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Gd-DISTANCE IN GRAPHS

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ABSTRACT

The distance between two vertices u and v in a graph is the number of edges in a shortest path connecting them. This is also known as the geodesic distance. In this paper we introduce the concept of Gd-distance by considering the degrees of various vertices presented in the path, in addition to the distance of the path we study some properties with this new distance. We define the eccentricities of vertices, radius and diameter of G with respect to the Gd-distance. First we prove that the new distance is a metric on the set of vertices of G. We compare the geodesic distance and Gd-distance of two vertices u, v of V.

Keywords: *Geodesic, Gd-distance, Gd-Eccentricity, Gd-Radius and Gd-Diameter.*

1. INTRODUCTION

By a graph G, we mean a non-trivial finite undirected connected graph without multiple edges and loops.

Following standard notation V (G) or V is the vertex set of G and E (G) or E is the edge set of G = G(V, E). Let u, v be two vertices of G, the standard or usual distance d (u, v) between u and v is the length of the shortest or geodesic u-v path in G.

Anto Kinsley *et.al* [1] introduced the concept of Dd-distance in graph as follows for two vertices u, v in a graph G, Dd-length of a u-v path is defined as

$$D^{Dd}(u, v) = D(u, v) + \deg(u) + \deg(v)$$

In this article, we introduce a new distance, which we call as Gd-distance between any two vertices of a graph G and study some of its properties. This distance is significantly different from other distances.

Anto Kinsley et.al introduced the concept of Dd-distance by considering the length of the longest path between u and v in addition to degree of end vertices.

2. Gd-DISTANCE

Definition2.1: *If u, v are vertices of a connected graph G, Gd-length of a u-v path is defined as*

$$d^{Gd}(u, v) = d(u, v) + deg(u) + deg(v)$$

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Example 2.2:

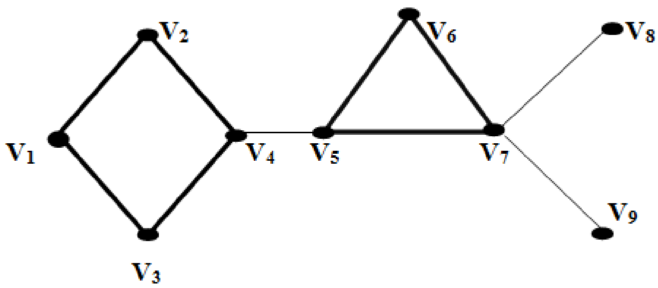

Figure-2.1 G

In fig 1.1 if u = v_1, v = v_8 or v_9

Then, $d^{Gd}(u, v) = d(u, v) + deg(u) + deg(v)$
$= 5+2+1 = 8$

Theorem 2.3: *If G is any connected graph, then the* d^{Gd} *distance is a metric on the set of vertices of G*

Proof: Let G be a connected graph and u, v ∈ V(G)

Then, it is clear by definition that $d^{Gd}(u, v) \geq 0$ and $d^{Gd}(u, v) = 0$

This implies that u = v.

Also we have, $d^{Gd}(u, v) = d^{Gd}(v, u)$

Thus it remains to prove that d^{Gd} satisfies the triangle inequality

Case-(i): Let u, v, w ∈ V(G)

Let P and Q be u-w and w-v shortest or geodesic paths in G respectively such that $d^{Gd}(u, w) = l_{Gd}(P)$ and $d^{Gd}(w, v) = l_{Gd}(Q)$

Let R = PUQ be the u-v shortest or geodesic path obtained by joining P and Q at w of length greater than one. Then

If w ∈ shortest or geodesic path

$$\begin{aligned} d^{Gd}(u, w) + d^{Gd}(w, v) &= [\, l_{Gd}(P) + deg(u) + deg(w)] + [l_{Gd}(Q) + deg(w) + deg(v)] \\ &= l_{Gd}(PUQ) + deg(u) + deg(v) + 2deg(w) \\ &> d^{Gd}(u, v) \end{aligned}$$

If w ∉ shortest path

$$\begin{aligned} d^{Gd}(u, w) + d^{Gd}(w, v) &= [\, l_{Gd}(P) + deg(u) + deg(w)] + [l_{Gd}(Q) + deg(w) + deg(v)] \\ &= l_{Gd}(PUQ) + deg(u) + deg(v) + 2deg(w) \\ &> d^{Gd}(u, v) \end{aligned}$$

Therefore, $d^{Gd}(u, w) + d^{Gd}(w, v) > d^{Gd}(u, v)$

Case-(ii): Suppose R = PUQ be the u-v shortest or geodesic path of length one. Then we take a vertex other than u and v such that either u = w or w = v

Subcase-(i):
Suppose u = w that implies $d^{Gd}(u, w) = 0$

$$\begin{aligned} \text{Therefore, } d^{Gd}(u, w) + d^{Gd}(w, v) &= 0 + [\, l_{Gd}(Q) + deg(w) + deg(v)] \\ &= l_{Gd}(PUQ) + deg(u) + deg(v) \\ &= d^{Gd}(u, v) \end{aligned}$$

Subcase-(ii):

Suppose v = w

Similar to subcase (i)

Thus the triangular inequalities hold.

