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DOUBLE PATH - UNIONS OF E-CORDIAL GRAPHS

MUKUND V BAPAT
Hindale, Devgad, Sindhudurg, Maharashtra–India, 416630.

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ABSTRACT

Path unions are obtained by attaching a single copy each of given graph to the vertices of a path Pm. It is denoted by Pm(G). A double path unions is obtained by attaching two copies of given graph to each vertex of path Pm. It is denoted by Pm(2-G). We show that Pm(2-C_3) , Pm(2-C_4), P_m(2-kite) are families of E- cordial graphs.

Key words: *path, path union, labeling, cordial, etc.*

Subject Classification: *05C78.*

1. INTRODUCTION

In 1997 Yilmaz and Cahit introduced weaker version of edge graceful labeling E-cordial labeling. [4]. Let G be a (p, q) graph. f:E→{0,1} Define f on V by f(v) =$\sum\{f(vu)(vu)\epsilon E(G)\}(mod\ 2)$.The function f is called as E-cordial labeling if |vf(0)-vf(1)|≤1 and |ef(0)-ef(1)|≤1 where v_f(i) is the number of vertices labeled with i =0,1. And e_f(i) is the number of edges labeled with i= 0,1,We follow the convention that v_f(0,1) =(a, b) for v_f(0)=a and vf(1)=b further e_f(0,1)=(x,y)for e_f(0)=x and e_f(1)=y. A graph that admits E-cordial labeling is called as E-cordial graph. Yilmaz and Cahit prove that trees Tn are E-cordial iff for n not congruent to 2(mod 4), Kn are E-cordial iff n not congruent to 2(mod 4), Fans Fn are E-cordial iff for n not congruent to 1(mod 4) etc. Yilmaz and Cahit observe that A graph on n vertices can not be E-cordial if n is congruent to 2 (mod 4). One should refer Dynamic survey of graph labeling by Joe Gallian [2] for more results on E-cordial graphs.

In this paper we consider path unions on two copies of same graph .It is called as double path union .It is obtained by attaching two copies of agraph at each vertex of path Pm .Note that this graph has |E(G)| = m-1 + m.(|E(G)|) and |V(G)| = m. This pathunion is denoted by Pm(2-G). We discuss e- cordiality for G =C3,C4, kite [5]

The graphs we consider are finite, undirected, simple and connected. For terminology and definitions we refer Harary [3] and Dynamic survey of graph labeling by Joe Gallian [2].

2. THEOREMS PROVED

2.1 pathunion of Double C_3 (i.e. G = P_m(2-C_3) is E- Cordial iff m is odd number.

Proof: We define G as V(G) = {v_1,v_2,…,v_m} U {$u_{i,1}$,$u_{i,2}$,$u_{i,3}$,$u_{i,4}$} and E(G) = {e_i=(v_iv_{i+1}) / I = 1,2,,,..m} U {$c_{i,1}$=($v_iu_{i,1}$), $c_{i,2}$ = ($u_{i,1}u_{i,2}$), $c_{i,3}$ = ($u_{i,2}v_i$), $c_{i,4}$ = ($v_ic_{i,3}$), $c_{i,5}$ = ($u_{i,3}u_{i,4}$), $c_{i,6}$ = ($u_{i,4}v_i$)}.

To obtain a desired E-cordial function f: E(G)→{0,1}. We show labeled copies of two units as shown in figure 2.2 and 2.3 below. We use to connect them too obtain a path- union on larger m.

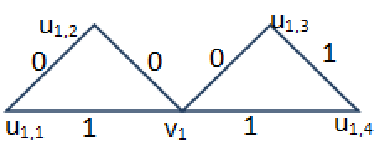

Figure 2.1: v_f(0,1) = (3,3), e_f(0,1) = (3,3) P_1(2-C_3) E- cordial.

Corresponding Author: Mukund V Bapat
Hindale, Devgad, Sindhudurg, Maharashtra–India, 416630.

