

DOUBLE PATH - UNIONS OF E-CORDIAL GRAPHS

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(Received On: 30-01-18; Revised & Accepted On: 12-02-18)

ABSTRACT

Path unions are obtained by attaching a single copy each of given graph to the vertices of a path P_m . It is denoted by $P_m(G)$. A double path unions is obtained by attaching two copies of given graph to each vertex of path P_m . It is denoted by $P_m(2-G)$. We show that $P_m(2-C_3)$, $P_m(2-C_4)$, $P_m(2-kite)$ are families of E-cordial graphs.

Key words: path, path union, labeling, cordial, etc.

Subject Classification: 05C78.

1. INTRODUCTION

In 1997 Yilmaz and Cahit introduced weaker version of edge graceful labeling E-cordial labeling. [4]. Let G be a (p, q) graph. $f: E \rightarrow \{0,1\}$ Define f on V by $f(v) = \sum \{f(vu) \mid (vu) \in E(G)\} \pmod{2}$. The function f is called as E-cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(i)$ is the number of vertices labeled with $i = 0,1$. And $e_f(i)$ is the number of edges labeled with $i = 0,1$. We follow the convention that $v_f(0,1) = (a, b)$ for $v_f(0) = a$ and $v_f(1) = b$ further $e_f(0,1) = (x, y)$ for $e_f(0) = x$ and $e_f(1) = y$. A graph that admits E-cordial labeling is called as E-cordial graph. Yilmaz and Cahit prove that trees T_n are E-cordial iff for n not congruent to $2 \pmod{4}$, K_n are E-cordial iff n not congruent to $2 \pmod{4}$, Fans F_n are E-cordial iff for n not congruent to $1 \pmod{4}$ etc. Yilmaz and Cahit observe that A graph on n vertices can not be E-cordial if n is congruent to $2 \pmod{4}$. One should refer Dynamic survey of graph labeling by Joe Gallian [2] for more results on E-cordial graphs.

In this paper we consider path unions on two copies of same graph. It is called as double path union. It is obtained by attaching two copies of a graph at each vertex of path P_m . Note that this graph has $|E(G)| = m-1 + m \cdot (|E(G)|)$ and $|V(G)| = m$. This path union is denoted by $P_m(2-G)$. We discuss e-cordiality for $G = C_3, C_4$, kite [5]

The graphs we consider are finite, undirected, simple and connected. For terminology and definitions we refer Harary [3] and Dynamic survey of graph labeling by Joe Gallian [2].

2. THEOREMS PROVED

2.1 path union of Double C_3 (i.e. $G = P_m(2-C_3)$) is E-Cordial iff m is odd number.

Proof: We define G as $V(G) = \{v_1, v_2, \dots, v_m\} \cup \{u_{i,1}, u_{i,2}, u_{i,3}, u_{i,4}\}$ and $E(G) = \{e_i = (v_i v_{i+1}) \mid i = 1, 2, \dots, m\} \cup \{c_{i,1} = (v_i u_{i,1}), c_{i,2} = (u_{i,1} u_{i,2}), c_{i,3} = (u_{i,2} v_i), c_{i,4} = (v_i c_{i,3}), c_{i,5} = (u_{i,3} u_{i,4}), c_{i,6} = (u_{i,4} v_i)\}$.

To obtain a desired E-cordial function $f: E(G) \rightarrow \{0,1\}$. We show labeled copies of two units as shown in figure 2.2 and 2.3 below. We use to connect them too obtain a path- union on larger m .

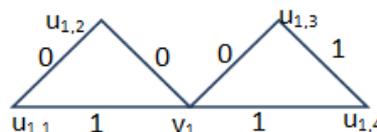


Figure 2.1: $v_f(0,1) = (3,3)$, $e_f(0,1) = (3,3)$ $P_1(2-C_3)$ E-cordial.

