International Journal of Mathematical Archive-9(2), 2018, 48-51
Available online through www.ijma.info ISSN 2229 – 5046

MATHEMATICAL MODELING OF GROUND WATER FLOW AND ANALYSIS BY ADOMAIN DECOMPOSITION METHOD

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(Received On: 18-12-17; Revised & Accepted On: 17-01-18)

ABSTRACTS

***I**n View of the Adomian decomposition method, we solve some models of nonlinear partial differential equations. Which givens ground water flow in porous media. We found the existence of exact solutions for those models. The numerical results show the efficiency and accuracy of this method.*

Keywords: *Adomian decomposition method, nonlinear partial differential equations.*

INTRODUCTION

Nonlinear partial differential equations can be found in wide variety scientific and engineering applications. Many important mathematical models can be expressed in terms of nonlinear partial differential equations. The most general form of nonlinear partial differential equation is given by:

$$F(u, u_t, u_x, u_y, x, y, t) = 0 \tag{1a}$$

With initial and boundary conditions

$$u(x, y, 0) = \emptyset(x, y), \forall x, y \in \Omega, \Omega \in R^2 \tag{1b}$$

$$u(x, y, t) = f(x, y, t), \forall x, y \in \partial\Omega \tag{1c}$$

where Ω is the solution region and $\partial\Omega$ is the boundary of Ω.

In recent years, much research has been focused on the numerical solution of nonlinear partial equations by using numerical methods and developing these methods (Al-Saif, 2007; Leveque, 2006; Rossler & Husner, 1997; Wescot & Rizwan-Uddin, 2001). In the numerical methods, which are commonly used for solving these kind of equations large size or difficult of computations is appeared and usually the round-off error causes the loss of accuracy.

The Adomian decomposition method which needs less computation was employed to solve many problems (Celik *et al.*, 2006; Javidi & Golbabai, 2007). Therefore, we applied the Adomian decomposition method to solve some models of nonlinear partial equation, this study reveals that the Adomian decomposition method is very efficient for nonlinear models, and it results give evidence that high accuracy can be achieved.

MATHEMATICAL METHODOLOGY

The principle of the Adomian decomposition method (ADM) when applied to a general nonlinear equation is in the following form (Celik *et al.*, 2006; Seng *et al.*, 1996):

$$\mathrm{Lu} + \mathrm{Ru} + \mathrm{Nu} = \mathrm{g} \tag{2}$$

The linear terms decomposed into *Lu*+*Ru*, while the nonlinear terms are represented by *Nu* , where *L* is an easily invertible linear operator, *R* is the remaining linear part. By inverse operator L, with $L^{-1}(\cdot) = \int_0^t (\cdot)dt$. Equation (2) can be hence as;

$$u = L^{-1}(g) - L^{-1}(Ru) - L^{-1}(Nu) \tag{3}$$

The decomposition method represents the solution of equation (3) as the following infinite series:

$$u = \sum_{n=0}^{\infty} u_n \tag{4}$$

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The nonlinear operator $Nu = \Psi(u)$ is decomposed as:

$$Nu = \sum_{n=0}^{\infty} A_n \tag{5}$$

where A_n are Adomian's polynomials, which are defined as (Seng et al., 1996):

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\psi\left(\sum_{i=1}^{n}\lambda^i u_i\right)\right]_{\lambda=0} \qquad n = 0,1,2,\dots \tag{6}$$

Substituting equations (4) and (5) into equation (3), we have

$$u = \sum_{n=0}^{\infty} u_n = u_0 - L^{-1}\left(R\left(\sum_{n=0}^{\infty} u_n\right)\right) - L^{-1}\left(\sum_{n=0}^{\infty} A_n\right) \tag{7}$$

Consequently, it can be written as:

$$\left.\begin{aligned} &u_0 = \varphi + L^{-1}(g) \\ &u_1 = -L^{-1}(R(u_0)) - L^{-1}(A_0) \\ &u_2 = -L^{-1}(R(u_1)) - L^{-1}(A_1) \\ &\quad \dots\dots \\ &u_n = -L^{-1}(R(u_{n-1})) - L^{-1}(A_{n-1}) \end{aligned}\right\} \tag{8}$$

where φ is the initial condition,

Hence all the terms of u are calculated and the general solution obtained according to