Hence d^{Gd} is a metric on the vertex set

Corollary 2.4: *For any three vertices u, v, w of a graph G,* $d^{Gd}(u, v) \leq d^{Gd}(u, w) + d^{Gd}(w, v) - deg(w)$

Proof: Let G be a connected graph.

$$d^{Gd}(u, w) + d^{Gd}(w, v) = l_{Gd}(PUQ) + deg(u) + deg(v) + 2deg(w)$$
$$\geq d^{Gd}(u,v) + 2deg(w)$$

$$d^{Gd}(u, w) + d^{Gd}(w, v) \geq d^{Gd}(u, v) + deg(w)$$

Therefore, $d^{Gd}(u, v) \leq d^{Gd}(u, w) + d^{Gd}(w, v) - deg(w)$.

Proposition 2.5: *In a tree T, two distinct vertices u, v are adjacent if and only if* $d^{Gd}(u, v) = deg(u) + deg(v) + 1$

Proof: Let u, v are adjacent

This implies that $d(u, v) = 1$

By definition, $d^{Gd}(u, v) = d(u, v) + deg(u) + deg(v)$
$= deg(u) + deg(v) + 1$

Conversely, if $d^{Gd}(u, v) = deg(u) + deg(v) + 1$

This implies that, $d(u, v) = 1$
u, v are adjacent.

Proposition 2.6: *If G is a connected graph with two distinct vertices u, v are adjacent then* $d^{Gd}(u, v) \leq 3(n-1)$

Proof: Let u, v are adjacent.

This implies that $d(u, v) \leq n-1$

By definition, $d^{Gd}(u, v) = d(u, v) + deg(u) + deg(v)$
$\leq n-1 + deg(u) + deg(v)$

Since $d(u, v) \leq n-1$, $deg(u) \leq n-1$, $deg(v) \leq n-1$
$\leq 3(n-1)$ for any vertices u & v

Therefore, $d^{Gd}(u, v) \leq 3(n-1)$

3. Gd-ECCENTRICITY, Gd-RADIUS AND Gd-DIAMETER

Definition 3.1: *The Gd-eccentricity of any vertex V $e^{Gd}(v)$, is defined as the maximum distance from V to any other vertex.*
(ie) $e^{Gd}(v) = max\{ d^{Gd}(u,v): u \in V(G)\}$

Definition 3.2: *Any vertex u for which* $d^{Gd}(u, v) = d^{Gd}(v)$ *is called Gd-eccentric vertex of V. Further, a vertex u is said to be Gd- eccentric vertex of G if it is the Gd-eccentric vertex of some vertex.*

Definition 3.3: *The Gd-radius denoted by $r^{GD}(G)$ is the minimum Gd-eccentricity among all vertices of G.*
(ie) $r^{Gd}(G) = min\{e^{Gd}(v): v \in V(G)\}$

Definition 3.4: *The Gd-diameter denoted by $d^{Gd}(G)$ is the maximum Gd-eccentricity among all vertices of G.*
(ie) $d^{Gd}(G) = max\{e^{Gd}(v): v \in V(G)\}$

Definition 3.5: *The Gd-center of G, $c^{Gd}(G)$ is the subgraph induced by the set of all vertices of minimum Gd-eccentricity*

Definition 3.6: *A graph is called Gd-self centered if $c^{Gd}(G) = G$ (or) equivalently $r^{Gd}(G) = d^{Gd}(G)$*

Similarly, the set of all vertices of maximum Gd-eccentricity is the periphery of G.

Remark 3.7: *For any non-trivial connected Graph G,* $r^{Gd}(G) \leq d^{Gd}(G) \leq 2r^{Gd}(G)$

Proof: By definition of radius and diameter, $r^{Gd}(G) \leq d^{Gd}(G)$

We have to claim other inequality

Let u and v be two vertices with $d^{Gd}(u,v) = d^{Gd}(G)$

Let $w \in c^{Gd}(G)$

Hence $e^{Gd}(w) = r^{Gd}(G)$ and so $d^{Gd}(w,x) \leq r^{Gd}(G)$ for any other vertex of x of G

$$\begin{aligned} \text{Now } d^{Gd}(G) = d^{Gd}(u,v) &\leq d^{Gd}(u,w) + d^{Gd}(w,v) \\ &\leq r^{Gd}(G) + r^{Gd}(G) \\ &= 2r^{Gd}(G) \end{aligned}$$

Hence $\quad d^{Gd}(G) \leq 2r^{Gd}(G)$

It completes the proof.

