

ON ADJACENT VERTEX DISTINGUISHING
TOTAL COLORING OF THE FAMILY OF LADDER GRAPH

K.THIRUSANGU*

Department of Mathematics, SIVET College, Chennai – 600073, India.

R. EZHILARASI

Ramanujan Institute for Advanced Study in Mathematics,
University of Madras, Chennai – 600 005, India.

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ABSTRACT

A total k -coloring of the graph G is adjacent vertex distinguishing, if a mapping $f : V(G) \cup E(G) \rightarrow \{1, 2, \dots, k\}$, $k \in \mathbb{Z}^+$ such that any two adjacent or incident elements in $V(G) \cup E(G)$ have different colors and $C_f(u) \neq C_f(v)$ whenever $uv \in E(G)$, where $C_f(v)$ is the color class of the vertex v (with respect to f), we denote $C_f(v)$ as $C(v)$.

$$C(v) = \{f(v)\} \cup \{f(vw) \mid vw \in E(G)\}$$
$$\bar{C}(v) = \{1, 2, \dots, k\} \setminus C(v)$$

In this paper, we present an algorithm to obtain the adjacent vertex distinguishing total coloring of the family of ladder graph. Also, we prove the existence of the adjacent vertex distinguishing total coloring of some family of ladder graph in detail.

Keywords: adjacent vertex distinguishing total coloring, ladder graph, triangular ladder, diagonal ladder.

AMS Mathematics Subject Classification: 05C85, 05C15, 05C62, 05C76.

1. INTRODUCTION

A graph G consists of a set of vertices $V(G)$ and a set of edges $E(G)$. For every vertex $u, v \in V(G)$, the edge connecting two vertices is denoted by $uv \in E(G)$. All graphs considered here are finite, simple and undirected. The degree of a vertex v is denoted by $\deg(v)$ or $d(v)$. Let $\Delta(G)$ denote the maximum degree of a graph G . The minimum number of colors required to give an adjacent vertex distinguishing total coloring (abbreviated as AVDTTC) to the graph G is denoted by $\chi_{avt}(G)$. For all other standard terminology and concepts of graph theory, we refer [1], [2], [3]. For graphs G_1 and G_2 , $G_1 \cup G_2$ denotes their union, and is defined as $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$. If a simple graph G has two adjacent vertices of maximum degree, then $\chi_{avt}(G) \geq \Delta(G) + 2$. Otherwise, if a simple graph G does not have two adjacent vertices of maximum degree, then $\chi_{avt}(G) = \Delta(G) + 1$. The well-known AV DTC conjecture, made by Zhang et al [10] says that every simple graph G has $\chi_{avt}(G) \leq \Delta(G) + 3$. The adjacent vertex distinguishing total chromatic number of tensor product of graphs, triangular snake, Quadrilateral snake family of graph has been obtained in the literature [6], [7], [8]. Also, AVDTTC of line and splitting graph of some graph and line graph of snake graph family has been obtained in the literature [5], [9]. The definitions and other information which are useful for the present investigation are given below.

Corresponding Author: K.Thirusangu*

Department of Mathematics, SIVET College, Chennai – 600073, India.

Definition 1.1: The ladder graph denoted by L_n is obtained from two path P_n with $\{v_1, v_2, \dots, v_n\}$ and $\{u_1, u_2, \dots, u_n\}$ vertices by joining the vertices $u_i v_i$ for $1 \leq i \leq n$.

$$V[L_n] = \left\{ \bigcup_{i=1}^n u_i \cup v_i \right\} \text{ and } E[L_n] = \left\{ \left(\bigcup_{i=1}^{n-1} (u_i u_{i+1} \cup v_i v_{i+1}) \right) \cup \left(\bigcup_{i=1}^n u_i v_i \right) \right\}$$

Here $d(v_1) = d(v_n) = d(u_1) = d(u_n) = 2$ and $d(v_i) = d(u_i) = 3$ for $2 \leq i \leq n - 1$.

