)e^{-b(t-K\tau)},$$
$$K\tau < t \le (K+1)\tau$$

where u($k\tau$) be the initial value at time $k\tau$. Using the second equation of system (3), we deduce the stroboscopic map such that

$$u((k+1)\tau = (1-\theta)\left\{\frac{a}{b} + \left(u(k\tau) - \frac{a}{b}\right)e^{-b\tau}\right\} = \mathrm{f}(\mathrm{u}(k\tau)) \tag{5}$$

where $\mathrm{f(u)} = (1-\theta)\{\frac{a}{b} + \left(u - \frac{a}{b}\right)e^{-b\tau}\}$.

It is easy to know that system (4) has unique positive equilibrium.
$u^* = a(1-\theta)(1-e^{-b\tau})/b(1-(1-\theta)e^{-b\tau})$ which satisfies $u < f(u) < u^*$; $u^* < f(u) < u$ if $u > u^*$. It implies the corresponding periodic solution of (3)

$$\overline{u_e}\,(t) = \frac{a}{b} + \left(u^* - \frac{a}{b}\right)e^{-b(t-K\tau)},\ K\tau < t \le (K+1)\tau$$

is globally asymptotically stable.

Lemma 2: Consider the following delay differential equation $\frac{dx}{dt} = a_1x(t-\omega) - a_2x(t)$, Where $0 < a_1 < a_2$; $\omega > 0$. Then limit t tends to Zero, x(t) = 0.

We firstly demonstrate the existence of a malicious object free periodic solution in which infectious are entirely absent from the population permanently, i.e. I(t) = 0 for all $t \ge 0$. Under this condition, the growth of susceptible individuals, recovered individuals and total population must satisfy the following impulsive system

$$\frac{dS}{dT} = \mu - \mu S(t) + \delta R(t)$$
$$\frac{dR}{dt} = -(\mu+\delta)R(t);\ t \ne K\tau, k\epsilon N \tag{6}$$
$$\frac{dN}{dt} = \mu - \mu N(t)$$
$$S(t^+) = (1-\theta)S(t)$$
$$R(t^+) = R(t) + \theta S(t);\ t = K\tau$$
$$N(t^+) = N(t)$$

From the third and sixth equation of (6) we obtain limit t tends to infinity, N(t) = 1. If I(t) = 0, it follows from the second and sixth equations of system (1) that $limit\ t \to \infty$, E(t) = 0. Therefore, we have the following limit system of (5)

$$R(t) = 1 - S(t)$$
$$S'(t) = (\mu+\delta)\big(1 - S(t)\big);\ t \ne k\tau \tag{7}$$
$$S(t) = (1-\theta)S(t);\quad t = k\tau$$

According to Lemma 1, we know that periodic solution of system (7) is of the form

$$\overline{S_e}(t) = 1 - \frac{\theta e^{-(\mu+\delta)(t-k\tau)}}{1-(1-\theta)e^{-(\mu+\delta)\tau}},\ K\tau < t \le (K+1)\tau \tag{8}$$

Theorem 1: If $R^*<1$, then the infection free periodic solution of (2) is globally attractive, where

$$R^* = \frac{\beta e^{-\mu\omega}(1-e^{-(\mu+\delta)\tau})}{(r+\mu+\alpha)\big(1-(1-\theta)\big)e^{-(\mu+\delta)\tau}}$$

Proof: Since $R^*<1$, we can choose $\varepsilon_1 > 0$ sufficiently small such that

$$\frac{\beta e^{-\mu\omega}(1-e^{-(\mu+\delta)\tau})}{(r+\mu+\alpha)(1-(1-\theta))e^{-(\mu+\delta)\tau}} < (r+\mu+\alpha) \tag{9}$$

From the first equation of system (2), we have $\frac{dS}{dt} < (\mu+\delta)(1-S(t))$

So, we consider the following comparison impulsive differential system

$$\frac{dx}{dt} = (\mu+\delta)(1-x(t)); t \neq k\tau \tag{10}$$
$$x(t^+) = (1-\theta)x(t); \quad t = k\tau$$

