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ACYCLIC AND STAR COLORING OF DUPLICATE GRAPH OF LADDER GRAPH

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ABSTRACT

In this paper we present acyclic and star coloring algorithms to color the vertices of Duplicate graph of ladder L_m. Also we obtain the chromatic numbers of the same.

Keywords: *acyclic coloring, star coloring, duplicate graph, ladder graph.*

INTRODUCTION

A proper coloring of a graph G is the coloring of the vertices of G such that no two neighbors in G are assigned the same color. Thoughout this paper, by a graph we mean a finite, undirected, simple graph and the term coloring is used to denote vertex coloring of graphs.

A acyclic coloring of a graph G is the proper vertex coloring such that the subgraph induced by 2 colors α and β is a forest. The notion of acyclic chromatic number was introduced by B.Grunbaum in 1973. The acyclic chromatic number of a graph G = G(V, E) is the minimum number of colors which are necessary to color G acyclically and is denoted by a(G).

A star coloring of a graph G is the proper vertex coloring in which every path on 4 vertices uses atleast 3 distinct colors. Equivalently, in star coloring, the induced subgraph formed by the vertices of any 2 colors has connected components that are stars. The notion of star chromatic number was introduced by B.Grunbaum in 1973. The star chromatic number, $\chi_s(G)$ of G, is the least number of colors needed to star color G.

A ladder graph L_m is a planar undirected graph with 2m vertices and 3m-2 edges. It is obtained as the cartesian product of two path graphs, one of which has only one edge: $L_{m,1} = P_m \times P_1$, where m is the number of rungs in the ladder.

A duplicate graph of G is DG = (V_1, E_1) where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and f: $V \to V'$ is bijective (for $v \in V$, we write $f(v) = v'$) and the edge set E_1 of DG is defined as. The edge uv is in E if and only if both uv′ and u′v are edges in E_1.

Acyclic Coloring of DG(L_m)

Coloring Algorithm

Input: DG(L_m), $m \geq 3$
$V \leftarrow \{v_1, v_2, \ldots, v_m, v', v'_1, v'_2, \ldots, v'_m\}$
if $m \equiv 0 \pmod 3$
{
for (k = 1 to $2\left\lfloor \frac{m+1}{3} \right\rfloor$)

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{
v_{3k-2}, v'_{3k-2} $\leftarrow$ 1;
v_{3k-1}, v'_{3k-1} $\leftarrow$ 2;
v_{3k}, v'_{3k} $\leftarrow$ 3;
}
}

else if m ≡ 1(mod 3)
{
for (k = 1 to $\left\lfloor \frac{2m+1}{3} \right\rfloor$)
{
v_{3k-2}, v'_{3k-2} $\leftarrow$ 1;
v_{3k-1}, v'_{3k-1} $\leftarrow$ 2;
}
for (k = 1 to $2\left\lfloor \frac{m+1}{3} \right\rfloor$)
{
v_{3k}, v'_{3k} $\leftarrow$ 3;
}
}

else
{
for (k = 1 to $\left\lfloor \frac{2m+1}{3} \right\rfloor$)
{
v_{3k}, v'_{3k} $\leftarrow$ 3;
v_{3k-1}, v'_{3k-1} $\leftarrow$ 2;
}

for (k = 1 to $2\left\lfloor \frac{m+1}{3} \right\rfloor$)
{
v_{3k-2}, v'_{3k-2} $\leftarrow$ 1;
}
}

Output: vertex colored DG(L_m)

Theorem: The acyclic chromatic number of DG(L_m) is **a[DG(L_m)] = 3, m ≥ 3**.

Proof: Color the vertices of DG(L_m) as given in the algorithm.

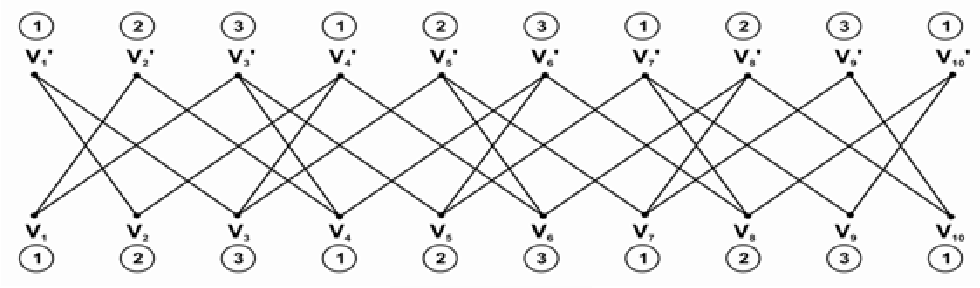
CASE-(I): When m ≡ 0(mod 3)

The color class of 1 is {v_{3k-2}, v'_{3k-2}; $1 \le k \le 2\left\lfloor \frac{m+1}{3} \right\rfloor$}. The color class of 2 is {v_{3k-1}, v'_{3k-1}; $1 \le k \le 2\left\lfloor \frac{m+1}{3} \right\rfloor$}. The color class of 3 is {v_{3k}, v'_{3k}; $1 \le k \le 2\left\lfloor \frac{m+1}{3} \right\rfloor$}.

Case-(i): Consider the color classes 1 and 2. The induced subgraph of these color classes is the union of paths P_3 and P_4.