Definition 1.2: The graph [4] $L_n AK_1$ is obtained from a ladder L_n by join u_i and v_i with the new vertices u'_i and v'_i respectively, for $1 \leq i \leq n$. The resultant graph is $L_n AK_1$ whose vertex set and edge set are

$$V[L_n AK_1] = \left\{ \bigcup_{i=1}^n (u_i \cup v_i \cup u'_i \cup v'_i) \right\}$$

$$\text{and } E[L_n AK_1] = \left\{ \left(\bigcup_{i=1}^{n-1} (v_i v_{i+1} \cup u_i u_{i+1}) \right) \cup \left(\bigcup_{i=1}^n (u_i v_i \cup u_i u'_i \cup v_i v'_i) \right) \right\}.$$

Here $d(v_1) = d(u_1) = d(v_n) = d(u_n) = 3$, $d(v'_i) = d(u'_i) = 1$ for $1 \leq i \leq n$ and $d(v_i) = d(u_i) = 4$ for $2 \leq i \leq n - 1$.

Definition 1.3: The Triangular ladder TL_n is obtained from a ladder L_n by adding edges $u_i v_{i+1}$ for $1 \leq i \leq n - 1$.

Definition 1.4: The graph $TL_n AK_1$ is obtained from a ladder TL_n by join u_i and v_i with the new vertices u'_i and v'_i respectively, for $1 \leq i \leq n$. The resultant graph is $TL_n AK_1$ whose vertex set and edge set are

$$V[TL_n AK_1] = \left\{ \bigcup_{i=1}^n (u_i \cup v_i \cup u'_i \cup v'_i) \right\}$$

$$\text{and } E[TL_n AK_1] = \left\{ \left(\bigcup_{i=1}^{n-1} (v_i v_{i+1} \cup u_i u_{i+1} \cup u_i v_{i+1}) \right) \cup \left(\bigcup_{i=1}^n (u_i v_i \cup u_i u'_i \cup v_i v'_i) \right) \right\}.$$

Here $d(v_1) = d(u_n) = 3$, $d(v_n) = d(u_1) = 4$, $d(v'_i) = d(u'_i) = 1$ for $1 \leq i \leq n$ and $d(v_i) = d(u_i) = 5$ for $2 \leq i \leq n - 1$.

Definition 1.5: The diagonal ladder DL_n is a ladder with additional edges $u_i v_{i+1}$ and $u_{i+1} v_i$ for $1 \leq i \leq n - 1$.

Definition 1.6: The graph $DL_n AK_1$ is obtained from a ladder DL_n by join u_i and v_i with the new vertices u'_i and v'_i respectively, for $1 \leq i \leq n$. The resultant graph is $DL_n AK_1$ whose vertex set and edge set are

$$V[DL_n AK_1] = \left\{ \bigcup_{i=1}^n (u_i \cup v_i \cup u'_i \cup v'_i) \right\}$$

$$\text{and } E[DL_n AK_1] = \left\{ \left(\bigcup_{i=1}^{n-1} (v_i v_{i+1} \cup u_i u_{i+1} \cup u_i v_{i+1} \cup u_{i+1} v_i) \right) \cup \left(\bigcup_{i=1}^n (u_i v_i \cup u_i u'_i \cup v_i v'_i) \right) \right\}.$$

Here $d(v_1) = d(v_n) = d(u_n) = d(u_1) = 4$, $d(v'_i) = d(u'_i) = 1$ for $1 \leq i \leq n$ and $d(v_i) = d(u_i) = 6$ for $2 \leq i \leq n - 1$.

2. RESULTS

In this section, we present an algorithm to obtain the AVDTC of Ladder graph L_n , $L_n AK_1$, Triangular ladder TL_n , $TL_n AK_1$ and the diagonal ladder DL_n , $DL_n AK_1$. Also, we discuss their color classes in detail. Let $\{v_1, v_2, \dots, v_n\}$ be the vertices of the graph G . Define $f : V(G) \cup E(G) \rightarrow \{1, 2, \dots, k\}$. Here $f(v_i)$ denotes the color of the vertex v_i and $f(v_i v_j)$ denotes the color of the edge $v_i v_j$.

Algorithm 2.1: Procedure: Adjacent vertex distinguishing total coloring of ladder graph L_n , for $n \geq 3$.

Input: $G(V[L_n], E[L_n])$

for $1 \leq i \leq n$ do

$f(u_i v_i) = 5$

if $i \equiv 0 \pmod{2}$

$f(u_i) \leftarrow 1; f(v_i) \leftarrow 2$

else

$f(u_i) \leftarrow 2; f(v_i) \leftarrow 1$

end for

for $1 \leq i \leq n - 1$ do

if $i \equiv 1 \pmod{2}$

$f(u_i u_{i+1}) = f(v_i v_{i+1}) \leftarrow 3$

else

$f(u_i u_{i+1}) = f(v_i v_{i+1}) \leftarrow 4$

end for

end procedure

Output: Adjacent vertex distinguishing total colored of L_n , for $n \geq 3$.

