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# ON ADJACENT VERTEX DISTINGUISHING TOTAL COLORING OF THE FAMILY OF LADDER GRAPH 

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#### Abstract

A total k-coloring of the graph $G$ is adjacent vertex distinguishing, if a mapping $f: V(G) \cup E(G) \rightarrow\{1,2, \cdots, k\}$, $k \in \mathbb{Z}^{+}$such that any two adjacent or incident elements in $V(G) \cup E(G)$ have different colors and $C_{f}(u) \neq C_{f}(v)$ whenever $u v \in E(G)$, where $C_{f}(v)$ is the color class of the vertex $v$ (with respect to $f$ ), we denote $C_{f}(v)$ as $C(v)$. $$
\begin{aligned} & C(v)=\{\{\mathrm{f}(\mathrm{v})\} \cup\{f(v w)\} \mid v w \in E(G)\} \\ & \bar{C}(v)=\{1,2, \cdots, k\} \mid C(v) \end{aligned}
$$


In this paper, we present an algorithm to obtain the adjacent vertex distinguishing total coloring of the family of ladder graph. Also, we prove the existence of the adjacent vertex distinguishing total coloring of some family of ladder graph in detail.

Keywords: adjacent vertex distinguishing total coloring, ladder graph, triangular ladder, diagonal ladder.
AMS Mathematics Subject Classification: 05C85, 05C15, 05C62, 05C76.

## 1. INTRODUCTION

A graph $G$ consists of a set of vertices $V(G)$ and a set of edges $E(G)$. For every vertex $u, v \in V(G)$, the edge connecting two vertices is denoted by $u v \in E(G)$. All graphs considered here are finite, simple and undirected. The degree of a vertex v is denoted by $\operatorname{deg}(\mathrm{v})$ or $\mathrm{d}(\mathrm{v})$. Let $\Delta(G)$ denote the maximum degree of a graph $G$. The minimum number of colors required to give an adjacent vertex distinguishing total coloring (abbreviated as AVDTC) to the graph $G$ is denoted by $\chi_{\text {avt }}(G)$. For all other standard terminology and concepts of graph theory, we refer [1], [2], [3]. For graphs $G_{1}$ and $G_{2}, G_{1} \cup G_{2}$ denotes their union, and is defined as $V\left(G_{1} \cup G_{2}\right)=V\left(G_{1}\right) \cup V\left(G_{2}\right)$ and $E\left(G_{1} \cup G_{2}\right)=E\left(G_{1}\right) \cup E\left(G_{2}\right)$. If a simple graph $G$ has two adjacent vertices of maximum degree, then $\chi_{\text {avt }}(G) \geq \Delta(G)+2$. Otherwise, if a simple graph $G$ does not have two adjacent vertices of maximum degree, then $\chi_{\text {avt }}(G)=\Delta(G)+1$. The well-known AV DTC conjecture, made by Zhang et al [10] says that every simple graph $G$ has $\chi_{\text {avt }}(G) \leq \Delta(G)+3$. The adjacent vertex distinguishing total chromatic number of tensor product of graphs, triangular snake, Quadrilateral snake family of graph has been obtained in the literature [6], [7], [8]. Also, AVDTC of line and splitting graph of some graph and line graph of snake graph family has been obtained in the literature [5], [9]. The definitions and other information which are useful for the present investigation are given below.

[^0]Definition 1.1: The ladder graph denoted by $L_{n}$ is obtained from two path $P_{n}$ with $\left\{v_{1}, v_{2}, \cdots, v_{n}\right\}$ and $\left\{u_{1}, u_{2}, \cdots, u_{n}\right\}$ vertices by joining the vertices $u_{i} v_{i}$ for $1 \leq i \leq n$.
$V\left[L_{n}\right]=\left\{\bigcup_{i=1}^{n} u_{i} \cup v_{i}\right\}$ and $E\left[L_{n}\right]=\left\{\left(\bigcup_{i=1}^{n-1}\left(u_{i} u_{i+1} \cup v_{i} v_{i+1}\right)\right) \cup\left(\bigcup_{i=1}^{n} u_{i} v_{i}\right)\right\}$
Here $d\left(v_{1}\right)=d\left(v_{n}\right)=d\left(u_{1}\right)=d\left(u_{n}\right)=2$ and $d\left(v_{i}\right)=d\left(u_{i}\right)=3$ for $2 \leq i \leq n-1$.
Definition 1.2: The graph [4] $L_{n} A K_{1}$ is obtained from a ladder $L_{n}$ by join $u_{i}$ and $v_{i}$ with the new vertices $u^{\prime}{ }_{i}$ and $v_{i}^{\prime}$ respectively, for $1 \leq i \leq n$. The resultant graph is $L_{n} A K_{1}$ whose vertex set and edge set are