By lemma (2), we can obtain the unique periodic solution of system (10).

$$\overline{S_e}(t) = \overline{x_e}(t) = 1 - \frac{\theta e^{-(\mu+\delta)(t+k\tau)}}{1-(1-\theta)e^{-(\mu+\delta)\tau}};\ k\tau < t \leq (k+1)\tau$$

Is globally asymptotically stable. Let (S(t), I(t)) be the solution of system (2) with initial values (3) and $S(0^+) = S_0 > 0$, x(t) be the solution of system (10) with initial value $x(0^+) = S_0$. By the comparison theorem in impulsive differential equation, there exists an integer $k_1 > 0$ such that

$$S(t) < S(t) < \overline{x_e}(t) + \varepsilon_1, \quad \tau < kt \leq (k+1)\tau, k > k_1$$

i.e. $$S(t) < \overline{x_e}(t) \leq \frac{1-e^{-(\mu+\delta)\tau}}{1-(1-\theta)e^{-(\mu+\delta)\tau}} + \varepsilon_1 \tag{11}$$
$$\triangleq S^M, \quad k\tau < t \leq (k+1)\tau \qquad k > k_1$$

where $S_e(t)$ is defined in (8). Further, from the second of system (2) we can write

$$\frac{dI}{dt} \leq \beta e^{-\mu\omega} S^M I(t-\omega) - (\alpha+r+\mu)I(t), \quad t > k_1 + \omega \tag{12}$$

From inequality (8), we have

$$\frac{\beta e^{-\mu\omega}(1-e^{-(\mu+\delta)\tau})}{(1-(1-\theta))e^{-(\mu+\delta)\tau}} < (r+\mu+\alpha)$$

By lemma (2), we have $\lim_{t\to\infty} y(t) = 0$. By comparison theorem and non negativity of I(t), we obtain

$$\lim_{t\to\infty} I(t) = 0 \tag{13}$$

There, for any sufficiently small $\varepsilon_1 \epsilon (0,1)$, there exists an integer $k_2 > k_1 + \frac{\omega}{\tau}$ satisfying I(t)< ε_1 for all t > $k_2\tau$. From the fifth equation of system (1), we have

$$N'(t) \geq \mu - \mu N(t) - \alpha\varepsilon_1; t > k_2\tau. \tag{14}$$

Consider the following comparison system

$$\frac{dz}{dt} = (\mu - \alpha\varepsilon_1) - \mu z(t); \quad t > k_2\tau.$$

Since $\lim_{t\to\infty} \frac{dz}{dt} = 0$

$$\therefore z(t) = \frac{\mu - \alpha\varepsilon_1}{\mu}$$

By comparison theorem, there is an integer $k_3 > k_2$ such that

$$N(t) \geq \frac{\mu - \alpha\varepsilon_1}{\mu} - \varepsilon_1, \text{ for all } t > k_3\tau. \tag{15}$$

Because ε_1 can be arbitrary small and sup $\lim_{t\to\infty} N(t) \leq 1$.

Hence, $\lim_{t\to\infty} = 1.$ (16)

From equation (13) and (16), there exists an integer $k_4 > k_3$ such that

$$I(t) < \varepsilon_1, \text{ and } N(t) > 1-\varepsilon_1 \tag{17}$$

For all t > $k_4\tau$.

When t > $k_4\tau$, the second and sixth equations of $\frac{dE}{dt} \leq \beta\varepsilon_1 - \mu E(t)$. This implies that there exists an integer $k_5 > k_4$ such that

$$E(t) \leq \frac{\beta\varepsilon_1}{\mu} + \varepsilon_1, \text{ for all } t > k_4\tau. \tag{18}$$