Theorem 2.1: The ladder graph L_n admits AV DT C and $\chi_{avt}(L_n) = 5$ for $n \geq 3$.

Proof: By definition (1.1), we have $|V(L_n)| = 2n$ and $|E(L_n)| = 3n - 2$. We color the graph L_n by defining a function $f: V(L_n) \cup E(L_n) \rightarrow \{1, 2, \dots, 5\}$ by using the algorithm (2.1). Now the color classes for $n \geq 3$, we have $C(v_1) = \{1, 3, 5\}$, $C(u_1) = \{2, 3, 5\}$

$$C(u_n) = \begin{cases} \{1, 3, 5\}, & \text{if } n \equiv 0 \pmod{2} \\ \{2, 4, 5\}, & \text{if } n \equiv 1 \pmod{2} \end{cases} \text{ and } C(v_n) = \begin{cases} \{2, 3, 5\}, & \text{if } n \equiv 0 \pmod{2} \\ \{1, 4, 5\}, & \text{if } n \equiv 1 \pmod{2} \end{cases}$$

For $2 \leq i \leq n - 1$

$$\bar{C}(u_i) = \begin{cases} \{2\}, & \text{if } i \equiv 0 \pmod{2} \\ \{1\}, & \text{if } i \equiv 1 \pmod{2} \end{cases} \text{ and } \bar{C}(v_i) = \begin{cases} \{1\}, & \text{if } i \equiv 0 \pmod{2} \\ \{2\}, & \text{if } i \equiv 1 \pmod{2} \end{cases}$$

Clearly, the color classes of any two adjacent vertices are different.

Hence $\chi_{avt}(L_n) = 5$ for $n \geq 3$.

Algorithm 2.2: Procedure: Adjacent vertex distinguishing total coloring of $L_n AK_1$, for $n \geq 3$

Input: $G(V[L_n AK_1], E[L_n AK_1])$

for $1 \leq i \leq n$ do

$f(u'_i) = f(v'_i) \leftarrow 3; f(u_i v_i) \leftarrow 5; f(u_i u'_i) = f(v_i v'_i) \leftarrow 6$

if $i \equiv 1 \pmod{2}$

$f(u_i) \leftarrow 1; f(v_i) \leftarrow 2$

else

$f(u_i) \leftarrow 2; f(v_i) \leftarrow 1$

end for

for $1 \leq i \leq n - 1$ do

if $i \equiv 1 \pmod{2}$

$f(u_i u_{i+1}) = f(v_i v_{i+1}) \leftarrow 3$

else

$f(u_i u_{i+1}) = f(v_i v_{i+1}) \leftarrow 4$

end for

end procedure

Output: Adjacent vertex distinguishing total colored of L_nAK_1 , for $n \geq 3$.

Theorem 2.2: The graph L_nAK_1 admits AV DT C and $\chi_{avt}(L_nAK_1) = 6$ for $n \geq 3$.

Proof: By definition (1.2), we have the vertex set and edge set of L_nAK_1 and $|V(L_nAK_1)| = 4n$ and $|E(L_nAK_1)| = 5n - 2$. Now the color classes for $n \geq 3$, using algorithm (2.2) we have $C(v_1) = \{2, 3, 5, 6\}$, $C(u_1) = \{1, 3, 5, 6\}$

$$C(u_n) = \begin{cases} \{2, 3, 5, 6\}, & \text{if } n \equiv 0 \pmod{2} \\ \{1, 4, 5, 6\}, & \text{if } n \equiv 1 \pmod{2} \end{cases} \text{ and } C(v_n) = \begin{cases} \{1, 3, 5, 6\}, & \text{if } n \equiv 0 \pmod{2} \\ \{2, 4, 5, 6\}, & \text{if } n \equiv 1 \pmod{2} \end{cases}$$

For $2 \leq i \leq n - 1$

$$\bar{C}(u_i) = \begin{cases} \{1\}, & \text{if } i \equiv 0 \pmod{2} \\ \{2\}, & \text{if } i \equiv 1 \pmod{2} \end{cases} \text{ and } \bar{C}(v_i) = \begin{cases} \{2\}, & \text{if } i \equiv 0 \pmod{2} \\ \{1\}, & \text{if } i \equiv 1 \pmod{2} \end{cases}$$

Clearly, the color classes of any two adjacent vertices are different.