$$
\begin{aligned}
& \qquad V\left[L_{n} A K_{1}\right]=\left\{\bigcup_{i=1}^{n}\left(u_{i} \cup v_{i} \cup u_{i}^{\prime} \cup v_{i}^{\prime}\right)\right\} \\
& \text { and } E\left[L_{n} A K_{1}\right]=\left\{\left(\bigcup_{i=1}^{n-1}\left(v_{i} v_{i+1} \cup u_{i} u_{i+1}\right)\right) \cup\left(\bigcup_{i=1}^{n}\left(u_{i} v_{i} \cup u_{i} u_{i}^{\prime} \cup v_{i} v_{i}^{\prime}\right)\right)\right\} .
\end{aligned}
$$

Here $d\left(v_{1}\right)=d\left(u_{1}\right)=d\left(v_{n}\right)=d\left(u_{n}\right)=3, d\left(v_{i}^{\prime}\right)=d\left(u_{i}^{\prime}\right)=1$ for $1 \leq i \leq n$ and $d\left(v_{i}\right)=d\left(u_{i}\right)=4$ for $2 \leq i \leq n-1$.
Definition 1.3: The Triangular ladder $\mathrm{TL}_{\mathrm{n}}$ is obtained from a ladder $\mathrm{L}_{\mathrm{n}}$ by adding edges $u_{i} v_{i+1}$ for $1 \leq i \leq n-1$.
Definition 1.4: The graph $\mathrm{TL}_{n} \mathrm{AK}_{1}$ is obtained from a ladder $\mathrm{TL}_{n}$ by join $u_{i}$ and $v_{i}$ with the new vertices $u_{i}^{\prime}$ and $v_{i}^{\prime}$ respectively, for $1 \leq i \leq n$. The resultant graph is $\mathrm{T}_{\mathrm{n}} \mathrm{AK}_{1}$ whose vertex set and edge set are

$$
V\left[T L_{n} A K_{1}\right]=\left\{\bigcup_{i=1}^{n}\left(u_{i} \cup v_{i} \cup u_{i}^{\prime} \cup v_{i}^{\prime}\right)\right\}
$$

and $E\left[T L_{n} A K_{1}\right]=\left\{\left(\bigcup_{i=1}^{n-1}\left(v_{i} v_{i+1} \cup u_{i} u_{i+1} \cup u_{i} v_{i+1}\right)\right) \cup\left(\bigcup_{i=1}^{n}\left(u_{i} v_{i} \cup u_{i} u_{i}^{\prime} \cup v_{i} v_{i}^{\prime}\right)\right)\right\}$.
Here $d\left(v_{1}\right)=d\left(u_{n}\right)=3, d\left(v_{n}\right)=d\left(u_{1}\right)=4, d\left(v_{i}^{\prime}\right)=d\left(u_{i}^{\prime}\right)=1$ for $1 \leq i \leq n$ and $d\left(v_{i}\right)=d\left(u_{i}\right)=5$ for $2 \leq i \leq n-1$
Definition 1.5: The diagonal ladder $\mathrm{DL}_{\mathrm{n}}$ is a ladder with additional edges $u_{i} v_{i+1}$ and $u_{i+1} v_{i}$ for $1 \leq i \leq n-1$.
Definition 1.6: The graph $\mathrm{DL}_{\mathrm{n}} \mathrm{AK}_{1}$ is obtained from a ladder $\mathrm{DL}_{\mathrm{n}}$ by join $u_{i}$ and $v_{i}$ with the new vertices $u_{i}^{\prime}$ and $v_{i}^{\prime}$ respectively, for $1 \leq i \leq n$. The resultant graph is $\mathrm{DL}_{\mathrm{n}} \mathrm{AK}_{1}$ whose vertex set and edge set are

$$
\begin{aligned}
& \qquad V\left[D L_{n} A K_{1}\right]=\left\{\bigcup_{i=1}^{n}\left(u_{i} \cup v_{i} \cup u_{i}^{\prime} \cup v_{i}^{\prime}\right)\right\} \\
& \text { and } E\left[D L_{n} A K_{1}\right]=\left\{\left(\bigcup_{i=1}^{n-1}\left(v_{i} v_{i+1} \cup u_{i} u_{i+1} \cup u_{i} v_{i+1} \cup u_{i+1} v_{i}\right)\right) \cup\left(\bigcup_{i=1}^{n}\left(u_{i} v_{i} \cup u_{i} u_{i}^{\prime} \cup v_{i} v_{i}^{\prime}\right)\right)\right\} .
\end{aligned}
$$