Therefore, from inequalities (17) and (18), the first equation of system (1)

$$\frac{dS(t)}{dt} = -\beta S(t)I(t) + \mu\big(1 - S(t)\big) + \delta R(t)$$

$$\frac{dS(t)}{dt} = -\beta S(t)I(t) + \mu\big(1 - s(t)\big) + \delta(N(t) - S(t) - E(t) - I(t))$$

$$\geq \left(\mu + \delta\left(1 - 3\varepsilon_1 - \frac{\beta\varepsilon_1}{\mu}\right)\right) - (\mu + \delta + \beta\varepsilon_1)S(t), \quad for\ all\ t > k_5\tau$$

Consider the following comparison impulsive differential equation for $t > k\tau$ and k > k_5,

$$u(t) = \left(\mu + \delta\left(1 - 3\varepsilon_1 - \varepsilon_1\beta\frac{1}{\mu}\right)\right) - (\mu + \delta + \beta\varepsilon_1)u(t), \quad t \neq k\tau \tag{19}$$

$$u(t^+) = (1 - \theta)u(t), \ t = k\tau$$

By lemma1, we have the unique periodic solution of the system (19)

$$\bar{u}(t) = \frac{\mu+\delta\left(1-3\varepsilon_1-\frac{\beta\varepsilon_1}{\mu}\right)}{\mu+\delta+\beta\varepsilon_1} + \left(\mu^* - \frac{\mu+\delta\left(1-3\varepsilon_1-\frac{\beta\varepsilon_1}{\mu}\right)}{\mu+\delta+\beta\varepsilon_1}\right)$$

where $u^* = \left\{\frac{\mu+\delta\left(1-3\varepsilon_1-\frac{\beta\varepsilon_1}{\mu}\right)}{\mu+\delta+\beta\varepsilon_1}\right\}\left\{\frac{(1-\theta)\left(1-e^{-(\mu+\delta+\beta\varepsilon_1)\tau}\right)}{1-(1-\theta)e^{-(\mu+\delta+\beta\varepsilon_1)\tau}}\right.$

Which is globally asymptotically stable.

According to the comparison theorem of impulsive differential equations, there exists an integer $k_6 > k_5$ such that

$$S(t) > u_e(t) - \varepsilon_1, \tau < \mathrm{k}t \leq (k+1)\tau, k > k_6 \tag{20}$$

Since $\varepsilon_1 and\ \varepsilon_0$ are arbitrarily small, it follows from (10), (18) and (20) that $t \overset{lim}{\longrightarrow} \infty\ S(t) = S_e(t)$, $t \overset{lim}{\longrightarrow} \infty\ E(t) = 0$.

This implies

$$\overline{S_e}(t) = \overline{x_e}(t) = 1 - \frac{\theta e^{-(\mu+\delta)(t+k\tau)}}{1-(1-\theta)e^{-(\mu+\delta)\tau}};\ \mathrm{k}\tau < t \leq (k+1)\tau$$

Is globally attractive. Therefore, infection free solution $(\overline{S}_e(t),0)$ is globally attractive. The proof of above theorem (1) is complete.

Denote $\theta^* = \frac{(\beta e^{-\mu\omega}-r-\mu-\alpha)(e^{-(\mu+\delta)\tau}-1)}{(\mu+\alpha+r)}$

$$\tau^* = \frac{1}{\mu}\log\left(1 + \frac{\theta(r+\mu+\alpha)}{(\beta e^{-\mu\omega} - r - \mu - \alpha)}\right)$$

$$\omega^* = -\frac{1}{\mu}\log\left\{\frac{(r+\mu+\alpha)\big(1-(1-\theta)\big)}{\beta(1-e^{-(\mu+\delta)\tau})}\varepsilon^{-(\mu+\delta)\tau}\right\}$$

Corollary 1: (a) If $\beta e^{-\mu\omega} \leq r + \mu + \alpha$, then the infection free periodic solution $(\overline{S_e}(t),0)$ is globally attractive. (b) If $\beta e^{-\mu\omega} > r + \mu + \alpha$, then the infection free periodic solution $(\overline{S_e}(t),0)$ is globally attractive provided that $\theta > \theta^*$ or $\tau > \tau^*$.