Theorem 3.8: *If u, v are two adjacent vertices of a connected graph G, with* $e^{Gd}(u) \geq e^{Gd}(v)$ *then* $e^{Gd}(u) - e^{Gd}(v) \leq n$.

Proof: Let w be an eccentric vertex of u such that $d^{Gd}(u, w) = e^{Gd}(u)$

$$\begin{aligned} \text{Then } e^{Gd}(u) &\leq d^{Gd}(u, v) + d^{Gd}(v, w) - \deg(v) \\ &\leq d^{Gd}(u, v) + e^{Gd}(v) - \deg(v) \end{aligned}$$

Since $\quad e^{Gd}(v) \geq d^{Gd}(v, w)$

$\quad d^{Gd}(u, v) \leq \deg(u) + \deg(v) + 1$

since u and v are adjacent

$$\begin{aligned} e^{Gd}(u) - e^{Gd}(v) &\leq d^{Gd}(u, v) - \deg(v) \\ &= \deg(u) + 1 \\ &\leq n-1+1, \text{ Since } \deg(u) \leq n-1 \\ &\leq n \end{aligned}$$

Theorem 3.9: *For every tree* $u, v \in T$, $d^{Gd}(u, v) > d(u, v)$

Proof: We know that, $d^{Gd}(u, v) = d(u, v) + \deg(u) + \deg(v)$

$d^{Gd}(u, v) > d(u, v)$

Theorem 3.10:

(i) *For a cycle* C_n, $d^{Gd}(u, v) = 5$ *for every adjacent vertices* $\forall u, v \in C_n$

(ii) *For a path* P_n, $n \geq 3$ *with any two adjacent vertices u and v*

Then $d^{Gd}(u, v) = \begin{cases} 5 & \text{if } u \text{ and } v \text{ are internal vertices} \\ 4 & \text{Otherwise} \end{cases}$

(iii) For a complete graph K_n, $d^{Gd}(u, v) = 2n-1 \quad \forall u, v \in K_n$

Proof:

(i) For a cycle C_n,

If u and v are adjacent,

Then $d(u, v) = 1$, $\deg(u) = 2$, $\deg(v) = 2$

By definition, $d^{Gd}(u, v) = d(u, v) + \deg(u) + \deg(v) = 5$

The proof is complete.

(ii) For a path P_n, $n \geq 3$

(a) If u and v are adjacent and internal vertices

Then d(u, v) = 1, deg(u) = 2, deg(v) = 2

By definition, d^{Gd} (u, v) = d(u, v) + deg(u) + deg(v) = 5

(b) If u and v are adjacent and either u or v is a external vertex

Then d(u, v) = 1, either deg(u) = 1 and deg(v) = 2 (or) deg(u) =2 and deg(v) =1

By definition, d^{Gd} (u, v) = 4

(iii) For a complete graph, K_n

Always, d(u, v) = 1, deg(u) = n-1 and deg(v) = n-1

Then by definition,
d^{Gd} (u, v) = d(u, v) + deg(u) + deg(v)
= 1+ (n-1) + (n-1)
= 2n-1.
The proof is complete.

Theorem 3.11: *For any connected graph G, for all u, v belongs to G, d^{Gd} (u, v) = 3, u, v $\in$ G, then G becomes a path of length 1 with two vertices.*

Proof: We have d^{Gd} (u, v) = d(u, v)+deg(u)+deg(v)

Since d^{Gd} (u, v) = 3
d(u, v) = 3 - deg(u) - deg(v)

Therefore, deg(u) $\leq$ 1 and deg(v) $\leq$ 1

Now, deg(u)+deg(v) = 2

Therefore, d(u, v) =1

This implies u and v are the two vertices of length 1.

Theorem 3.12: *For any connected graph G, if u and v are end vertices of G and d(u, v) = n-1 then d^{Gd}* (u, v) = n+1

Proof: For any connected graph,

Suppose u and v are end vertices
deg(u) = 1, deg(v) = 1 and given d(u, v) = n-1

Then by definition d^{Gd} (u, v) = d(u, v) + deg(u) + deg(v)
= n+1

The proof is complete.

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