Here $d\left(v_{1}\right)=d\left(v_{n}\right)=d\left(u_{n}\right)=d\left(u_{1}\right)=4, d\left(v_{i}^{\prime}\right)=d\left(u_{i}^{\prime}\right)=1$ for $1 \leq i \leq n$ and $d\left(v_{i}\right)=d\left(u_{i}\right)=6$ for $2 \leq i \leq n-1$

## 2. RESULTS

In this section, we present an algorithm to obtain the AVDTC of Ladder graph $L_{n}, \mathrm{~L}_{\mathrm{n}} \mathrm{AK}$, Triangular ladder $\mathrm{TL}_{\mathrm{n}}$, $\mathrm{TL}_{\mathrm{n}} \mathrm{AK}_{1}$ and the diagonal ladder $\mathrm{DL}_{\mathrm{n}}, \mathrm{DL}_{\mathrm{n}} \mathrm{AK}_{1}$. Also, we discuss their color classes in detail. Let $\left\{\mathrm{v}_{1}, \mathrm{v}_{2}, \ldots, \mathrm{v}_{\mathrm{n}}\right\}$ be the vertices of the graph $G$. Define $f: V(G) \cup E(G) \rightarrow\{1,2, \cdots, k\}$. Here $f\left(\mathrm{v}_{\mathrm{i}}\right)$ denotes the color of the vertex $\mathrm{v}_{\mathrm{i}}$ and $f\left(v_{i} v_{j}\right)$ denotes the color of the edge $v_{i} v_{j}$.

Algorithm 2.1: Procedure: Adjacent vertex distinguishing total coloring of ladder graph $L_{n}$, for $\mathrm{n} \geq 3$.
Input: $\mathrm{G}\left(\mathrm{V}\left[L_{n}\right], \mathrm{E}\left[L_{n}\right]\right)$
for $1 \leq \mathrm{i} \leq \mathrm{n}$ do
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$f\left(u_{i} v_{i}\right)=5$
if $\mathrm{i} \equiv 0(\bmod 2)$

$$
\mathrm{f}\left(\mathrm{u}_{\mathrm{i}}\right) \leftarrow 1 ; \mathrm{f}\left(\mathrm{v}_{\mathrm{i}}\right) \leftarrow 2
$$

else

$$
\mathrm{f}\left(\mathrm{u}_{\mathrm{i}}\right) \leftarrow 2 ; \mathrm{f}\left(\mathrm{v}_{\mathrm{i}}\right) \leftarrow 1
$$

end for
for $1 \leq \mathrm{i} \leq \mathrm{n}-1$ do
if $\mathrm{i} \equiv 1(\bmod 2)$

$$
\mathrm{f}\left(\mathrm{u}_{\mathrm{i}} \mathrm{u}_{\mathrm{i}+1}\right)=\mathrm{f}\left(\mathrm{v}_{\mathrm{i}} \mathrm{v}_{\mathrm{i}+1}\right) \leftarrow 3
$$

else

$$
\mathrm{f}\left(\mathrm{u}_{\mathrm{i}} \mathrm{u}_{\mathrm{i}+1}\right)=\mathrm{f}\left(\mathrm{v}_{\mathrm{i}} \mathrm{v}_{\mathrm{i}+1}\right) \leftarrow 4
$$

end for
end procedure
Output: Adjacent vertex distinguishing total colored of $L_{n}$, for $n \geq 3$.
Theorem 2.1: The ladder graph $L_{n}$ admits AV DT C and $\chi_{\text {avt }}\left(L_{n}\right)=5$ for $n \geq 3$.

