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MHD HEAT TRANSFER SLIP-FLOW BETWEEN TWO PARALLEL POROUS WALLS IN A ROTATING SYSTEM WITH HALL CURRENTS

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ABSTRACT

The paper deals with Hall effect on MHD heat transfer slip-flow of an ionized gas between two parallel porous walls in a rotating system, treating the transport properties of the fluid as constant when the bounding side walls are maintained at constant and equal temperatures. The governing equation of heat transfer is solved by using the slip conditions in two cases, that is when the side walls are made up (i) insulating (non-conducting) and (ii) conducting porous materials. The solutions to temperature distribution, mean temperature and rate of heat transfer coefficients at the side walls are obtained analytically. Also, the temperature profiles are plotted in support of different sets of values of the governing parameters involved. The heat transfer characteristic is discussed by analyzing the parameters namely Hartmann number and Taylor number (rotation); Hall, porous and slip parameters. The solutions to the temperature distribution are found to be the independent of the ratio of electron pressure to the total pressure in case of nonconducting porous side walls, it is noticed that an increase in Hall parameter and Taylor number are to diminish the temperature distribution. An increase in slip parameter enhances the temperature distribution. Also, it is seen that an increase in porous parameter is to increase the temperature distribution everywhere except at near to the lower wall.

Key words: MHD, Hall currents, Rotating system, Slip flow, Porous boundaries.

LIST OF SYMBOLS

a_1, a_2, a_3, \dots	Functions/real constants involved in the equations and solutions
$b_1, b_2, \dots, d_1, d_2, \dots$	Functions/real constants involved in the equations and solutions
C_1, C_2	Functions/real constants involved in the equations and solutions
C _p	Specific heat at constant pressure
E _x , E _z , <i>H</i> _a	Applied electric fields in x- and z- components respectively. Hartmann number
H_0	Applied uniform transverse magnetic field
h I_x, I_z J_x, J_z k_1, k_2 m $m_1, m_2, m_3,$	Channel width Non-dimensionalized current densities Current densities in x- and z- components respectively. Functions/real constants Hall parameter Functions/real constants involved in the equations and solutions
m _x , m _z Nu	Non-dimensionalized electric fields Nusselt number

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MHD Heat Transfer Slip-Flow Between Two Parallel Porous Walls in a Rotating System with ... / IJMA- 9(2), Feb.-2018.

P _r	Prandtl number
р	Pressure
p_e	Electron pressure
q(y)	
\overline{Q}	Complex conjugate of Q
S T	Ratio of electron pressure to the total pressure
l T	Taylor number (Rotation parameter)
I_F	Temperature
$T_{_{W}}$	Constant temperature
и	x- component of velocity distributions in the fluid, known as Primary velocity distribution
<i>u</i> _p	$= \left(-\frac{dp}{dx}\right) \frac{h^2}{\rho v}$ the characteristic velocity
V ₀	Constant suction velocity
W	z- component of velocity distributions in the fluid, known as Secondary velocity distribution
$(\mathbf{x}, \mathbf{y}, \mathbf{z})$	Space co-ordinates in rectangular Cartesian co-ordinate system
<u>B</u>	Magnetic flux density vector
\underline{E}	Electric field vector
J	Current density vector
\overline{V}	Fluid velocity vector
μ	Constant magnetic permeability
$\frac{dp}{dx}$	Constant pressure gradient
ω _e	Gyration frequency of electron
τ	Mean collision time between electron and ion
$ au_{e}$	Mean collision time between electron and neutral particles
β	Slip parameter
λ	Porous parameter (suction number)
$\sigma_{_0}$	Electrical conductivity
ρ	Density
$\sigma_{_1}, \ \sigma_{_2}$	Modified electrical conductivities of parallel and normal to the direction of electric field
θ	Non-dimensional form of temperature distribution
$ heta_m$	Mean temperature
ω	Frequency of oscillation
Ω	Angular velocity, where $\overline{\Omega} = (0, \Omega, 0)$

1. INTRODUCTION

The study of magnetohydrodynamic flows through channels in a rotating frame reference has drawn the attention on many researchers namely, Agarwal (1961), Vidyanidhi (1969), Nanda and Mahanty (1970), Soundalgekar (1974), Jana *et al.* (1977), Jana and Datta (1980) and Seth *et al.* (1982), Sheng and Leong (2012) and many more in view of its wide applications in cosmically studies such as in the study of stars, planets and in geophysical fluid dynamics. Also it is well known in literature that when a system consisting of electrically conducting fluid masses of low density is subjected to the action of a very strong magnetic field, the Hall currents enter into the system. And these Hall currents tend to modify the mechanical behavior of the fluid flow to a considerable extent. Hence, the resulting effects due to Hall currents in hydromagnetic fluid flow has gained considerable impetus and have been studied during several decades by many investigators under varied conditions and in different geometrical configurations (Nayyar *et al.* (1956), Cowling (1957), Sato (1961), Tani (1962), Sutton (1965), Pop (1998), Debnath (1979), Rao (1981), Bharali (1982), Niranjan (1990)). It is also observed in the literature that the MHD flow behavior in the channel flows with porous boundaries has been influenced significantly by the presence of Coriolis force, hydromagnetic force and Hall currents. Several investigations have been appeared in the literature due to their applications for many engineering's and technological fields, in which the works of Raju and Rao (1993), Takhar *et al.* (2002), Ghosh *et al.* (2009), Hazem (2009), Gupta *et al.* (2011), Das *et al.* (2013), Khaled (2015) and many more are of worth mentioning.