Corollary 2: (a) If $\beta e^{-(\mu+\delta)\tau} \leq (r + \mu + \alpha)(1 - (1-\theta)e^{-(\mu+\delta)\tau}$, then the periodic solution $(\overline{S_e}(t),0)$ is globally attractive. (b) If $\beta e^{-(\mu+\delta)\tau} > (r + \mu + \alpha)(1 - (1-\theta)e^{-(\mu+\delta)\tau}$, then the infection free periodic solution $\overline{S_e}(t),0)$ is globally attractive provided that $\omega > \omega^*$.

Remark: theorem 1 determines the global attractively of system (1) in G for the case R^*<1. Its epidemiological implication is that the infectious population vanishes. Corollaries 1and 2 imply that the infectious population will disappear if the length of latent period of the infectious nodes large enough.

PERMANENCE

In this section we say the infectious population is endemic if the infectious population persists a certain level for sufficiently large time. The endemicity of the infectious population can be well captured and studied through the notion of uniform persistence and permanence.

Definition 1: System (1) is said to be uniformly persistent if there is $\eta > 0$ (independent of the initial data) such that every solution (S(t), I(t)) with initial conditions (3) of system (2) satisfies

$$\lim_{t \to \infty} \inf S(t) \geq \eta; \quad \lim_{t \to \infty} \inf I(t) \geq \eta$$

Definition 2: System (1) is said to be permanent if there exists a compact region $G_0 \epsilon$ inf G such that every solution of (2) with initial conditions (3) will eventually enter and remain in region G_0.

We denote $R_* = \frac{\beta e^{-\mu\omega}(1-\theta)\left(1-e^{-(\mu+\delta)\tau}\right)}{\left(1-(1-\theta)e^{-(\mu+\delta)\tau}\right)(r+\alpha+\mu)}$; $\quad I^* = \frac{\mu e^{-\mu\omega}(1-\theta)\left(1-e^{-(\mu+\delta)\tau}\right)}{\left(1-(1-\theta)e^{-(\mu+\delta)\tau}\right)(r+\alpha+\mu)} - \frac{\mu}{\beta}$ (21)

Theorem: suppose $R_*>1$, then there is a positive constant q such that each positive solution (S(t), I(t)) of system (2) satisfies $I(t) \geq q$ if t is large.

Proof: The second equation of (2) can be written as

$$\frac{dI}{dt} = \beta e^{-\mu\omega} S(t-\omega)I(t-\omega) - (r+\mu+\alpha)I(t) + \beta e^{-\mu\omega}S(t)I(t) - \beta e^{-\mu\omega}S(t)I(t)$$
$$= \{\beta e^{-\mu\omega}S(t) - (r+\mu+\alpha)\}I(t) - \beta e^{-\mu\omega}\frac{d}{dt}\int_{t-\omega}^{t} S(\theta)I(\theta)d\theta$$

Let us consider any positive solution (S(t), I(t)) of system (2). According to this solution, we define

$$V(t)=I(t)+\beta e^{-\mu\omega}\int_{t-\omega}^{t} S(\theta)\, I(\theta)d\theta \quad (22)$$

The derivative of V(t) along the solution (2) is

$$\frac{dV}{dt} = (\alpha + r + \mu)I(t)\left(\frac{\beta e^{-\mu\omega}}{(r+\mu+\alpha)}S(t) - 1\right) \quad (23)$$

Since $R^*>1$, we see that $I^* > 0$ and there exists sufficiently small $\epsilon > 0$ such that

$$\left(\frac{\beta e^{-\mu\omega}}{(\mu+r+\alpha)}\left(\frac{\mu}{\mu+\delta+\beta I^*} \times \frac{(1-\theta)(1-e^{-(\mu+\delta+\beta I^*)\tau}}{1-(1-\theta)e^{-(\mu+\delta+\beta I^*)\tau}} - \epsilon\right) > 1 \quad (24)$$

We claim that it is impossible that $I(t) \leq I^*$ for all $t \geq t_0$ (t_0 is non negative constant). Suppose the contrary, then as $t \geq t_0$.