Proof: By definition (1.1), we have $\left|V\left(L_{n}\right)\right|=2 n$ and $\left|E\left(L_{n}\right)\right|=3 n-2$. We color the graph $L_{n}$ by defining a function $f: V\left(L_{n}\right) \cup E\left(L_{n}\right) \rightarrow\{1,2, \ldots, 5\}$ by using the algorithm (2.1). Now the color classes for $n \geq 3$, we have $C\left(v_{1}\right)=\{1,3,5\}, C\left(u_{1}\right)=\{2,3,5\}$
$C\left(u_{n}\right)=\left\{\begin{array}{l}\{1,3,5\}, \text { if } n \equiv 0(\bmod 2) \\ \{2,4,5\}, \text { if } n \equiv 1(\bmod 2)\end{array}\right.$ and $C\left(v_{n}\right)=\left\{\begin{array}{l}\{2,3,5\}, \text { if } n \equiv 0(\bmod 2) \\ \{1,4,5\}, \text { if } n \equiv 1(\bmod 2)\end{array}\right.$
For $2 \leq \mathrm{i} \leq \mathrm{n}-1$
$\bar{C}\left(u_{i}\right)=\left\{\begin{array}{l}\{2\}, \text { if } i \equiv 0(\bmod 2) \\ \{1\}, \text { if } i \equiv 1(\bmod 2)\end{array}\right.$ and $\bar{C}\left(v_{i}\right)=\left\{\begin{array}{l}\{1\}, \text { if } i \equiv 0(\bmod 2) \\ \{2\}, \text { if } i \equiv 1(\bmod 2)\end{array}\right.$
Clearly, the color classes of any two adjacent vertices are different.
Hence $\chi_{\text {avt }}\left(L_{n}\right)=5$ for $\mathrm{n} \geq 3$.
Algorithm 2.2: Procedure: Adjacent vertex distinguishing total coloring of $\mathrm{L}_{\mathrm{n}} \mathrm{AK}_{1}$, for $\mathrm{n} \geq 3$
Input: $G\left(V\left[L_{n} A K_{1}\right], E\left[L_{n} A K_{1}\right]\right)$
for $1 \leq \mathrm{i} \leq \mathrm{n}$ do
$\mathrm{f}\left(\mathrm{u}_{\mathrm{i}}\right)=\mathrm{f}\left(\mathrm{v}_{\mathrm{i}}{ }_{\mathrm{i}}\right) \leftarrow 3 ; \mathrm{f}\left(\mathrm{u}_{\mathrm{i}} \mathrm{v}_{\mathrm{i}}\right) \leftarrow 5 ; \mathrm{f}\left(\mathrm{u}_{\mathrm{i}} \mathrm{u}_{\mathrm{i}}{ }_{\mathrm{i}}\right)=\mathrm{f}\left(\mathrm{v}_{\mathrm{i}} \mathrm{v}_{\mathrm{i}}{ }_{\mathrm{i}}\right) \leftarrow 6$
if $\mathrm{i} \equiv 1(\bmod 2)$

$$
\mathrm{f}\left(\mathrm{u}_{\mathrm{i}}\right) \leftarrow 1 ; \mathrm{f}\left(\mathrm{v}_{\mathrm{i}}\right) \leftarrow 2
$$

else

$$
\mathrm{f}\left(\mathrm{u}_{\mathrm{i}}\right) \leftarrow 2 ; \mathrm{f}\left(\mathrm{v}_{\mathrm{i}}\right) \leftarrow 1
$$

end for
for $1 \leq \mathrm{i} \leq \mathrm{n}-1$ do
if $\mathrm{i} \equiv 1(\bmod 2)$

$$
\mathrm{f}\left(\mathrm{u}_{\mathrm{i}} \mathrm{u}_{\mathrm{i}+1}\right)=\mathrm{f}\left(\mathrm{v}_{\mathrm{i}} \mathrm{v}_{\mathrm{i}+1}\right) \leftarrow 3
$$

else

$$
\mathrm{f}\left(\mathrm{u}_{\mathrm{i}} \mathrm{u}_{\mathrm{i}+1}\right)=\mathrm{f}\left(\mathrm{v}_{\mathrm{i}} \mathrm{v}_{\mathrm{i}+1}\right) \leftarrow 4
$$