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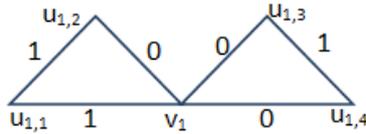


Figure-2.2: Unit A $v_f(0,1) = (1,4)$, $e_f(0,1) = (3,3)$

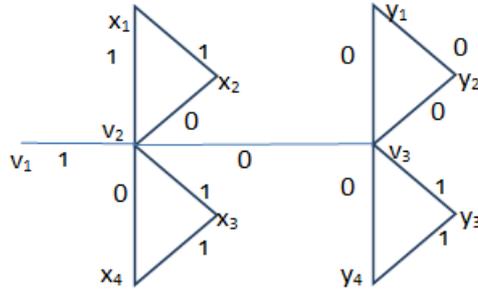


Figure-2.3: Unit B $v_f(0,1) = (5,6)$, $e_f(0,1) = (7,7)$ The label of v_3 is 1. Here v_1 from C will be attached

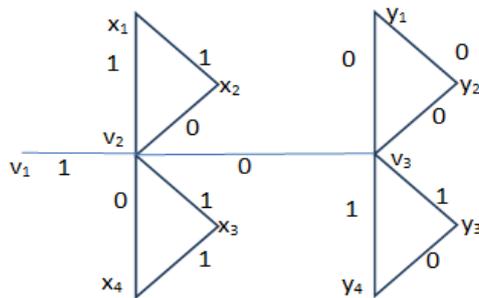


Figure-2.4: Unit c $v_f(0,1) = (5, 6)$, $e_f(0,1) = (7,7)$ The label of v_3 is 0 where v_1 from type B will be attached

Unit B is attached at unit A with vertex v_1 to obtain $P_3(2-C_3)$. To obtain $P_5(2-C_3)$ unit B is attached at point v_3 of unit B

| Table to use for Pathunion of length $m > 1$. For $m = 1$ see fig 5.1 | | | | |
|--|---------------|------------------------|------------------------|---------------|
| m | New type used | Resultant $V_f(0,1) =$ | Resultant $e_f(0,1) =$ | Remarks |
| 1 | A | (1,4) | (3,3) | Starting unit |
| 3 | B | (7,8) | (10,10) | E-cordial |
| 5 | C | (13,12) | (17,17) | E-cordial |
| 7 | B | (17,18) | (24,24) | E-cordial |
| 9 | C | (23,22) | (31,31) | E-cordial |
| 11 | B | (27,28) | (38,38) | E-cordial |

Then sequence of unit C, B, C, B... is followed. We get $P_m(2-C_3)$ for $m = 2x + 1, x = 0, 1, 2, \dots$

To obtain a path union on P_{m+2} we first obtain a path union on P_m . (m is odd number) The process is recursive. The label numbers observed are $e_f(0,1) = (3 + 7x, 3 + 7x)$ for edges for all m and $v_f(0,1) = (10k + 3, 10k + 2)$ for $m > 3$ and $m \equiv 1 \pmod{4}$, $k = \frac{m-1}{4}$.

If $m \equiv 3 \pmod{4}$, $k = \frac{m-3}{4}$ and $v_f(0,1) = (10k + 7, 10k + 8)$ ($m \geq 3$)

2.2 pathunion of Double C_4 (i.e. $G = P_m(2-C_4)$) is E- Cordial iff m is not congruent to 2 (mod 4). We below give different types of structure used to form $P_m(2-C_4)$. They are of type C, A, D and B.