$$\frac{dN}{dt} = \mu - \mu N(t) - \alpha I \geq (\mu - \alpha I^*) - \mu N(t) \quad (25)$$
$$\frac{dE}{dt} = \beta S(t)I(t) - \beta e^{-\mu\omega}S(t-\omega)I(t-\omega) - \mu E(t)$$

This implies $\frac{dE}{dt} \leq \beta I^* - \mu E(t)$ (26)

By (25) and (26), there exists $T_1 \geq \omega$ such that

$N(t) \geq 1 - \frac{\alpha}{\mu}I^* - \frac{\epsilon}{2}$; $\quad E(t) \geq \frac{\beta I^*}{\mu} + \frac{\epsilon}{2}$ for all $t \geq t_0 + T_1$. Hence, when $t \geq t_0 + T_1$ and $t \neq k\tau$ $(k\epsilon N)$

$$\frac{dS(t)}{dt} = \mu - \mu S(t) - \beta S(t)I(t) + \delta(N(t) - S(t) - E(t) - I(t))$$
$$\geq \left(\mu + \delta\left(1 - \frac{\alpha}{\mu}I^* - \frac{\beta I^*}{\mu} - I^* - \epsilon\right)\right) - (\mu + \beta I^* + \delta)S(t)$$

Consider the following comparison system

$$\frac{dV}{dt} = \left(\mu + \delta\left(1 - \frac{\alpha}{\mu}I^* - \beta\frac{I^*}{\mu} - I^* - \epsilon\right)\right) - (\mu + \beta I^* + \delta)V(t); \quad t \neq k\tau \quad (27)$$
$$V(t) = (1-\theta)V(t); \quad t = k\tau$$

By Lemma 1, we obtain

$$V_e(t) = \frac{\left(\mu+\delta\left(1-\frac{\alpha}{\mu}I^*-\beta\frac{I^*}{\mu}-I^*-\epsilon\right)\right)}{(\mu+\beta I^*+\delta)} + \left(V^* - \frac{\left(\mu+\delta\left(1-\frac{\alpha}{\mu}I^*-\beta\frac{I^*}{\mu}-I^*-\epsilon\right)\right)}{(\mu+\beta I^*+\delta)}\right)e^{-(\mu+\beta I^*+\delta)(t-k\tau)};$$

$k\tau < t \leq (k+1)\tau$

where $V^* = \frac{\left(\mu+\delta\left(1-\frac{\alpha}{\mu}I^*-\beta\frac{I^*}{\mu}-I^*-\epsilon\right)\right)(1-\theta)}{(\mu+\beta I^*+\delta)(1-(1-\theta))(e^-(\mu+\beta I^*+\delta)\tau))}(1 - e^-(\mu + \beta I^* + \delta)\tau)$

is the unique globally asymptotically stable periodic solution of system (27). There exists a T^* greater than T_1 satisfying

$$S(t) > V_e(t) - \varepsilon \geq V^* - \varepsilon = \eta$$
$$for\ all\ t \geq t_0 + T^* \triangleq t_1 \quad (28)$$

By (22) and (28), we have

$$\frac{dV}{dt} \geq (r+\mu+\alpha)\left(\frac{\beta e^{-\mu\omega}}{r+\mu+\alpha}\eta - 1\right)$$
$$for\ all\ t \geq t_1;\quad t \neq k\tau, t-\omega \neq k\tau \tag{29}$$

Set $I_l = I(t)$ where min of $t\epsilon[t_1, t_1+\omega]$

We have to show that $I(t) \geq I_l$ for all $t \geq t_1$. If it is not true, then there exists a $T_0 \geq 0$ such that $I(t) \geq I_l$ for all $t_1 \leq t \leq t_1+\omega+T_0$, $I(t_1+\omega+T_0) = I_l$ and $\frac{dI(t1+\omega+T_0)}{dt} \leq 0$. However, the second equation of system (2) and (28) imply that

$$I(t_1+\omega+T_0) > (r+\mu+\alpha)\left(\frac{\beta e^{-\mu\omega}}{r+\mu+\alpha}\eta - 1\right)I_l > 0.$$