end for
end procedure

Output: Adjacent vertex distinguishing total colored of $\mathrm{L}_{\mathrm{n}} \mathrm{AK}_{1}$, for $\mathrm{n} \geq 3$.
Theorem 2.2: The graph $L_{n} A K_{1}$ admits AV DT C and $\chi_{\text {avt }}\left(L_{n} A K_{1}\right)=6$ for $n \geq 3$.
Proof: By definition (1.2), we have the vertex set and edge set of $L_{n} A K_{1}$ and $\left|V\left(L_{n} A K_{1}\right)\right|=4 n$ and $\left|E\left(L_{n} A K_{1}\right)\right|=5 n-2$. Now the color classes for $n \geq 3$, using algorithm (2.2) we have
$C\left(\mathrm{v}_{1}\right)=\{2,3,5,6\}, C\left(\mathrm{u}_{1}\right)=\{1,3,5,6\}$
$C\left(u_{n}\right)=\left\{\begin{array}{l}\{2,3,5,6\}, \text { if } n \equiv 0(\bmod 2) \\ \{1,4,5,6\}, \text { if } n \equiv 1(\bmod 2)\end{array}\right.$ and $C\left(v_{n}\right)=\left\{\begin{array}{l}\{1,3,5,6\}, \text { if } n \equiv 0(\bmod 2) \\ \{2,4,5,6\}, \text { if } n \equiv 1(\bmod 2)\end{array}\right.$
For $2 \leq \mathrm{i} \leq \mathrm{n}-1$
$\overline{\mathrm{C}}\left(\mathrm{u}_{\mathrm{i}}\right)=\left\{\begin{array}{l}\{1\}, \text { if } \mathrm{i} \equiv 0(\bmod 2) \\ \{2\}, \text { if } \mathrm{i} \equiv 1(\bmod 2)\end{array}\right.$ and $\overline{\mathrm{C}}\left(\mathrm{v}_{\mathrm{i}}\right)=\left\{\begin{array}{l}\{2\}, \text { if } \mathrm{i} \equiv 0(\bmod 2) \\ \{1\}, \text { if } i \equiv 1(\bmod 2)\end{array}\right.$
Clearly, the color classes of any two adjacent vertices are different.
$\therefore \quad \chi_{\text {avt }}\left(L_{n} A K_{1}\right)=6$, for $\mathrm{n} \geq 3$.
Algorithm 2.3: Procedure: Adjacent vertex distinguishing total coloring of triangular ladder $\mathrm{TL}_{\mathrm{n}}$, for $n \geq 3$.
Input:

$$
\begin{aligned}
& V\left[T L_{n}\right] \leftarrow\left\{\bigcup_{i=1}^{n} u_{i} \cup v_{i}\right\} \\
& E\left[T L_{n}\right] \leftarrow\left\{\left(\bigcup_{i=1}^{n-1}\left(v_{i} v_{i+1} \cup u_{i} u_{i+1} \cup u_{i} v_{i+1}\right)\right) \cup\left(\bigcup_{i=1}^{n}\left(u_{i} v_{i}\right)\right\}\right.
\end{aligned}
$$

for $1 \leq i \leq n$ do
$f\left(u_{i} v_{i}\right) \leftarrow 5$
if $i \equiv 1(\bmod 2)$

$$
f\left(u_{i}\right) \leftarrow 3 ; f\left(v_{i}\right) \leftarrow 1
$$

else

$$
f\left(u_{i}\right) \leftarrow 4 ; f\left(v_{i}\right) \leftarrow 2
$$

end for
for $1 \leq i \leq n-1$ do
$f\left(u_{i} v_{i+1}\right) \leftarrow 6$
if $i \equiv 1(\bmod 2)$

$$
f\left(u_{i} u_{i+1}\right) \leftarrow 1 ; f\left(v_{i} v_{i+1}\right) \leftarrow 3
$$

else

$$
f\left(u_{i} u_{i+1}\right) \leftarrow 2 ; f\left(v_{i} v_{i+1}\right) \leftarrow 4
$$

end for
end procedure
Output: Adjacent vertex distinguishing total colored of $\mathrm{TL}_{\mathrm{n}}$, for $n \geq 3$.
Theorem 2.3: The triangular ladder $\mathrm{TL}_{\mathrm{n}}$ admits AVDTC and

$$
x_{\text {avt }}\left(T L_{n}\right)=6, \text { for } n \geq 3 .
$$

Proof: The vertex set and edge set of $T L_{n}$ as given in the algorithm (2.3), we have the color classes for $n \geq 3$.
$C\left(v_{1}\right)=\{1,3,5\}, C\left(u_{1}\right)=\{1,3,5,6\}$
$C\left(u_{n}\right)=\left\{\begin{array}{l}\{1,4,5\}, \text { if } n \equiv 0(\bmod 2) \\ \{2,3,5\}, \text { if } n \equiv 1(\bmod 2)\end{array}\right.$ and $C\left(v_{n}\right)=\left\{\begin{array}{l}\{2,3,5,6\}, \text { if } n \equiv 0(\bmod 2) \\ \{1,4,5,6\}, \text { if } n \equiv 1(\bmod 2)\end{array}\right.$
For $2 \leq i \leq n-1$
$\bar{C}\left(u_{i}\right)=\left\{\begin{array}{l}\{3\}, \text { if } i \equiv 0(\bmod 2) \\ \{4\}, \text { if } i \equiv 1(\bmod 2)\end{array}\right.$ and $\bar{C}\left(v_{i}\right)=\left\{\begin{array}{l}\{1\}, \text { if } i \equiv 0(\bmod 2) \\ \{2\}, \text { if } i \equiv 1(\bmod 2)\end{array}\right.$
Clearly, the color classes of any two adjacent vertices are different.
$\therefore \quad x_{\text {avt }}\left(T L_{n}\right)=6, \quad n \geq 3$.