$\therefore \chi_{avt}(L_nAK_1) = 6$, for $n \geq 3$.

Algorithm 2.3: Procedure: Adjacent vertex distinguishing total coloring of triangular ladder TL_n , for $n \geq 3$.

Input: $V[TL_n] \leftarrow \left\{ \bigcup_{i=1}^n u_i \cup v_i \right\}$

$$E[TL_n] \leftarrow \left\{ \left(\bigcup_{i=1}^{n-1} (v_i v_{i+1} \cup u_i u_{i+1} \cup u_i v_{i+1}) \right) \cup \left(\bigcup_{i=1}^n (u_i v_i) \right) \right\}$$

for $1 \leq i \leq n$ do

$f(u_i v_i) \leftarrow 5$

if $i \equiv 1 \pmod{2}$

$f(u_i) \leftarrow 3; f(v_i) \leftarrow 1$

else

$f(u_i) \leftarrow 4; f(v_i) \leftarrow 2$

end for

for $1 \leq i \leq n - 1$ do

$f(u_i v_{i+1}) \leftarrow 6$

if $i \equiv 1 \pmod{2}$

$f(u_i u_{i+1}) \leftarrow 1; f(v_i v_{i+1}) \leftarrow 3$

else

$f(u_i u_{i+1}) \leftarrow 2; f(v_i v_{i+1}) \leftarrow 4$

end for

end procedure

Output: Adjacent vertex distinguishing total colored of TL_n , for $n \geq 3$.

Theorem 2.3: The triangular ladder TL_n admits AVDT C and

$$\chi_{avt}(TL_n) = 6, \text{ for } n \geq 3.$$

Proof: The vertex set and edge set of TL_n as given in the algorithm (2.3), we have the color classes for $n \geq 3$.

$$C(v_1) = \{1, 3, 5\}, C(u_1) = \{1, 3, 5, 6\}$$

$$C(u_n) = \begin{cases} \{1, 4, 5\}, & \text{if } n \equiv 0 \pmod{2} \\ \{2, 3, 5\}, & \text{if } n \equiv 1 \pmod{2} \end{cases} \text{ and } C(v_n) = \begin{cases} \{2, 3, 5, 6\}, & \text{if } n \equiv 0 \pmod{2} \\ \{1, 4, 5, 6\}, & \text{if } n \equiv 1 \pmod{2} \end{cases}$$

For $2 \leq i \leq n - 1$

$$\bar{C}(u_i) = \begin{cases} \{3\}, & \text{if } i \equiv 0 \pmod{2} \\ \{4\}, & \text{if } i \equiv 1 \pmod{2} \end{cases} \text{ and } \bar{C}(v_i) = \begin{cases} \{1\}, & \text{if } i \equiv 0 \pmod{2} \\ \{2\}, & \text{if } i \equiv 1 \pmod{2} \end{cases}$$

Clearly, the color classes of any two adjacent vertices are different.

$\therefore \chi_{avt}(TL_n) = 6$, $n \geq 3$.

Algorithm 2.4: Procedure: Adjacent vertex distinguishing total coloring of TL_nAK_1 , for $n \geq 3$.

Input: $G(V[TL_nAK_1], E[TL_nAK_1])$
 for $1 \leq i \leq n$ do
 $f(u_i v_i) \leftarrow 5; f(u'_i) \leftarrow 1; f(v'_i) \leftarrow 3; f(u_i u'_i) = f(v_i v'_i) \leftarrow 7$
 if $i \equiv 1 \pmod{2}$
 $f(u_i) \leftarrow 3; f(v_i) \leftarrow 1$
 else
 $f(u_i) \leftarrow 4; f(v_i) \leftarrow 2$
 end for
 for $1 \leq i \leq n - 1$ do
 $f(u_i v_{i+1}) = 6$
 if $i \equiv 1 \pmod{2}$
 $f(u_i u_{i+1}) \leftarrow 1; f(v_i v_{i+1}) \leftarrow 3$
 else
 $f(u_i u_{i+1}) \leftarrow 2; f(v_i v_{i+1}) \leftarrow 4$
 end for
 end procedure

Output: Adjacent vertex distinguishing total colored of TL_nAK_1 , for $n \geq 3$.