The heat transfer flow problems of the slip-flow regime have also been attracted by several investigators in view of their rich potential applications for engineering and many industrial manufacturing processes (Matthews (2008), Hazem (2006), Zaman (2013)). Since in many of the practical problems, the particle adjacent to a solid surface no longer takes the velocity of the surface but there is a stagnant layer of fluid close to the wall allowing the fluid to slip (sees Navier (1827)). The slip velocity is proportional to the shear stress, the normal velocity remains zero and the fluid still behave the Navier-Stokes equation. For no-slip conditions, the fluid will have zero velocity relative to the boundary. The existence of slip phenomenon at the boundaries and the interface has been observed in the problems related to the flows of rarefied gasses (low density), hypersonic flows of a chemically reacting binary mixture, rough surfaces and many such types (see Street (1963) and Eduard-Paloka (2001)). Partial slips also occur to fluids with particulate such as emulsions, suspensions, polymer solutions etc. A lot of research contributions to the problems of this type have been reported on the literature by many authors, namely Schaaf and Chambre (1961), Lance and Rogers (1962), Street (1963), Sastry and Bhadram (1972), Tamada and Miura (1978), Bhatt and Sacheti (1979), Michel and Stephen (1994), Makinde and Osalusi (2006), Raju (2007), Matthews and James (2008), Mostafa (2012), Ghara et al. (2013), Faiza (2013), Raju and Neela (2016) and others. The effects of Hall currents in an MHD flow of an ionized gas under varied conditions and of different geometrical considerations in slip flow regime represent an area of rapid growth in the contemporary research, but still, there is a few problems which are yet to be investigated in different conditions. So, in this paper an attempt is made to study the temperature distribution due to magnetohydrodynamic (MHD) slip-flow of an ionized gas between two parallel porous walls in a rotating frame of reference, taking into account the effects of Hall currents, Hartmann number, rotation and porous parameters. The governing equations of flow and heat transfer are formulated and simplified. The resulting linear differential equations are solved and obtained the solutions to temperature distribution of two cases of study - when the side walls are made up (i) insulating (non-conducting) and (ii) conducting porous materials. The mean temperature and the rate of heat transfers coefficients at the side walls are also determined. The temperature profiles are plotted and the behavior of heat transport is discussed by analyzing the governing parameters. This paper is arranged as follows. Section 1 gives the brief introduction. In section-2, the basic equations of flow and energy with boundary conditions are given. Section 3 presents the solutions to the problem of two cases of study. Section 4 deals with the results and discussion of temperature distribution based on the profiles, which are displayed in figures 2 to 16; while section 5 presents the conclusion. This theoretical study may bear several practical applications of many engineering's and industrial manufacturing processes, such as in aerodynamic heating and in the problems of engineering applications, for example in rotating MHD generators, Hall accelerators and thermo-nuclear power reactors and polymer solutions etc.

2. FORMULATION OF THE PROBLEM

A steady flow of an ionized gas (electrically conducting gas) between two parallel porous walls infinite in extent along x- and z- directions is considered, when both the fluid and side walls are in a state of rigid rotation with uniform angular velocity $\overline{\Omega}$ about y-axis normal to the side walls. Fig.1 shows the co-ordinate system and flow model. The x-axis is taken in the direction of hydrodynamic pressure gradient in the plane parallel to the channel walls, but not in the direction of flow and y- axis is at right angles to it. The fluid is subjected to a constant suction \mathbf{v}_0 applied normal to the side walls. A parallel uniform magnetic field H₀ is applied in the y – direction by taking the Hall currents in to account. The height of the channel is denoted by 2h (that is, $y = \pm h$) and the width is assumed to be very large in comparison with the channel height 2h. Since, the side walls are infinitely large in extent along x- and z- directions, so all physical quantities except pressure will depend on y only. It is also assumed that, the induced magnetic field is negligible in comparison with the applied field under the assumption that the magnetic Reynolds number is small. The fluid velocity \overline{V} , magnetic field \overline{B} , electric field \overline{E} and the current density \overline{J} may reasonably be assumed as $\overline{V} = (u, v_0, w)$, $\overline{B} = (0, H_0, 0)$, $\overline{E} = (E_x, 0, E_z)$, $\overline{J} = (J_x, 0, J_z)$ and $\overline{\Omega} = (0, \Omega, 0)$ in the equations of momentum and current.