This is a contradiction. Thus, $I(t) \geq I_l$ for all $t > t_1$, consequently $t>t_1$, we have that

$$\frac{dV}{dt} \geq (r+\mu+\alpha)\left(\frac{\beta e^{-\mu\omega}}{r+\mu+\alpha}\eta - 1\right)I_l > 0,\quad (t \neq k\tau, \omega \neq k\tau)$$

Since V(t) is continuous on [0, +∞] and these points at which V(t) is not derivable are at most countable, this implies $V(t) \rightarrow \infty$ as $t \rightarrow \infty$. This is contrary to the boundedness of V(t). Hence, the claim is proved. From the claim, we will discuss the following two possibilities

(1) $I(t) \geq I^*$ for large t.
(2) I(t) oscillates about I^* for large t.

Finally, we will show that $I(t) \geq I^* e^{-(r+\alpha+\mu)(T^*+\omega)} \triangleq q$ as t is large sufficiently. Evidently, we consider the case (2). Let t_1 and t_2 be large sufficiently times satisfying:

$$I(t_1) = (I_2) = I^*;\quad I(t) < I^* \text{ as } t\epsilon(t_1, t_2).$$

If $t_2 - t_1 \leq T^* + \omega$, since $\frac{dI}{dt} \geq -(\mu+\alpha+r)I(t)\alpha$ and $I(t_1) = I^*$ which implies $I(t) \geq q$ for all $t\epsilon[t_1, t_1+T^*+\omega]$. Thus, proceeding exactly as the proof for above claim, we see that $I(t) \geq q$ for all $t\epsilon[t_1, t_1+T^*+\omega]$ and $S(t) > \eta$ for all $t\epsilon[t_1+T^*, t_2]$. Next, we will prove that $I(t) \geq q$ is not true, then there is a $T_1 \geq 0$ such that $I(t) \geq q$ for all $t \epsilon [t_1, t_1+T^*+\omega+T_1]$, $I(t_1+T^*+\omega+T_1) = q$ and $I'(t_1+T^*+\omega+T_1) \leq 0$. Using the second equation of system (2) as $t = t_1+T^*+\omega+T_1$, we further obtain

$$\frac{dI}{dt} = \beta e^{-\mu\omega} S(t-\omega)I(t-\omega) - (r+\mu+\alpha) \geq (\beta e^{-\mu\omega}\eta - (\mu+\alpha+r)q > 0.$$

This is a contradiction. So, $(I) \geq q$ is valid for all $t\epsilon[t_1, t_2]$.

Theorem 3: Suppose $R_* > 1$. Then system (2) is permanent.

Proof: Let (S(t), I(t), R(t), N(t)) be any solution of system (2). From the first equation of system (2), we have

$$S'(t) \geq \mu - (\mu+\beta)S(t).$$

We consider the following auxiliary comparison system

$$\frac{dx}{dt} = \mu - (\mu+\beta)x(t), t \neq k\tau; k\epsilon N \tag{30}$$
$$x(t^+) = (1-\theta)x(t);\quad t = k\tau.$$

By Lemma 1, we get that

$$\underset{t\longrightarrow\infty}{lim} \inf S(t) \geq p; \tag{31}$$

where $p = \frac{\mu}{\mu+\beta} \times \frac{(1-\theta)(1-e^{-(\mu+\beta)\tau})}{1-(1-\theta)e^{-(\mu+\beta)\tau}}$

From the theorem (2) and the third equation of system(2), we have

$$R'(t) = r \times q - (\mu+\delta)R(t).$$

It is easy to see that $\underset{t\longrightarrow\infty}{lim} \inf R(t) \geq \omega;\quad \omega = \frac{rq}{\mu+\delta}$

Set $G_0 = \left\{(S, I, R, N): S \geq p; I \geq q; R \geq \omega; S+I+R \leq 1; \frac{\mu}{\mu+\alpha} \leq N \leq 1\right\}$

By the theorem (2) and (3), we know that the set G_0 is globally attractive, i.e., every solution of (2) with initial condition (3) will eventually enter and remain in the region G_0. Therefore, the system (2) is permanent.