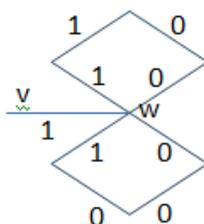


Figure-2.5: Type C $v_f(0,1) = (4,4)$, $e_f(0,1) = (5,4)$; label of w

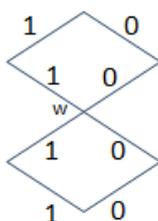


Figure-2.6: Type A $v_f(0,1) = (5,2)$, $e_f(0,1) = (4,4)$

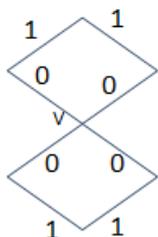


Figure-2.7: $m=1$ $v_f(0,1) = (3,4)$, $e_f(0,1) = (4,4)$

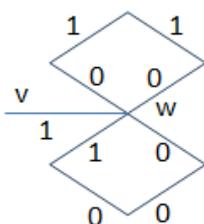


Figure-2.8: Type D $v_f(0,1) = (4,4)$, $e_f(0,1) = (5,4)$; label of $w = 0$

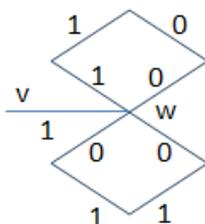


Figure-2.9: Type B $v_f(0,1) = (4,4)$, $e_f(0,1) = (4,5)$; label of $w = 0$

Vertex v of one structure is identified with vertex w of suitable other copy to obtain a path of more length. We below give a scheme how to use different types of structures above to obtain a path-union of bigger length. To obtain a path union on P_{m+1} we first obtain a path union on P_m . The process is recursive.

| m | New type used | Resultant $V_f(0,1)=$ | Resultant $e_f(0,1)$ | Remarks |
|---|---------------|-----------------------|----------------------|---------------|
| 1 | A | (3,4) | (4,4) | E-cordial |
| 2 | C | (8,6) | (9,8) | Not E-cordial |
| 3 | B | (11,10) | (13,13) | E-cordial |
| 4 | B | (14,14) | (17,18) | E-cordial |
| 5 | D | (17,18) | (22,22) | E-cordial |
| 6 | C | (22,20) | (27,26) | Not E-cordial |
| Further sequence of B, B, D, C is repeated... | | | | |

$V_f(0,1) = (14x,14x)$ and $e_f(0,1) = (13+18x,13+18x)$, $e_f(0,1) = (17+18x,18+18x)$ for $m = 4x$.

$V_f(0,1) = (3+14x,4+14x)$, $e_f(0,1) = (4+18x,4+18x)$ for $m = 4x + 1$ and for $m > 1$

$V_f(0,1) = (8+14x,6+14x)$ and $e_f(0,1) = (9+18x,9+18x)$ for $m = 4x + 2$

$V_f(0,1) = (11+14x,10+14x)$ and $e_f(0,1) = (13+18x,13+18x)$ for $m = 4x + 3$

Thus G is E - cordial graph.

2.3 Path union of double kite given by $P_m(2\text{-kite})$ is E.-cordial.

Proof: We define The graph as follows : $V(G) = \{v_1, v_2, \dots, v_m\} \cup \{u_{i,1}, u_{i,2}, u_{i,3} / i = 1, 2, \dots, m-1\} \cup \{u_{i,4}, u_{i,5}, u_{i,6}\}$, $E(G) = \{e_i = (v_i v_{i+1}) / i = 1, 2, \dots, m-1\} \cup \{c_{i,1} = (v_i u_{i,1}), c_{i,2} = (u_{i,1} u_{i,2}), c_{i,3} = (u_{i,2} u_{i,3}), c_{i,4} = (u_{i,3} v_i), c_{i,5} = (u_{i,1} u_{i,3}) / i = 1, 2, \dots, m-1\} \cup \{c_{i,6} = (v_i u_{i,4}), c_{i,7} = (u_{i,4} u_{i,5}), c_{i,8} = (u_{i,5} u_{i,6}), c_{i,9} = (u_{i,6} v_i), c_{i,10} = (u_{i,4} u_{i,6}) / i = 1, 2, \dots, m-1\}$ To define E-cordial labeling we construct different units related to $P_1(2\text{-kite})$. We give a scheme to connect these units to obtain path of given length m .

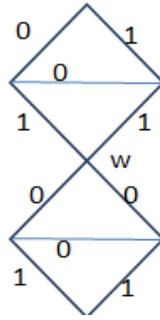


Figure-2.10: $P_1(2\text{-kite})$. Labeled copy. $v_f(0,1)=(4,3), e_f(0,1)=(5,5)$