Theorem 2.4: The graph TL_nAK_1 admits AVDT C and $\chi_{avt}(TL_nAK_1) = 7$ for $n \geq 3$.

Proof: By definition (1.4), we have $|V(TL_nAK_1)| = 4n$ and $|E(TL_nAK_1)| = 6n - 3$. We have the color classes for $n \geq 3$, using algorithm (2.4)

$C(v_i) = \{1, 3, 5, 7\}$, $C(u_i) = \{1, 3, 5, 6, 7\}$, $C(u'_i) = \{1, 7\}$, $C(v'_i) = \{3, 7\}$ for $1 \leq i \leq n$.

$C(u_n) = \begin{cases} \{1, 4, 5, 7\}, & \text{if } n \equiv 0 \pmod{2} \\ \{2, 3, 5, 7\}, & \text{if } n \equiv 1 \pmod{2} \end{cases}$ and $C(v_n) = \begin{cases} \{2, 3, 5, 6, 7\}, & \text{if } n \equiv 0 \pmod{2} \\ \{1, 4, 5, 6, 7\}, & \text{if } n \equiv 1 \pmod{2} \end{cases}$

For $2 \leq i \leq n - 1$

$\bar{C}(u_i) = \begin{cases} \{3\}, & \text{if } i \equiv 0 \pmod{2} \\ \{4\}, & \text{if } i \equiv 1 \pmod{2} \end{cases}$ and $\bar{C}(v_i) = \begin{cases} \{1\}, & \text{if } i \equiv 0 \pmod{2} \\ \{2\}, & \text{if } i \equiv 1 \pmod{2} \end{cases}$

Clearly, the color classes of any two adjacent vertices are different.

$\therefore \chi_{avt}(TL_nAK_1) = 7, n \geq 3$.

Algorithm 2.5: Procedure: Adjacent vertex distinguishing total coloring of Diagonal ladder DL_n , for $n \geq 3$.

Input: $V[DL_n] \leftarrow \left\{ \bigcup_{i=1}^n u_i \cup v_i \right\}$
 $E[DL_n] \leftarrow \left\{ \left(\bigcup_{i=1}^{n-1} (v_i v_{i+1} \cup u_i u_{i+1} \cup u_i v_{i+1} \cup u_{i+1} v_i) \right) \cup \left(\bigcup_{i=1}^n (u_i v_i) \right) \right\}$

for $1 \leq i \leq n$ do
 $f(u_i v_i) \leftarrow 5$
 if $i \equiv 1 \pmod{2}$
 $f(u_i) \leftarrow 3; f(v_i) \leftarrow 1$
 else
 $f(u_i) \leftarrow 4; f(v_i) \leftarrow 2$
 end for
 for $1 \leq i \leq n - 1$ do
 $f(u_i v_{i+1}) \leftarrow 6; f(u_{i+1} v_i) \leftarrow 7$
 if $i \equiv 1 \pmod{2}$
 $f(u_i u_{i+1}) \leftarrow 1; f(v_i v_{i+1}) \leftarrow 3$
 else
 $f(u_i u_{i+1}) \leftarrow 2; f(v_i v_{i+1}) \leftarrow 4$
 end for
 end procedure

Output: Adjacent vertex distinguishing total colored of DL_n , for $n \geq 3$.

Theorem 2.5: The Diagonal ladder DL_n admits AV DTC and $\chi_{avt}(DL_n) = 7$, for $n \geq 3$.

Proof: The vertex set and edge set of DL_n is given in the algorithm (2.5).

By definition (1.5), we have $|V(DL_n)| = 2n$ and $|E(DL_n)| = 5n - 4$. We have the color classes for $n \geq 3$
 $C(v_1) = \{1, 3, 5, 7\}$, $C(u_1) = \{1, 3, 5, 6\}$

$$C(u_n) = \begin{cases} \{1, 4, 5, 7\}, & \text{if } n \equiv 0 \pmod{2} \\ \{2, 3, 5, 7\}, & \text{if } n \equiv 1 \pmod{2} \end{cases} \quad \text{and} \quad C(v_n) = \begin{cases} \{2, 3, 5, 6\}, & \text{if } n \equiv 0 \pmod{2} \\ \{1, 4, 5, 6\}, & \text{if } n \equiv 1 \pmod{2} \end{cases}$$