3. BASIC EQUATIONS OF FLOW WITH BOUNDARY CONDITIONS AND MATHEMATICAL ANALYSIS OF THE PROBLEM

The fundamental equations to be solved are the equations of motion and current for the steady flow of neutral fully ionized gas valid under the above assumptions on par with Spitzer [38] and Sato [33] are expressed as

$$-\left\{1-s\left(1-\frac{\sigma_{1}}{\sigma_{0}}\right)\right\}\frac{dp}{dx}+\rho v_{0}\frac{du}{dy}+\rho v\frac{d^{2}u}{dy^{2}}+H_{0}\{-\sigma_{1}(E_{z}+uH_{0})+\sigma_{2}(E_{x}-wH_{0})\}=2\rho\Omega w,$$
(1)

$$-\{1-s(1-\frac{\sigma_1}{\sigma_0})\}\frac{dp}{dx} + \rho v_0 \frac{du}{dy} + \rho v \frac{d^2 u}{dy^2} + H_0\{-\sigma_1(E_z + uH_0) + \sigma_2(E_x - wH_0)\} = 2\rho\Omega w,$$
(2)

in the above equations, Ω represents the angular velocity with which the whole system is rotated about y-axis and s = p_e/p is the ratio of the electron pressure to the total pressure. ρ is the density, V the kinematic viscosity, H_0 is the applied uniform transverse magnetic field, v_0 is constant suction velocity, σ_0 is the conductivity which is defined as a © 2018, IJMA. All Rights Reserved 165

coefficient of proportionality between current density and the collision term in the equation of motion of charged particles. σ_0 , σ_1 are the modified conductivities parallel and normal to the direction of electric field. The value of s is 1/2 for neutral fully-ionized plasma and approximately zero for a weakly-ionized gas. u, w and E_x and E_z are x- and z- components of velocity \overline{V} and electric field \overline{E} respectively. Also,

$$\sigma_1 = \frac{\sigma_0}{1+m^2}, \sigma_2 = \frac{\sigma_0 m}{1+m^2} \quad \text{and} \quad m = \omega_e / \left(\frac{1}{\tau} + \frac{!}{\tau_e}\right), \tag{3}$$

where ω_e is the gyration frequency of electron, τ and τ_e are the mean collision time between electron and ion, electron and neutral particles respectively. The expression for 'm' as given above in eq(3) is valid in the case of partially– ionized gas agrees with that of fully–ionized gas when τ_e approaches infinity.

Then the two equations (1) and (2) have been non-dimensionalised, using the characteristic length h and velocity $u_p = -\left(\frac{dp}{dx}\right)\left(\frac{h^2}{\rho v}\right)$. Using the notations u, w for u/u_p , w/u_p and y for y/h, we obtain the non-dimensional

equations as

$$\frac{d^2 u}{dy^2} + \lambda \frac{du}{dy} - \frac{H_a^2}{1+m^2} (m_z + u) + \frac{mH_a^2}{1+m^2} (m_x - w) + k_1 = 2T^2 w,$$
⁽⁴⁾

$$\frac{d^2w}{dy^2} + \lambda \frac{dw}{dy} + \frac{H_a^2}{1+m^2}(m_x - w) + \frac{mH_a^2}{1+m^2}(m_z + u) + k_2 = -2T^2u,$$
(5)

where, $k_1 = 1 - s \left(1 - \frac{1}{1 + m^2} \right)$, $k_2 = -s \left(\frac{m}{1 + m^2} \right)$, $m_x = E_x/(H_0 u_P)$, $m_Z = E_z/(H_0 u_P)$, Hartmann number H_a is

defined as $H_a^2 = \frac{H_o^2 h^2 \sigma_o}{\rho v}$, T^2 (Taylor number) = $\frac{\Omega h^2}{v}$ and λ (Suction number) = $(hv_0)/v$) (6)

Writing
$$q = u + iw$$
, $k = k_1 + ik_2$, $E = m_x + im_z$; equations (4) and (5) can be written in complex form as:

$$\frac{d^2q}{dy^2} + \lambda \frac{dq}{dy} + \left(\frac{-H_a^2}{1+m^2} + i\frac{mH_a^2}{1+m^2} + 2iT^2\right)q = -k - i\frac{H_a^2}{1+m^2}E - \frac{mH_a^2}{1+m^2}E$$
(7)

and which is to be solved subject to the slip boundary conditions as given by

$$q = \mp \beta \frac{dq}{dy} \quad \text{at} \quad y = \pm 1, \tag{8}$$

where β is the first order velocity slip parameter.

Also, I_x and I_z defined by $J_x/(\sigma_0 H_0 u_p)$ and $J_z/(\sigma_0 H_0 u_p)$ respectively, are given in complex notation as

$$I = I_{x} + iI_{z} = \left(\frac{m+i}{1+m^{2}}\right) \left(q - iH_{a} - \frac{s}{H_{a}^{2}}\right) + \frac{is}{H_{a}^{2}}$$
(9)

The non-dimensional electric field m_x and m_z are to be determined by boundary conditions at large x and z. The solutions are obtained in two cases of study, that is when the side walls are made up of non-conducting (insulating) and conducting porous materials, in turn which are used to determine the temperature distribution using the energy equation as described below.