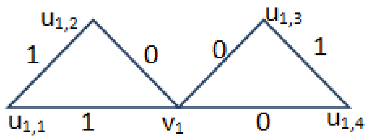

Figure-2.2: Unit A $v_f(0,1)$ =(1,4), $e_f(0,1)$=(3,3)

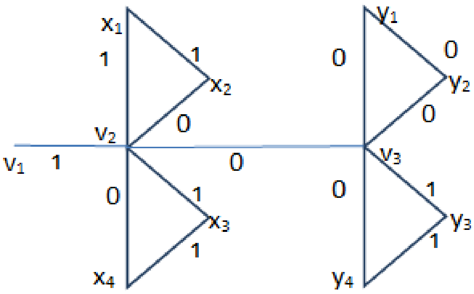

Figure-2.3: Unit B $v_f(0,1)$ = (5,6), $e_f(0,1)$ = (7,7) The label of v_3 is 1. Here v_1 from C will be attached

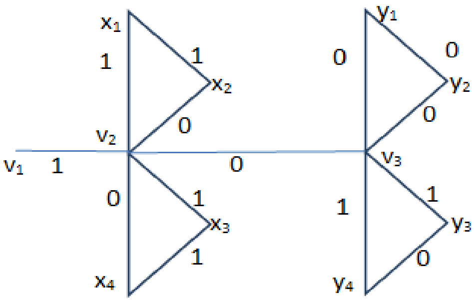

Figure-2.4: Unit c $v_f(0,1)$ = (5, 6), $e_f(0,1)$ = (7,7) The label of v_3 is 0 where v_1 from type B will be attached

Unit B is attached at unit A with vertex v_1 to obtain $P_3(2\text{-}C_3)$. To obtain $P_5(2\text{-}C_3)$ unit B is attached at point v_3 of unit B .

Table to use for Pathunion of length m>1 .For m= 1 see fig 5.1				
m	New type used	Resultant $V_f(0,1)$ =	Resultant $e_f(0,1)$ =	Remarks
1	A	(1,4)	(3,3)	Starting unit
3	B	(7,8)	(10,10)	E-cordial
5	C	(13,12)	(17,17)	E-cordial
7	B	(17,18)	(24,24)	E-cordial
9	C	(23,22)	(31,31)	E-cordial
11	B	(27,28)	(38,38)	E-cordial

Then sequence of unit C, B, C, B… is followed. We get $Pm(2\text{-}C_3)$ for m=2x+1,x=0,1,2,,,…

To obtain a path union on P_{m+2} we first obtain a path union on P_m. (m is odd number) The process is recursive. The label numbers observed are $e_f(0,1)$=(3 +7x,3+7x) for edges for all m and $v_f(0,1)$ = (10k+3,10k+2) for m>3 and m≡ 1(mod4), k=$\frac{m-1}{4}$.

If m≡3 (mod 4), k =$\frac{m-3}{4}$ and vf(0,1) =(10k+7,10k+8) (m≥3)

2.2 pathunion of Double C_4 (i.e. G = $Pm(2\text{-}C_4)$ is E- Cordial iff m is not congruent to 2 (mod 4). We below give different types of structure used to form Pm(2-C4).They are of type C, A, D and B.