Case-(ii): Consider the color classes 1 and 3. The induced subgraph of these color classes is a collection of P_4.

Case-(iii): Consider the color classes 2 and 3. The induced subgraph of these color classes is the union of paths P_3 and P_4.



a[DG(L$_5$)]=3

CASE-(II): When m ≡ 1(mod 3)

The color class of 1 is $\{v_{3k-2}, v'_{3k-2};\ 1 \le k \le \left\lfloor \frac{2m+1}{3} \right\rfloor\}$. The color class of 2 is $\{v_{3k-1}, v'_{3k-1};\ 1 \le k \le \left\lfloor \frac{2m+1}{3} \right\rfloor\}$. The color class of 3 is $\{v_{3k}, v'_{3k};\ 1 \le k \le 2\left\lfloor \frac{m+1}{3} \right\rfloor\}$.

Case-(i): Consider the color classes 1 and 2. The induced subgraph of these color classes is the union of paths P_3 and P_4.

Case-(ii): Consider the color classes 1 and 3. The induced subgraph of these color classes is a collection of P_4.

Case-(iii): Consider the color classes 2 and 3. The induced subgraph of these color classes is the union of paths P_3 and P_4.

CASE-(III): When m ≡ 2(mod 3)

The color class of 1 is $\{v_{3k-2}, v'_{3k-2};\ 1 \le k \le 2\left\lfloor \frac{m+1}{3} \right\rfloor\}$. The color class of 2 is $\{v_{3k-1}, v'_{3k-1};\ 1 \le k \le \left\lfloor \frac{2m+1}{3} \right\rfloor\}$. The color class of 3 is $\{v_{3k}, v'_{3k};\ 1 \le k \le 2\left\lfloor \frac{m+1}{3} \right\rfloor\}$.

Case-(i): Consider the color classes 1 and 2. The induced subgraph of these color classes is the union of paths P_3 and P_4.

Case-(ii): Consider the color classes 2 and 3. The induced subgraph of these color classes is the union of paths P_3 and P_4.

Case-(iii): Consider the color classes 1 and 3. The induced subgraph of these color classes is a collection of P_4.

In all the cases, the induced subgraph of any two color classes is not a cycle.

Thus coloring given in the algorithm is an acyclic coloring.

Therefore **$a[DG(L_m)] = 3, m \ge 3$.**

Star Coloring of DG(L$_m$)

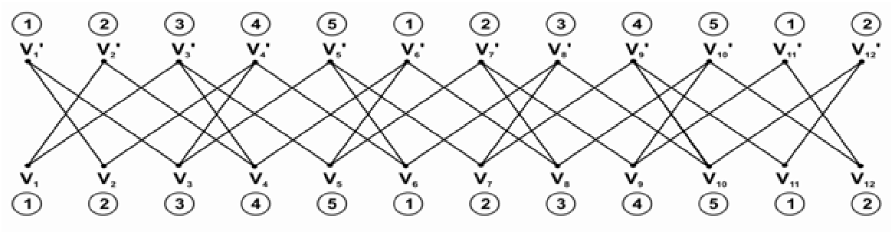
Coloring Algorithm

Input: $DG(L_m)$, $m \ge 3$
$V \leftarrow \{v_1, v_2, \ldots, v_m, v'_1, v'_2, \ldots, v'_m\}$
for k = 1 to n
{
    $v_k, v'_k \leftarrow (k-1)(\text{mod } 5)+1$;
}
end for
Output: vertex colored $DG(L_m)$.

Theorem: The star chromatic number of M(Y'{n+1}) is given by χ_s**[DG(L'$_m$)] = 5, m ≥ 4**.

Proof: Color the vertices of DG(L_m) as given in the algorithm.

Consider the duplicate graph of the ladder graph DG(L_m) with V(DG(Lm)) = <v, v_1, v_2, …, v_m, v', v'_1, v'_2, ..., v'_m>. The color class of 1 is {v_{5k-4}, v'_{5k-4}; $1 \le k \le \left\lfloor \frac{m}{2} \right\rfloor$}. The color class of 2 is {v_{5k-3}, v'_{5k-3}; $1 \le k \le \left\lfloor \frac{m+4}{3} \right\rfloor$}. The color class of 3 is {v_{5k-2}, v'_{5k-2}; $1 \le k \le \left\lfloor \frac{m+2}{3} \right\rfloor$}. The color class of 4 and 5 are {v_{5k-1}, v'_{5k-1}; $1 \le k \le \left\lfloor \frac{m-1}{2} \right\rfloor$} and {$v_{5k}$, v'_{5k}; $1 \le k \le \left\lfloor \frac{m+1}{3} \right\rfloor$}.



x_s**[DG(L$_6$)]=5**

Now we need to examine the subgraphs induced by <C_i, C_j> ($1 \le i < j \le 5$) a bichromatic path or not.

Consider the color classes <C_i, C_j > ($1 \le i < j \le 5$), the induced subgraph of these color classes is either $K_{1,1}$ graph and isolated vertices or $K_{1,1}$ graph

Therefore <C_i, C_j > has no bicolored path of length 3.

Thus the induced subgraph of any pair of the color classes is a collection of stars and hence the coloring given in the algorithm is star coloring.

CONCLUSION

In this paper, we obtained the exact value of the acyclic and star chromatic numbers of the Duplicate graph of ladder L_m.

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