Algorithm 2.4: Procedure: Adjacent vertex distinguishing total coloring of $\mathrm{TL}_{\mathrm{n}} \mathrm{AK}_{1}$, for $n \geq 3$.
Input: $G\left(V\left[T L_{n} A K_{1}\right], E\left[T L_{n} A K_{1}\right]\right)$
for $1 \leq i \leq n d o$

$$
f\left(u_{i} v_{i}\right) \leftarrow 5 ; f\left(u_{i}^{\prime}\right) \leftarrow 1 ; f\left(v_{i}^{\prime}\right) \leftarrow 3 ; f\left(u_{i} u_{i}^{\prime}\right)=f\left(v_{i} v_{i}^{\prime}\right) \leftarrow 7
$$

$$
\text { if } i \equiv 1(\bmod 2)
$$

$$
f\left(u_{i}\right) \leftarrow 3 ; f\left(v_{i}\right) \leftarrow 1
$$

else

$$
f\left(u_{i}\right) \leftarrow 4 ; f\left(v_{i}\right) \leftarrow 2
$$

end for
for $1 \leq i \leq n-1$ do

$$
\begin{aligned}
& f\left(u_{i} v_{i+1}\right)=6 \\
& \text { if } i \equiv 1(\bmod 2)
\end{aligned}
$$

$$
f\left(u_{i} u_{i+1}\right) \leftarrow 1 ; f\left(v_{i} v_{i+1}\right) \leftarrow 3
$$

else

$$
f\left(u_{i} u_{i+1}\right) \leftarrow 2 ; f\left(v_{i} v_{i+1}\right) \leftarrow 4
$$

end for
end procedure
Output: Adjacent vertex distinguishing total colored of $\mathrm{TL}_{n} \mathrm{AK}_{1}$, for $n \geq 3$.

Theorem 2.4: The graph $T L_{n} A K_{1}$ admits $A V D T C$ and $\chi_{\text {avt }}\left(T L_{n} A K_{1}\right)=7$ for $n \geq 3$.
Proof: By definition (1.4), we have $\left|V\left(T L_{n} A K_{1}\right)\right|=4 \mathrm{n}$ and $\left|E\left(T L_{n} A K_{1}\right)\right|=6 \mathrm{n}-3$. We have the color classes for $\mathrm{n} \geq 3$, using algorithm (2.4)
$C\left(v_{1}\right)=\{1,3,5,7\}, C\left(u_{1}\right)=\{1,3,5,6,7\}, C\left(u_{i}^{\prime}\right)=\{1,7\}, C\left(v_{i}^{\prime}\right)=\{3,7\}$ for $1 \leq \mathrm{i} \leq \mathrm{n}$.
$C\left(u_{n}\right)=\left\{\begin{array}{l}\{1,4,5,7\}, \text { if } n \equiv 0(\bmod 2) \\ \{2,3,5,7\}, \text { if } n \equiv 1(\bmod 2)\end{array}\right.$ and $C\left(v_{n}\right)=\left\{\begin{array}{l}\{2,3,5,6,7\}, \text { if } n \equiv 0(\bmod 2) \\ \{1,4,5,6,7\}, \text { if } n \equiv 1(\bmod 2)\end{array}\right.$
For $2 \leq i \leq n-1$
$\bar{C}\left(u_{i}\right)=\left\{\begin{array}{l}\{3\}, \text { if } i \equiv 0(\bmod 2) \\ \{4\}, \text { if } i \equiv 1(\bmod 2)\end{array}\right.$ and $\bar{C}\left(v_{i}\right)=\left\{\begin{array}{l}\{1\}, \text { if } i \equiv 0(\bmod 2) \\ \{2\}, \text { if } i \equiv 1(\bmod 2)\end{array}\right.$
Clearly, the color classes of any two adjacent vertices are different.