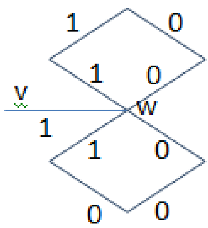

Figure-2.5: Type C $v_f(0,1)$ = (4,4), $e_f(0,1)$ = (5,4); label of w

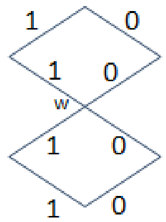

Figure-2.6: Type A $v_f(0,1) = (5,2)$, $e_f(0,1) = (4,4)$

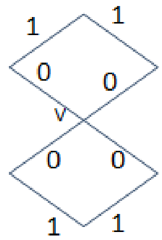

Figure-2.7: m =1 $v_f(0,1) = (3,4)$, $e_f(0,1) = (4,4)$

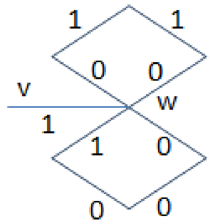

Figure-2.8: Type D $v_f(0,1) = (4,4)$, $e_f(0,1) = (5,4)$; label of w = 0

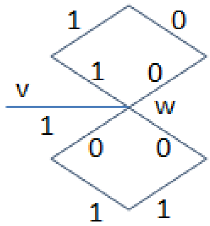

Figure-2.9: Type B $v_f(0,1) = (4,4)$, $e_f(0,1) = (4,5)$; label of w = 0

Vertex v of one structure is identified with vertex w of suitable other copy to obtain a path of more length. We below give a scheme how to use different types of structures above to obtain a path-union of bigger length. To obtain a path union on P_{m+1} we first obtain a path union on P_m. The process is recursive.

m	New type used	Resultant $V_f(0,1)$=	Resultant $e_f(0,1)$	Remarks
1	A	(3,4)	(4,4)	E-cordial
2	C	(8,6)	(9,8)	Not E-cordial
3	B	(11,10)	(13,13)	E-cordial
4	B	(14,14)	(17,18)	E-cordial
5	D	(17,18)	(22,22)	E-cordial
6	C	(22,20)	(27,26)	Not E-cordial
Further sequence of B, B, D, C is repeated…				

$V_f(0,1) = (14x,14x)$ and $e_f(0,1) = (13+18x,13+18x)$, $e_f(0,1) = (17+18x,18+18x)$ for m = 4x.
$V_f(0,1) = (3+14x,4+14x)$, $e_f(0,1) = (4+18x,4+18x)$ for m = 4x +1 and for m>1
$V_f(0,1) = (8+14x,6+14x)$ and $e_f(0,1) = (9+18x,9+18x)$ for m = 4x+2
$V_f(0,1) = (11+14x,10+14x)$ and $e_f(0,1) = (13+18x,13+18x)$ for m = 4x+3
Thus G is E – cordial graph.

2.3 Path union of double kite given by P_m(2-kite) is E.-cordial.

Proof: We define The graph as follows :$V(G) = \{v_1,v_2,..,v_m\}U\{u_{i,1},u_{i,2},u_{i,3}/ \ I = 1,2,..,m-1\}U\{u_{i,4},u_{i,5},u_{i,6}\}$, $E(G) = \{e_i=(v_iv_{i+1})/i = 1,2, ,,,,m-1\}U\{c_{i,1}=(v_iu_{i,1}),c_{i,2}=(u_{i,1}u_{i,2}),c_{i,3}=(u_{i,2}u_{i,3}),c_{i,4}=(u_{i,3}v_i),c_{i,5}=(u_{i,1}u_{i,3})/i=1,2,..,m-1\}U\{c_{i,6}=(v_iu_{i,4}),c_{i,7} = (u_{i,4}u_{i,5}),c_{i,8}=(u_{i,5}u_{i,6}),c_{i,9}=(u_{i,6}v_i),c_{i,10}=(u_{i,4}u_{i,6})/ \ i = 1,2,..,m-1\}$To define E-cordial labeling we construct different units related to P_1(2-kite).We give a scheme to connect these units to obtain path of given length m.