4. FORMULATION OF BASIC ENERGY EQUATION WITH BOUNDARY CONDITIONS

In many engineering problems, one can be interested in the quantity of heat flow as well as the pattern of temperature to which the heat flow generates under steady or unsteady conditions. For this purpose, we need to combine the physical law for the rate of heat transfer with the energy conservation equation. Using the fully developed steady flow as already obtained from the equation (9), the effect of flow parameters on the fluid's temperature and the heat transferred between the fluid and the porous walls is discussed. It is assumed that the thermal boundary conditions apply everywhere on the infinite channel walls and neglected the thermal conduction in the flow direction. The governing energy equation is simplified as:

$$\frac{1}{P_r} \frac{d^2\theta}{dy^2} + \lambda \frac{d\theta}{dy} + \left\{ \left(\frac{du}{dy} \right)^2 + \left(\frac{dw}{dy} \right)^2 + H_a^2 I^2 \right\} = 0,$$

$$I^2 = I_x^2 + I_z^2.$$
(10)
(11)

In addition to the dimensionless quantities as already defined in (6), we use $\theta = \frac{T_F - T_w}{(u_p^2 / c_p)}$ and $I_x + iI_z = \frac{J_x + iJ_z}{\sigma_0 H_0 u_p}$,

where c_p is the specific heat at constant pressure and P_r is the Prandtl number. Moreover, the wall temperature T_w is constant everywhere and the temperature T_F is finite everywhere in the fluid and is a function of y only.

The boundary conditions are

$$\theta = \mp \beta \frac{d\theta}{dy}$$
 at $y = \pm 1$ and $\frac{d\theta}{dy} = 0$ at $y = 0$. (12)

Solving the eq. (10) with the help of (11) and (12), we determine the temperature distribution, mean temperature and the rate of heat transfer in the fluid flow when both the side walls are made up of non-conducting and conducting porous materials by using the expressions for velocity fields of the eqs. (4 - 5).

5. SOLUTION OF THE PROBLEM

The solutions of the governing energy equation (10) are carried out separately as in the following two cases of study.

Non-Conducting (insulating) porous plates:

When the side walls are kept at large distance in z-direction and are made up of the non-conducting porous material, then the induced electric current does not go out of the channel but circulates in the fluid. So, an additional condition for the current defined in non-dimensional form is obtained by $\int_0^1 I_z dy = 0$. If the insulation at large x is also assumed, another relation is obtained as $\int_0^1 I_x dy = 0$. Constants in the solution are determined by the above two conditions and solutions for velocity distributions are obtained, which in turn are used to determine the temperature distributions from the following simplified equation:

$$\frac{1}{p_r}\frac{d^2\theta}{dy^2} + \lambda \frac{d\theta}{dy} + \frac{dQ}{dy}\frac{d\overline{Q}}{dy} + \frac{H_a^2}{1+m^2}(Q-1)(\overline{Q}-1) = 0$$
⁽¹³⁾

where $Q = q/q_m$, $q_m = u_m + iw_m$, \overline{Q} is the complex conjugate of $Q \cdot q$ is the solution of the velocity distributions in complex form, which is given by

$$q(y) = u(y) + iw(y) = c_1 e^{m_1 y} + c_2 e^{m_2 y} + \frac{a_2}{a_1}$$
, where $u(y) = \frac{q + \overline{q}}{2}$ and $w(y) = \frac{q - q}{2i}$ (14)

The mean velocity distributions in complex notation are

$$q_m = u_m + iw_m = c_1 a_{26} + c_2 a_{27} + \frac{a_2}{a_1}$$
, where $u_m = \frac{q_m + q_m}{2}$ and $w_m = \frac{q_m - q_m}{2i}$. (15)

Solving the equation (13) for temperature distribution θ by using the boundary conditions (12), the expressions for temperature distributions and the rate of heat transfer coefficients at the side walls are obtained as:

$$\theta = d_{36} + d_{37} e^{-\lambda \bar{y}} + b_{38} e^{b_{13}y} + b_{39} e^{b_{14}y} + b_{40} e^{b_{15}y} + b_{41} e^{b_{16}y} + b_{42} e^{b_{13}y} + b_{43} e^{b_{14}y} + b_{44} e^{m_1y} + b_{45} e^{m_1y} + b_{46} e^{b_{15}y} + b_{47} e^{b_{16}y} + b_{48} e^{m_2y} + b_{49} e^{m_2y} + b_{50} e^{b_1y} + b_{51} e^{b_2y} + b_{52} e^{b_1y} + b_{53} e^{b_2y} + b_{54} y$$
(16)

The mean temperature is given by $\theta_m = \int_{-\infty}^{\infty} \theta \, dy$