$$
\therefore \quad \chi_{\text {avt }}\left(T L_{n} A K_{1}\right)=7, \mathrm{n} \geq 3 .
$$

Algorithm 2.5: Procedure: Adjacent vertex distinguishing total coloring of Diagonal ladder $D L_{n}$, for $\mathrm{n} \geq 3$.
Input: $\quad V\left[D L_{n}\right] \leftarrow\left\{\bigcup_{i=1}^{n} u_{i} \cup v_{i}\right\}$
$E\left[D L_{n}\right] \leftarrow\left\{\left(\bigcup_{i=1}^{n-1}\left(v_{i} v_{i+1} \cup u_{i} u_{i+1} \cup u_{i} v_{i+1} \cup u_{i+1} v_{i}\right)\right) \cup\left(\bigcup_{i=1}^{n}\left(u_{i} v_{i}\right)\right\}\right.$
for $1 \leq \mathrm{i} \leq \mathrm{n}$ do
$f\left(u_{i} v_{i}\right) \leftarrow 5$
if $i \equiv 1(\bmod 2)$

$$
f\left(u_{i}\right) \leftarrow 3 ; f\left(v_{i}\right) \leftarrow 1
$$

else

$$
f\left(u_{i}\right) \leftarrow 4 ; f\left(v_{i}\right) \leftarrow 2
$$

end for
for $1 \leq \mathrm{i} \leq \mathrm{n}-1$ do
$f\left(u_{i} v_{i+1}\right) \leftarrow 6 ; f\left(u_{i+1} v_{i}\right) \leftarrow 7$
if $\mathrm{i} \equiv 1(\bmod 2)$

$$
f\left(u_{i} u_{i+1}\right) \leftarrow 1 ; f\left(v_{i} v_{i+1}\right) \leftarrow 3
$$

else

$$
f\left(u_{i} u_{i+1}\right) \leftarrow 2 ; f\left(v_{i} v_{i+1}\right) \leftarrow 4
$$

end for
end procedure
Output: Adjacent vertex distinguishing total colored of $D L_{n}$, for $\mathrm{n} \geq 3$.

Theorem 2.5: The Diagonal ladder $D L_{n}$ admits $A V D T C$ and $\chi_{a v t}\left(D L_{n}\right)=7$, for $\mathrm{n} \geq 3$.
Proof: The vertex set and edge set of $D L_{n}$ is given in the algorithm (2.5).
By definition (1.5), we have $\left|V\left(D L_{n}\right)\right|=2 n$ and $\left|E\left(D L_{n}\right)\right|=5 n-4$. We have the color classes for $\mathrm{n} \geq 3$
$C\left(v_{1}\right)=\{1,3,5,7\}, C\left(u_{1}\right)=\{1,3,5,6\}$
$C\left(u_{n}\right)=\left\{\begin{array}{l}\{1,4,5,7\}, \text { if } n \equiv 0(\bmod 2) \\ \{2,3,5,7\}, \text { if } n \equiv 1(\bmod 2)\end{array}\right.$ and $C\left(v_{n}\right)=\left\{\begin{array}{l}\{2,3,5,6\}, \text { if } n \equiv 0(\bmod 2) \\ \{1,4,5,6\}, \text { if } n \equiv 1(\bmod 2)\end{array}\right.$
For $2 \leq \mathrm{i} \leq \mathrm{n}-1$
$\overline{\mathrm{C}}\left(\mathrm{u}_{\mathrm{i}}\right)=\left\{\begin{array}{l}\{3\}, \text { if } \mathrm{i} \equiv 0(\bmod 2) \\ \{4\}, \text { if } \mathrm{i} \equiv 1(\bmod 2)\end{array}\right.$ and $\overline{\mathrm{C}}\left(\mathrm{v}_{\mathrm{i}}\right)=\left\{\begin{array}{l}\{1\}, \text { if } \mathrm{i} \equiv 0(\bmod 2) \\ \{2\}, \text { if } \mathrm{i} \equiv 1(\bmod 2)\end{array}\right.$
Clearly, the color classes of any two adjacent vertices are different.
$\therefore \chi_{\text {avt }}\left(D L_{n}\right)=7, \mathrm{n} \geq 3$.
Algorithm 2.6: Procedure: Adjacent vertex distinguishing total coloring of $D L_{n} A K_{1}$, for $\mathrm{n} \geq 3$.
Input: $\quad G\left(V\left[D L_{n} A K_{1}\right], E\left[D L_{n} A K_{1}\right]\right)$
for $1 \leq \mathrm{i} \leq \mathrm{n}$ do
$f\left(u_{i} v_{i}\right) \leftarrow 5 ; f\left(u^{\prime}\right)=f\left(v_{i}^{\prime}\right) \leftarrow 5 ; f\left(u_{i} u^{\prime}\right)=f\left(v_{i} v^{\prime}{ }_{i}\right) \leftarrow 8$ if $i \equiv 1(\bmod 2)$