ADM as $u = \sum u_n$. The convergent of this series has been proved in (Seng et al., 1996). n=0

However, for some problems (Celik et al., 2006) this series can't be determined, so we use an approximation of the solution from truncated series

$$U_M = \sum_{n=0}^{M} u_n \; with \; \lim_{M\to\infty} U_M = u$$

STATEMENT OF THE PROBLEM

In the investigated mathematical model, we consider that groundwater recharge takes place over a large basin of such geological configuration that the sides are limited by rigid boundaries and the bottom by a thick layer of water table. In this case, the flow is assumed vertically downwards through unsaturated porous media.

It is assumed that the diffusivity co-efficient is equivalent to its average value over the whole range of moisture content, and the permeability of the media is continuous linear function of the moisture content. The theoretical formulation of the problem of the problem yields a nonlinear partial differential equation.

MATHEMATICAL MODELING

From klute [89], the equation of continuity for an unsaturated medium is given by

$$\frac{\partial}{\partial t}(\rho_s \theta) = -\nabla \cdot M \tag{9}$$

Where ρ_s the bulk density of the medium is, θ is its moisture content on a dry weight basis, and M is the mass flux of moisture.

From Darcy's law [1, 2, 3] for the motion of water in a porous medium, we get

$$\vec{V} = -k\nabla\varphi \tag{10}$$

Where $\nabla\varphi$ represent the gradient of the moisture potential, the volume flux of moisture, and k the co-efficient of aqueous conductivity.

Combining equation (9) and (10) we obtain,

$$\frac{\partial}{\partial t}(\rho_s \theta) = \nabla \cdot (\rho k \nabla\varphi) \tag{11}$$

Where M = $\rho\vec{V}$, ρ is the flux density.

Since in the present case, we consider that the flow takes places only in the vertical direction, equation (11) reduce to,

$$\rho_s \frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left(\rho k \frac{\partial\psi}{\partial z}\right) - \frac{\partial}{\partial z}(\rho k g) \tag{12}$$

Where ψ the capillary pressure potential, g is the gravitational constant and $\varphi = \psi - gz$ [5-9] the positive direction of the z-axis is the same as that of the gravity.

Considering θ and ψ to be connected by a single valued function, we may write (12) as,

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial\theta}{\partial z}\right) - \frac{\rho}{\rho_s}\frac{\partial k}{\partial z} \tag{13}$$

Where $D = \frac{\rho}{\rho_s} k \frac{\partial\psi}{\partial\theta}$ and is called diffusivity co-efficient.

Replacing D by average value D_a and assuming$k = k_0\theta$, we have

$$\frac{\partial\theta}{\partial t} = D_a\frac{\partial^2\theta}{\partial z^2} - \frac{\rho}{\rho_s}k_0\frac{\partial\theta}{\partial z} \qquad (14)$$

Considering the water table to be situated at a depth L. and putting:

$$\frac{z}{L} = \xi, \quad \frac{tD_a}{L^2} = T, \quad \beta = \frac{\rho}{\rho_s}\frac{k_0}{D_a}$$

Solution of the Problem:

Consider the Problem

$$\frac{\partial\theta}{\partial T} = \frac{\partial^2\theta}{\partial\xi^2} - \beta\frac{\partial\theta}{\partial\xi}$$

with the initial Condition

$$\theta(\xi, 0) = e^{-\xi}$$

Solution: In this problem, we have

$$N\theta = \Psi(\theta) = \beta\frac{\partial\theta}{\partial\xi}$$

$$g(\xi, t) = 0$$

$$R\theta = 0$$

$$L\theta = \frac{\partial\theta}{\partial T}$$

And $\phi = \theta(\xi, 0) = e^{-\xi}$

By using equation (6) Adomain’s polynomials can be derived as follows:

$$\left.\begin{aligned} A_0 &= -\beta_0\frac{\partial\theta_0}{\partial\xi} \\ A_1 &= -\beta_1\frac{\partial\theta_0}{\partial\xi} - \beta_0\frac{\partial\theta_1}{\partial\xi} \\ A_2 &= -\beta_2\frac{\partial\theta_0}{\partial\xi} - \beta_1\frac{\partial\theta_1}{\partial\xi} - \beta_0\frac{\partial\theta_2}{\partial\xi} \\ A_3 &= -\beta_3\frac{\partial\theta_0}{\partial\xi} - \beta_2\frac{\partial\theta_1}{\partial\xi} - \beta_1\frac{\partial\theta_2}{\partial\xi} - \beta_0\frac{\partial\theta_3}{\partial\xi} \\ &\vdots \end{aligned}\right\} \qquad (15)$$

And so on. The rest of the polynomials can be constructed in similar manner.