$$=d_{36}-d_{37}\left(\frac{e^{-\lambda}-1}{\lambda}\right)+b_{38}\left(\frac{e^{b_{13}}-1}{b_{13}}\right)+b_{39}\left(\frac{e^{b_{14}}-1}{b_{14}}\right)+b_{40}\left(\frac{e^{b_{15}}-1}{b_{15}}\right)+b_{41}\left(\frac{e^{b_{16}}-1}{b_{16}}\right)+b_{42}\left(\frac{e^{b_{13}}-1}{b_{13}}\right)$$
$$+b_{43}\left(\frac{e^{b_{14}}-1}{b_{14}}\right)+b_{44}\left(\frac{e^{m_{1}}-1}{m_{1}}\right)+b_{45}\left(\frac{e^{m_{1}}-1}{m_{1}}\right)+b_{46}\left(\frac{e^{b_{15}}-1}{b_{15}}\right)+b_{47}\left(\frac{e^{b_{16}}-1}{b_{16}}\right)+b_{48}\left(\frac{e^{m_{2}}-1}{m_{2}}\right)$$
$$+b_{49}\left(\frac{e^{m_{2}}-1}{m_{2}}\right)+b_{50}\left(\frac{e^{b_{1}}-1}{b_{1}}\right)+b_{51}\left(\frac{e^{b_{2}}-1}{b_{2}}\right)+b_{52}\left(\frac{e^{b_{1}}-1}{b_{1}}\right)+b_{53}\left(\frac{e^{b_{2}}-1}{b_{2}}\right)+b_{53}\left(\frac{e^{b_{2}}-1}{b_{2}}\right)+b_{53}\left(\frac{e^{b_{2}}-1}{b_{2}}\right)+b_{54}\left(\frac{e^{b_{14}}-1}{b_{2}}\right)+b_{55}\left(\frac{e^{b_{14}}-1}{b_{2}}\right)+b_{56}\left(\frac{e^{b_{14}}-1}{b_{2}$$

The rate of heat transfer coefficient (Nusselt number) at the upper side wall is give by

$$N_{u} = \left(-\frac{d\theta}{dy}\right)_{y=1} = \lambda \, d_{37} \, e^{-\lambda} - d_{38} \, . \tag{18}$$

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The rate of heat transfer coefficient at the lower wall is give by

$$N_{u} = \left(\frac{d\theta}{dy}\right)_{y=-1} = -\lambda d_{37} e^{-\lambda} + d_{39}$$
⁽¹⁹⁾

Conducting porous plates

When the side walls are made up of conducting porous materials and short–circuited by an external conductor, the induced electric current flows out of the channel. In this case no electric potential exists between the side walls. If we assume zero electric field in the x– and z– directions, then we have $m_x = 0$, $m_z = 0$. Constants in the solution are determined by these two conditions. Solutions for velocity distributions are obtained, which in turn are used to determine the temperature distributions from the following simplified equation:

$$\frac{1}{p_r}\frac{d^2\theta}{dy^2} + \lambda \frac{d\theta}{dy} + \frac{dQ}{dy}\frac{d\overline{Q}}{dy} + H_a^2 \left[\frac{1}{1+m^2}(Q\overline{Q}) + \left(1 - \frac{1}{1+m^2}\right)\frac{s^2}{H_a^4}\frac{1}{q_m q_m} + \frac{is}{H_a^2}\frac{m}{1+m^2}\left(\frac{\overline{Q}}{q_m} - \frac{Q}{\overline{q_m}}\right)\right] = 0$$
(20)

where $Q = q/q_m$, $q_m = u_m + iw_m$, \overline{Q} is the complex conjugate of Q. q is the solution of the velocity distributions in complex form, which is given by

$$q(y) = u(y) + iw(y) = c_1 e^{m_1 y} + c_2 e^{m_2 y} + \frac{a_2}{a_1}, \text{ where } u(y) = \frac{q+q}{2} \text{ and } w(y) = \frac{q-q}{2i}.$$
 (21)

The mean velocity distributions in complex notation are given by

$$q_m = c_1 b_7 + c_2 b_8 + \frac{a_2}{a_1}$$
, where, $u_m = \frac{q_m + q_m}{2}$ and $w_m = \frac{q_m - q_m}{2i}$. (22)

Solving the equation (20) for temperature distribution θ with the help of the boundary conditions (12), the expressions for temperature distributions and the rate of heat transfer coefficients at the walls are obtained as:

$$\theta(y) = d_1 + d_2 e^{-\lambda y} + b_{51} e^{b_{19}y} + b_{52} e^{b_{20}y} + b_{53} e^{b_{21}y} + b_{54} e^{b_{22}y} + b_{55} e^{m_1 y} + b_{56} e^{m_2 y} + b_{57} e^{b_1 y} + b_{58} e^{b_2 y} + b_{48} e^{m_2 y} + b_{59} y$$
(23)

The mean temperature is given by

$$\begin{aligned}
\theta_{m} &= \int_{0}^{1} \theta \, dy \\
&= d_{1} - d_{2} \left(\frac{e^{-\lambda} - 1}{\lambda} \right) + b_{51} \left(\frac{e^{b_{19}} - 1}{b_{19}} \right) + b_{52} \left(\frac{e^{b_{20}} - 1}{b_{20}} \right) + b_{53} \left(\frac{e^{b_{21}} - 1}{b_{21}} \right) + b_{54} \left(\frac{e^{b_{22}} - 1}{b_{22}} \right) \\
&+ b_{55} \left(\frac{e^{m_{1}} - 1}{m_{1}} \right) + b_{56} \left(\frac{e^{m_{2}} - 1}{m_{2}} \right) + b_{57} \left(\frac{e^{b_{1}} - 1}{b_{1}} \right) + b_{58} \left(\frac{e^{b_{2}} - 1}{b_{2}} \right) + \frac{b_{59}}{2}
\end{aligned}$$
(24)