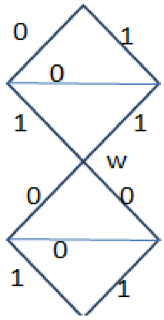

Figure-2.10: P_1(2-kite). Labeled copy. $v_f(0,1)=(4,3)$,$e_f(0,1)=(5,5)$

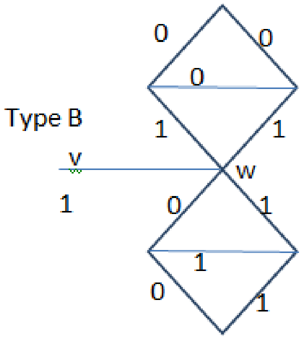

Figure-2.11: Labeled copy. $e_f(0,1)=(4,4)$, ef(0,1) = (5,6)

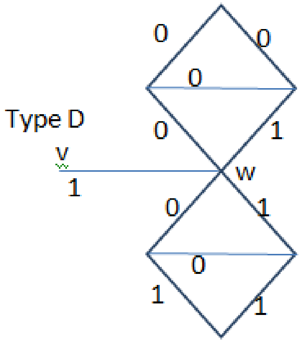

Figure-2.12: Labeled copy.$e_f(0,1) = (4,4)$, ef(0,1) = (6,5)

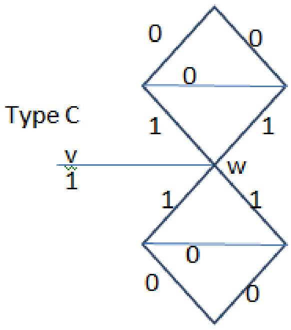

Figure-2.13: Labeled copy. $e_f(0,1) = (4,4)$, ef(0,1) = (6,5)

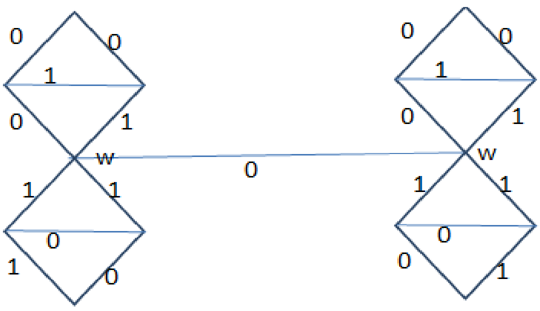

Figure-2.14: P_2(2-kite). Labeled copy. $e_f(0,1) = (6,8)$, ef(0,1) = (11,10).NOT E- cordial

m	Type Of label for m^{th} copy	$v_f(0,1)$	$e_f(0,1)$	Remarks
1	diagram	(3,4)	(5,5)	E-cordial
2	diagram	(6,8)	(11,10)	Not E-Cordial
3	B	(11,10)	(16,16)	E-cordial
4	B	(14,14)	(21,22)	E-cordial
5	D	(17,18)	(27,27)	E-cordial
6	C	(20,22)	(33,32)	Not E-Cordia
7	B	(25,24)	(38,38)	E-cordial
After this a sequence of B, D, C, B is followed repeatedly as required to obtain a path-union of required length.				

The vertex v on type B, D, C is used to identify with vertex w to obtain a path union of larger length. We below give a scheme how to use different types of structures above to obtain a path-union of bigger length. To obtain a path union on P_{m+1} we first obtain a path union on P_m. The process is recursive.
$m \equiv 0 \pmod 4$ $v_f(0,1) = (14x,14x)$, $e_f(0,1) = (21+22x, 22+22x)$ where $m = 4x$
$m \equiv 1 \pmod 4$ $v_f(0,1) = (14x+3,14x+4)$, $e_f(0,1) = (5+22x, 5+22x)$ where $m = 4x +1$
$m \equiv 2 \pmod 4$ $v_f(0,1) = (14x+6,14x+8)$, $e_f(0,1) = (11+22x,10+22x)$ where $m = 4x +2$ …
$m \equiv 3 \pmod 4$ $v_f(0,1) = (14x_{11},14x+10)$, $e_f(0,1) = (16+22x,16+22x)$ where $m = 4x +3$

CONCLUSIONS

We have established e-cordiality of double path union. By double path union we mean two copies of graph G are attached at every vertex of path Pm. It is necessary to investigate the concept for more graphs before coming across any conclusion.

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