By using Equation (8), we have

$$\left.\begin{aligned} \theta_0 &= e^{-\xi} \\ \theta_1 &= e^{-\xi}\left(\frac{t}{1}\right) \\ \theta_2 &= e^{-\xi}\left(\frac{t}{1}\right)^2 \\ \theta_3 &= e^{-\xi}\left(\frac{t}{1}\right)^3 \\ \theta_4 &= e^{-\xi}\left(\frac{t}{1}\right)^4 \\ &\vdots \\ \theta_n &= e^{-\xi}\left(\frac{t}{1}\right)^n \end{aligned}\right\} \qquad (16)$$

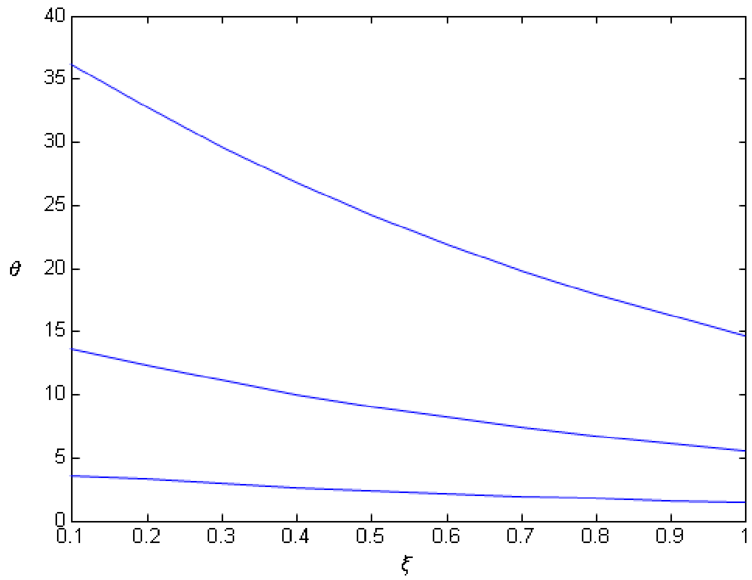
Substituting these individual terms in equation (4) obtain

$$\theta(\xi, t) = e^{-\xi}(1 + t + t^2 + t^3 + t^4 + \cdots + t^n)$$

This gives the exact solution.

Table1:

T	ξ	$\theta(\xi,t)$	t	ξ	$\theta(\xi,t)$	T	ξ	$\theta(\xi,t)$
1	0.1	3.6193	2	0.1	13.5726	3	0.1	**36.1935**
	0.2	3.2749		0.2	12.2810		0.2	**32.7492**
	0.3	2.9633		0.3	11.1123		0.3	**29.6327**
	0.4	2.6813		0.4	10.0548		0.4	**26.8128**
	0.5	2.4261		0.5	9.0980		0.5	**24.2612**
	0.6	2.1952		0.6	8.2322		0.6	**21.9525**
	0.7	1.9863		0.7	7.4488		0.7	**19.8634**
	0.8	1.7973		0.8	6.7399		0.8	**17.9732**
	0.9	1.6263		0.9	6.0985		0.9	**16.2628**
	1.0	1.4715		1.0	5.5182		1.0	**14.7152**



CONCLUSION

In this paper, we have applied the Adomian decomposition method for solving problem of nonlinear partial equations. We demonstrated that the decomposition procedure is quite efficient to determine the exact solutions. However, the method gives a simple powerful tool for obtaining the solutions without a need for large size of computations. It is also worth noting that the advantage of this method sometimes displays a fast convergence of the solutions. In addition, the numerical results which obtained by this method indicate a high degree of accuracy.

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Source of support: Nil, Conflict of interest: None Declared.