The rate of heat transfer coefficient at the upper side wall is

$$N_{u} = \left(-\frac{d\theta}{dy}\right)_{y=1} = \lambda d_{2} e^{-\lambda} - d_{40}$$
(25)

and the Nusselt number at lower wall is given by

$$N_{u} = \left(\frac{d\theta}{dy}\right)_{y=-1} = -\lambda d_{2} e^{-\lambda} + d_{41}$$
(26)

6. RESULTS AND DISCUSSION

The heat transferred aspects of slip-flow of an ionized gas in a horizontal channel bound by two parallel porous walls under the influence of an applied transverse magnetic field, taking Hall currents in to account are investigated analytically. This problem is considered in two cases, one for non-conducting porous side walls and other for conducting porous side walls. It is assumed that the magnetic Reynolds number is small. The transport properties of the fluid are taken to be constant and the bounding walls are maintained at constant and equal temperatures. The resulting differential equations are solved using the prescribed boundary conditions. Exact solutions are obtained for the temperature distributions, the rate of heat transferred coefficients in two cases of the study, by making use of the already obtained solutions for velocity distributions. The computational values of the distributions are determined to represent them graphically for various sets of values of the governing parameters involved and the profiles are shown in

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figures 2 to 16. Further the effect of flow parameters (such as the Hartmann number H_a , Hall parameter m, Taylor number T, porous parameter λ , and slip parameter β on heat transfer is discussed in the following sub-sections. It is noted that the expressions of the temperature distribution are found to be independent of the ratio of electron pressure to the total pressure in case of non-conducting porous side walls and depending on it for conducting porous side walls. It is also found that, the analysis of this paper is in good agreement with that of Raju and Rao (1993), when the side walls are non-porous with no-slip boundary conditions. Also, we recover the results of Raju and Neela (2016) when $\lambda = 0$ that is for non-porous walls.

Non-Conducting (insulating) porous plates

In this case, the profiles of the temperature distributions are shown in figs.2 to 6. Fig. 2 shows the effect of varying Hartmann number H_a on temperature distribution. It is observed, as H_a increases the temperature distribution is found to decrease except at near to the upper wall, where it increases when all other parameters are fixed, that is for fixed values of the Hall parameter m, Taylor number T, Porous parameter λ and slip parameter β . Fig. 3 exhibits the effect of varying the Hall parameter 'm' on temperature distribution. It is observed that, the temperature distribution decreases as the Hall parameter increases. Fig. 4 shows the effect of varying the Taylor number (Rotation parameter) T' on temperature distribution. It is found that, the temperature distribution decreases as T increases. The effect of slip parameter β is exhibited in Fig. 5. It is seen that, an increase in β increases the temperature distribution. The effect of porous parameter λ is shown in Fig. 6. As λ increases the temperature distribution is found to decrease, but when $\lambda = 5$ it increases at near to the lower wall.

Conducting porous plates

The graphs for temperature distributions are shown in Figs.7 to 16 for both cases, that is, when the ratio of electron pressure to the total pressure (ionization parameter), s = 0 from figs. 7 to 11 and s = 0.5 from figs.12 to 16.

A) For the ionization parameter, s = 0.

The effect of varying Hartmann number H_a on temperature distribution is shown in Fig. 7. It is observed that an increase in H_a increases the temperature distribution when the remaining parameters are fixed. Fig. 8 exhibits the effect of varying the Hall parameter 'm' on temperature distribution. It is observed that the temperature distribution decreases as the Hall parameter increases. Fig. 9 shows the effect of varying the Taylor number (Rotation parameter) 'T' on temperature distribution. It is found that the temperature distribution decreases as 'T' increases. The effect of slip parameter β is exhibited in Fig. 10. It is seen that, as β increases the temperature distribution is found to increase except at nearer to the upper wall, where it decreases. The effect of porous parameter λ is shown in Fig. 11. As λ increases the temperature distribution is found to decrease.

B) For the ionization parameter s = 0.5.

Fig. 12 shows the effect of varying Hartmann number H_a on temperature distribution. It is found that, as H_a increases the temperature distribution increases. Fig. 13 exhibits the effect of varying the Hall parameter 'm' on temperature distribution. It is observed that, the temperature distribution decreases as the Hall parameter increases. Fig. 14 shows the effect of varying the Taylor number 'T' on temperature distribution. It is found that, the temperature distribution increases as 'T' increases. The effect of slip parameter β is exhibited in Fig. 15. It is seen that, as β increases the temperature distribution decreases. The effect of porous parameter λ is shown in Fig. 16. It is observed that the temperature distribution decreases at the center of the channel as λ increases. But the same is increasing at the lower wall.

7. CONCLUSION

Magnetohydrodynamic heat transfer slip-flow of an ionized gas in a horizontal channel bounded by two parallel porous side walls under the action of an applied transverse magnetic field with Hall effect in a rotating frame of reference is studied theoretically. The transport properties of the fluid are taken to be constant and the bounding walls are maintained at constant and equal temperatures. The fundamental equation of energy is written down and the resulting differential equation is solved analytically to obtain the closed form solutions for temperature distribution, the mean temperature and the rate of heat transfer at the side walls. This problem is studied in two cases, that is when the two side walls are made up of non-conducting (insulating) and conducting porous materials. Profiles for the temperature distributions are plotted and discussed the effect of flow parameters, like the Hartmann number, Hall parameter, Taylor number, porous parameter and the slip parameter on the temperature fields. The expressions of the temperature distribution are found to be independent of the ratio of electron pressure to the total pressure in case of non-conducting porous side walls.