For $2 \leq i \leq n - 1$

$$\bar{C}(u_i) = \begin{cases} \{3\}, & \text{if } i \equiv 0 \pmod{2} \\ \{4\}, & \text{if } i \equiv 1 \pmod{2} \end{cases} \quad \text{and} \quad \bar{C}(v_i) = \begin{cases} \{1\}, & \text{if } i \equiv 0 \pmod{2} \\ \{2\}, & \text{if } i \equiv 1 \pmod{2} \end{cases}$$

Clearly, the color classes of any two adjacent vertices are different.

$\therefore \chi_{avt}(DL_n) = 7, n \geq 3$.

Algorithm 2.6: Procedure: Adjacent vertex distinguishing total coloring of DL_nAK_1 , for $n \geq 3$.

Input: $G(V[DL_nAK_1], E[DL_nAK_1])$

for $1 \leq i \leq n$ do

$$f(u_i v_i) \leftarrow 5; f(u'_i) = f(v'_i) \leftarrow 5; f(u_i u'_i) = f(v_i v'_i) \leftarrow 8$$

if $i \equiv 1 \pmod{2}$

$$f(u_i) \leftarrow 3; f(v_i) \leftarrow 1$$

else

$$f(u_i) \leftarrow 4; f(v_i) \leftarrow 2$$

end for

for $1 \leq i \leq n - 1$ do

$$f(u_i v_{i+1}) \leftarrow 6; f(u_{i+1} v_i) \leftarrow 7$$

if $i \equiv 1 \pmod{2}$

$$f(u_i u_{i+1}) \leftarrow 1; f(v_i v_{i+1}) \leftarrow 3$$

else

$$f(u_i u_{i+1}) \leftarrow 2; f(v_i v_{i+1}) \leftarrow 4$$

end for

end procedure

Output: Adjacent vertex distinguishing total colored of DL_nAK_1 , for $n \geq 3$.

Theorem 2.6: The graph DL_nAK_1 admits AV DTC and $\chi_{avt}(DL_nAK_1) = 8$ for $n \geq 3$.

Proof: By definition (1.6), we have $|V(DL_nAK_1)| = 4n$ and $|E(DL_nAK_1)| = 7n - 4$. We have colored the graph using algorithm (2.6). Now the color classes for $n \geq 3$,

$C(v_1) = \{1, 3, 5, 7, 8\}$, $C(u_1) = \{1, 3, 5, 6, 8\}$, $C(u'_i) = C(v'_i) = \{5, 8\}$, for $1 \leq i \leq n$

$$C(u_n) = \begin{cases} \{1, 4, 5, 7, 8\}, & \text{if } n \equiv 0 \pmod{2} \\ \{2, 3, 5, 7, 8\}, & \text{if } n \equiv 1 \pmod{2} \end{cases} \quad \text{and} \quad C(v_n) = \begin{cases} \{2, 3, 5, 6, 8\}, & \text{if } n \equiv 0 \pmod{2} \\ \{1, 4, 5, 6, 8\}, & \text{if } n \equiv 1 \pmod{2} \end{cases}$$

For $2 \leq i \leq n - 1$

$$\bar{C}(u_i) = \begin{cases} \{3\}, & \text{if } i \equiv 0 \pmod{2} \\ \{4\}, & \text{if } i \equiv 1 \pmod{2} \end{cases} \quad \text{and} \quad \bar{C}(v_i) = \begin{cases} \{1\}, & \text{if } i \equiv 0 \pmod{2} \\ \{2\}, & \text{if } i \equiv 1 \pmod{2} \end{cases}$$

Clearly, the color classes of any two adjacent vertices are different.

$\therefore \chi_{avt}(DL_nAK_1) = 8, n \geq 3$.

Remark: The adjacent vertex distinguishing total chromatic number of triangular snake and Quadrilateral snake family of graph has been obtained in the literature [6,7]. Also, this conjecture is true for the graphs T_nAK_1 , DT_nAK_1 , Q_nAK_1 and DQ_nAK_1 .

CONCLUSION

We found the adjacent vertex distinguishing total chromatic number of family of ladder graph such as ladder graph, triangular ladder and diagonal ladder. We are investigating to obtain the class of graph which admits adjacent vertex distinguishing total coloring.

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