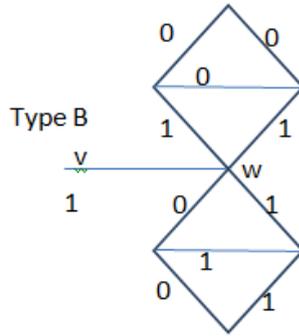


Figure-2.11: Labeled copy. $e_f(0,1)=(4,4), ef(0,1) = (5,6)$

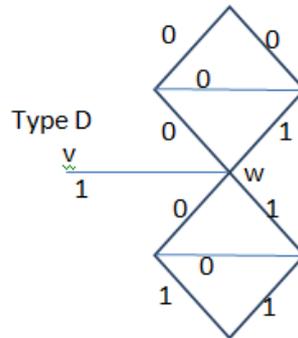


Figure-2.12: Labeled copy. $e_f(0,1) = (4,4), ef(0,1) = (6,5)$

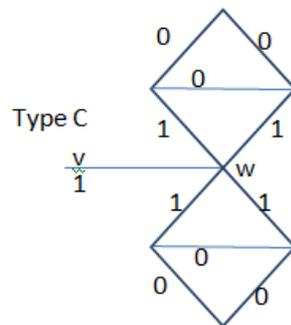


Figure-2.13: Labeled copy. $e_f(0,1) = (4,4), ef(0,1) = (6,5)$

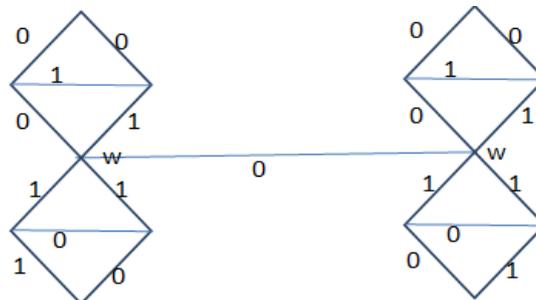


Figure-2.14: $P_2(2\text{-kite})$. Labeled copy. $e_f(0,1) = (6,8), ef(0,1) = (11,10)$. NOT E- cordial

| m | Type Of label for m th copy | $v_f(0,1)$ | $e_f(0,1)$ | Remarks |
|---|--|------------|------------|---------------|
| 1 | diagram | (3,4) | (5,5) | E-cordial |
| 2 | diagram | (6,8) | (11,10) | Not E-Cordial |
| 3 | B | (11,10) | (16,16) | E-cordial |
| 4 | B | (14,14) | (21,22) | E-cordial |
| 5 | D | (17,18) | (27,27) | E-cordial |
| 6 | C | (20,22) | (33,32) | Not E-Cordia |
| 7 | B | (25,24) | (38,38) | E-cordial |
| After this a sequence of B, D, C, B is followed repeatedly as required to obtain a path-union of required length. | | | | |

The vertex v on type B, D, C is used to identify with vertex w to obtain a path union of larger length. We below give a scheme how to use different types of structures above to obtain a path-union of bigger length. To obtain a path union on P_{m+1} we first obtain a path union on P_m . The process is recursive.

$$\begin{aligned}
 m \equiv 0 \pmod{4} \quad v_f(0,1) &= (14x, 14x), \quad e_f(0,1) = (21+22x, 22+22x) \quad \text{where } m = 4x \\
 m \equiv 1 \pmod{4} \quad v_f(0,1) &= (14x+3, 14x+4), \quad e_f(0,1) = (5+22x, 5+22x) \quad \text{where } m = 4x + 1 \\
 m \equiv 2 \pmod{4} \quad v_f(0,1) &= (14x+6, 14x+8), \quad e_f(0,1) = (11+22x, 10+22x) \quad \text{where } m = 4x + 2 \quad \dots \\
 m \equiv 3 \pmod{4} \quad v_f(0,1) &= (14x_{11}, 14x+10), \quad e_f(0,1) = (16+22x, 16+22x) \quad \text{where } m = 4x + 3
 \end{aligned}$$

CONCLUSIONS

We have established e-cordiality of double path union. By double path union we mean two copies of graph G are attached at every vertex of path P_m . It is necessary to investigate the concept for more graphs before coming across any conclusion.

REFERENCES

1. Bapat Mukund V. Ph.D.Thesis, University of Mumbai 2004.
2. Joe Gallian Dynamic survey of graph labeling 2016
3. Harary, Graph Theory, Narosa publishing, New Delhi
4. Yilmaz, Cahit, E-cordial graphs, Ars combina, 46, 251-256.
5. Introduction to Graph Theory by D. WEST, Pearson Education Asia.

Source of support: Nil, Conflict of interest: None Declared.

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