In case of non-conducting porous side walls, it is noticed that, an increase in Hartmann number diminishes the temperature distribution in the fluid region except at near to the upper wall, where it is found to increase, when all other parameters are fixed (that is for fixed values of the Hall parameter, Taylor number, Porous parameter and slip parameter). Also, an increase in Hall parameter and Taylor number are tend to diminish the temperature distribution. But, an increase in slip parameter is to enhance the temperature distribution.

In case of conducting porous side walls and when the ratio of electron pressure to the total pressure is zero, it is found that, an increase in Hartmann number is to raise the temperature distribution, when all other parameters are fixed. But an increase in Hall parameter and Taylor number are to diminish the temperature distribution. An increase in the slip parameter is to enhance the temperature distribution except at nearer to the upper wall, where it diminishes. It is also observed that, an increase in the values of porous parameter is to diminish the temperature.

In case of conducting porous side walls and when the ratio of electron pressure to the total pressure is half, it is seen that, an increase in Hartmann number raises the temperature distribution. The effect of raising the Hall parameter, Taylor number and slip parameter is to diminish the temperature distribution. Also, is found that, an increase in the values of porous parameter diminishes the temperature distribution at the center of the channel, but enhances the same at the lower wall. Although the validity of the obtained results is not verified practically, the fact is that the solutions satisfy all boundary and interface conditions and hence it is hoped that this theoretical study bears some conformity as is evident from the figures.

FIGURES:



Fig.-1: Co-ordinate system and physical model



→m=0

- m=2 + m=3

-m=5

115

Prof. T. Linga Raju¹ and V. Gowri Sankara Rao*² / MHD Heat Transfer Slip-Flow Between Two Parallel Porous Walls in a Rotating System with ... / IJMA- 9(2), Feb.-2018.













Prof. T. Linga Raju¹ and V. Gowri Sankara Rao*² / MHD Heat Transfer Slip-Flow Between Two Parallel Porous Walls in a Rotating System with ... / IJMA- 9(2), Feb.-2018.













Prof. T. Linga Raju¹ and V. Gowri Sankara Rao^{*2} / MHD Heat Transfer Slip-Flow Between Two Parallel Porous Walls in a Rotating System with ... / IJMA- 9(2), Feb.-2018.



APPENDIX

Constants/functions:

$$\begin{split} a_{1} &= \left[M^{2} \left(\frac{m i - 1}{1 + m^{2}}\right) + 2T^{2} i\right], \ a_{2} &= -\left[k + M^{2} E\left(\frac{i + m}{1 + m^{2}}\right)\right], \ a_{3} &= e^{-m_{1}} - m_{1}\beta e^{m_{1}}, \ a_{4} &= e^{-m_{2}} - m_{2}\beta e^{m^{2}}, \\ a_{5} &= e^{m_{1}} + \beta m_{1} e^{m_{1}}, \ a_{6} &= e^{m_{2} + \beta m_{2} e^{m_{2}}}, \ a_{7} &= a_{3}a_{6} - a_{5}a_{4}, \ a_{8} &= a_{6} - a_{4}, \ a_{9} &= a_{2}\left(-\frac{a_{8}}{a_{1}a_{7}}\right), \\ a_{10} &= -\frac{a_{9}a_{5} - \frac{a_{7}}{a_{6}}}{a_{6}}, \ a_{26} &= \frac{e^{m_{1}} - 1}{m_{1}}, \ a_{27} &= \frac{e^{m_{2}} - 1}{m_{2}}, \ b_{1} &= \overline{m}_{1}, \ b_{2} &= \overline{m}_{2}, \ b_{3} &= \overline{a}_{2}, \ b_{4} &= \overline{a}_{1}, \ b_{5} &= \overline{c}_{1}, \\ b_{6} &= \overline{c}_{2}, \ b_{7} &= c_{1}a_{26} + c_{2}a_{27} + \frac{a_{2}}{a_{1}}, \ b_{8} &= \overline{b}_{7}, \ b_{9} &= \frac{c_{1}m_{1}}{b_{7}}, \ b_{10} &= \frac{c_{2}m_{2}}{b_{7}}, \ b_{11} &= \frac{b_{5}b_{1}}{b_{8}}, \ b_{12} &= \frac{b_{6}b_{2}}{b_{8}}, \\ b_{13} &= m_{1} + b_{1}, \ b_{14} &= m_{1} + b_{2}, \ b_{15} &= m_{2} + b_{1}, \ b_{16} &= m_{2} + b_{2}, \ b_{17} &= -b_{9}b_{11}, \ b_{18} &= -b_{9}b_{12}, \ b_{9} &= -b_{10}b_{11}, \\ b_{20} &= -b_{10}b_{12}, \ b_{21} &= -\frac{M^{2}}{1 + m^{2}}, \ b_{22} &= \frac{1}{b_{7}b_{8}}, \ b_{23} &= b_{21}b_{22}c_{1}b_{3}, \ b_{24} &= b_{21}b_{22}c_{1}b_{6}, \ b_{25} &= b_{21}b_{22}c_{1}b_{8}, \ b_{30} &= -b_{21}b_{22}c_{2}b_{8}, \\ b_{51} &= b_{21}b_{22}\frac{a_{1}}{a_{4}}b_{3}, \ b_{32} &= b_{21}b_{22}c_{2}b_{6}, \ b_{39} &= b_{21}b_{22}c_{2}\frac{b_{3}}{b_{4}}, \\ b_{31} &= b_{21}b_{22}\frac{a_{1}}{a_{4}}b_{3}, \ b_{32} &= b_{21}b_{22}c_{2}b_{7}b_{5}, \ b_{34} &= -b_{21}b_{22}b_{7}b_{6}, \\ b_{55} &= b_{21}b_{22}\left(\frac{a_{2}}{a_{1}}\frac{b_{3}}{a_{4}} - \frac{a_{2}}{a_{1}}b_{8} - b_{7}\frac{b_{3}}{b_{4}} + b_{7}b_{8}\right\right), \ b_{36} &= \frac{b_{28}}{b_{12}^{2}}, \ b_{37} &= \frac{b_{28}}{b_{13}^{2}}, \ b_{38} &= \frac{b_{29}}{b_{13}^{2} + \lambda b_{13}}, \\ b_{44} &= \frac{b_{25}}{m_{1}^{2} + \lambda m_{1}}, \ b_{45} &= \frac{b_{29}}{b_{15}^{2} + \lambda b_{16}}, \ b_{42} &= \frac{b_{28}}{b_{13}^{2} + \lambda b_{13}}, \ b_{43} &= \frac{b_{29}}{b_{12}^{2} + \lambda b_{16}}, \ b_{48} &= \frac{b_{29}}{m_{2}^{2} + \lambda m_{2}}, \\ b_{44} &= \frac{b_{25}}{m_{1}^{2} + \lambda m_{1}}, \ b_{45} &= \frac{b$$