CONCLUSION

In this paper, we have studied the dynamical behavior of a delay SEIRS e- epidemic model with anti-malicious software and saturation incidence rate. We introduced two threshold values R^* and R_*, one for the global stability of the infection-free solution and another for the permanence of the endemic solution. We obtain if $R^*<1$then the malicious objects will be permanent which means that after some period of time the malicious objects will become endemic. Our results indicate that a long latent period of the malicious objects or a large number of malicious software will lead to eradication of the malicious objects. Numerical methods are employed to solve and simulate the system of equations developed with different parameters and behavior of susceptible, infected and recovered nodes with respect to time are observed, which is depicted in figure 2.The analysis of the graph says that susceptible and exposed nodes are decreasing initially with the increase of time, infectious nodes and recovered nodes are first increasing and then decreasing with the increase of time. Ultimately, They all are asymptotically stable.

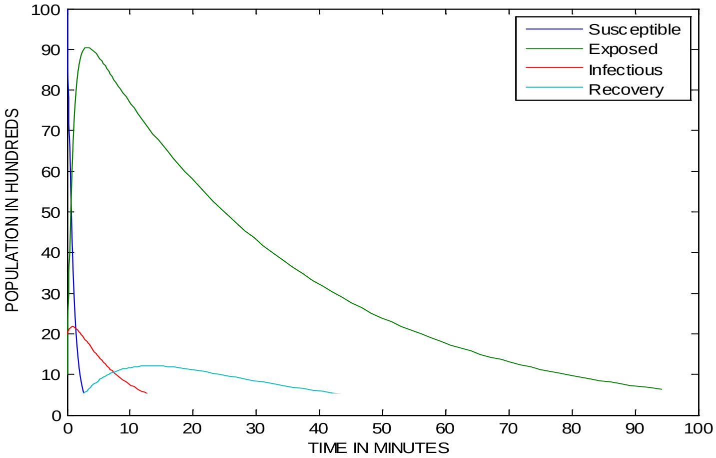

Figure-2: Dynamical behavior of susceptible, exposed, infected and recovered nodes with respect to time Dynamical behavior of system (1) with r $\beta = 0.05$; $\alpha = 0.001$; $\delta = 0.01$; $\mu = 0.03$; $r = 0.1$; $\omega = 0.3$; $E(0) = 1000$; $I(0) = 2000$; $S(0) = 10000$ and $N(0) = 15000$.

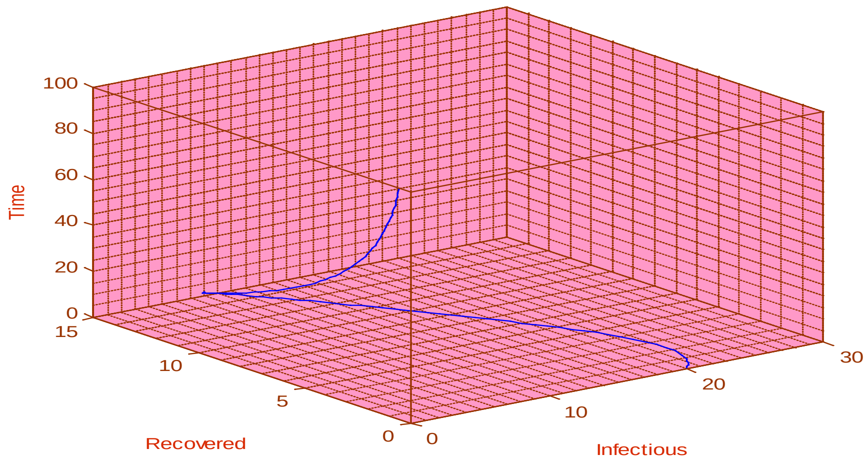

Figure-3: Dynamical behavior of infected nodes and recovered nodes with time

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Source of support: Nil, Conflict of interest: None Declared.