$$
f\left(u_{i}\right) \leftarrow 3 ; f\left(v_{i}\right) \leftarrow 1
$$

else

$$
f\left(u_{i}\right) \leftarrow 4 ; f\left(v_{i}\right) \leftarrow 2
$$

## end for

for $1 \leq \mathrm{i} \leq \mathrm{n}-1$ do

$$
f\left(u_{i} v_{i+1}\right) \leftarrow 6 ; f\left(u_{i+1} v_{i}\right) \leftarrow 7
$$

if $i \equiv 1(\bmod 2)$

$$
f\left(u_{i} u_{i+1}\right) \leftarrow 1 ; f\left(v_{i} v_{i+1}\right) \leftarrow 3
$$

else

$$
f\left(u_{i} u_{i+1}\right) \leftarrow 2 ; f\left(v_{i} v_{i+1}\right) \leftarrow 4
$$

end for
end procedure
Output: Adjacent vertex distinguishing total colored of $D L_{n} A K_{1}$, for $n \geq 3$.
Theorem 2.6: The graph $D L_{n} A K_{1}$ admits $A V D T C$ and $\chi_{\text {avt }}\left(D L_{n} A K_{1}\right)=8$ for $\mathrm{n} \geq 3$.
Proof: By definition (1.6), we have $\left|V\left(D L_{n} A K_{1}\right)\right|=4 \mathrm{n}$ and $\left|E\left(D L_{n} A K_{1}\right)\right|=7 \mathrm{n}-4$. We have colored the graph using algorithm (2.6). Now the color classes for $\mathrm{n} \geq 3$,
$C\left(v_{1}\right)=\{1,3,5,7,8\}, C\left(u_{1}\right)=\{1,3,5,6,8\}, C\left(u^{\prime}{ }_{j}\right)=C\left(v^{\prime}{ }_{i}\right)=\{5,8\}$, for $1 \leq \mathrm{i} \leq \mathrm{n}$
$C\left(u_{n}\right)=\left\{\begin{array}{l}\{1,4,5,7,8\}, \text { if } n \equiv 0(\bmod 2) \\ \{2,3,5,7,8\}, \text { if } n \equiv 1(\bmod 2)\end{array}\right.$ and $C\left(v_{n}\right)=\left\{\begin{array}{l}\{2,3,5,6,8\}, \text { if } n \equiv 0(\bmod 2) \\ \{1,4,5,6,8\}, \text { if } n \equiv 1(\bmod 2)\end{array}\right.$
For $2 \leq \mathrm{i} \leq \mathrm{n}-1$
$\overline{\mathrm{C}}\left(\mathrm{u}_{\mathrm{i}}\right)=\left\{\begin{array}{l}\{3\}, \text { if } \mathrm{i} \equiv 0(\bmod 2) \\ \{4\}, \text { if } \mathrm{i} \equiv 1(\bmod 2)\end{array} \quad\right.$ and $\quad \overline{\mathrm{C}}\left(\mathrm{v}_{\mathrm{i}}\right)=\left\{\begin{array}{l}\{1\}, \text { if } \mathrm{i} \equiv 0(\bmod 2) \\ \{2\}, \text { if } \mathrm{i} \equiv 1(\bmod 2)\end{array}\right.$
Clearly, the color classes of any two adjacent vertices are different.
$\therefore \chi_{\text {avt }}\left(D L_{n} A K_{1}\right)=8, \mathrm{n} \geq 3$.
Remark: The adjacent vertex distinguishing total chromatic number of triangular snake and Quadrilateral snake family of graph has been obtained in the literature [6,7]. Also, this conjecture is true for the graphs $T_{n} A K_{1}, D T_{n} A K_{1}, Q_{n} A K_{1}$ and $D Q_{n} A K_{1}$.

## CONCLUSION

We found the adjacent vertex distinguishing total chromatic number of family of ladder graph such as ladder graph, triangular ladder and diagonal ladder. We are investigating to obtain the class of graph which admits adjacent vertex distinguishing total coloring.

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