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$$b_{59} = b_{48} + b_{50}, c_1 = a_9, c_2 = a_{10}, d_1 = b_{75} - b_{76} b_{74}, d_2 = b_{76}, d_{36} = b_{58} - b_{63} b_{57}, d_{37} = b_{63}, \\ d_{38} = b_{13} b_{38} e^{b_{13}} + b_{14} b_{39} e^{b_{14}} + b_{15} b_{40} e^{b_{15}} + b_{16} b_{41} e^{b_{16}} + b_{42} b_{13} e^{b_{13}} + b_{14} b_{43} e^{b_{14}} + b_{14} m_1 e^{m_1} + b_{45} m_1 e^{m_1} \\ + b_{46} b_{15} e^{b_{15}} + b_{47} b_{16} e^{b_{16}} + b_{48} m_2 e^{m_2} + b_{49} m_2 e^{m_2} + b_{50} b_1 e^{b_1} + b_{51} b_2 e^{b_2} + b_{52} b_1 e^{b_1} + b_{53} b_2 e^{b_2} + b_{54} \\ d_{39} = b_{13} b_{38} e^{-b_{13}} + b_{14} b_{39} e^{-b_{14}} + b_{15} b_{40} e^{-b_{15}} + b_{16} b_{41} e^{-b_{16}} + b_{42} b_{13} e^{-b_{13}} + b_{14} b_{43} e^{-b_{14}} + b_{14} m_1 e^{-m_1} + b_{45} m_1 e^{-m_1} \\ + b_{46} b_{15} e^{-b_{15}} + b_{47} b_{16} e^{-b_{16}} + b_{48} m_2 e^{-m_2} + b_{49} m_2 e^{-m_2} + b_{50} b_1 e^{-b_1} + b_{51} b_2 e^{-b_2} + b_{52} b_1 e^{-b_1} + b_{53} b_2 e^{-b_2} + b_{54} \\ d_{40} = b_{60} e^{b_{19}} + b_{61} e^{b_{20}} + b_{62} e^{b_{21}} + b_{63} e^{b_{22}} + b_{64} e^{m_1} + b_{65} e^{-m_2} + b_{66} e^{b_1} + b_{67} e^{b_2} + b_{59} , \\ d_{41} = b_{60} e^{-b_{19}} + b_{61} e^{-b_{20}} + b_{62} e^{-b_{21}} + b_{63} e^{-b_{22}} + b_{64} e^{-m_1} + b_{65} e^{-m_2} + b_{66} e^{-b_1} + b_{67} e^{-b_2} + b_{59} \\ m_1 = \sqrt{a_1}, m_2 = -\sqrt{a_1} \end{cases}$$

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Prof. T. Linga Raju¹ and V. Gowri Sankara Rao^{*2}/

MHD Heat Transfer Slip-Flow Between Two Parallel Porous Walls in a Rotating System with ... / IJMA- 9(2), Feb.-2018.

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