

INTEGRAL EDGE SUM OF  $\Gamma(Z_n)$

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ABSTRACT

The first simplification of Beck's [2] zero divisor graph was introduced by D.F.Anderson and P.S.Livingston[1]. Their motivation was to give a better illustration of the zero divisor structure of the ring. In this paper, we find the integral sum of zero divisor graph and study their properties.

**Keywords:** Integral Edge Sum, Zero divisor graph.

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1. INTRODUCTION

Let  $R$  be a commutative ring and let  $Z(R)$  be its set of zero-divisors. We associate a graph  $\Gamma(R)$  to  $R$  with vertices  $\Gamma(R)^* = Z(R) - \{0\}$ , the set of non-zero divisors of  $R$  and for distinct  $u, v \in Z(R)^*$ , the vertices  $u$  and  $v$  are adjacent if and only if  $uv = 0$ . The zero divisor graph is very useful to find the algebraic structures and properties of rings. The idea of a zero divisor graph of a commutative ring was introduced by I. Beck in [2]. The first simplification of Beck's zero divisor graph was introduced by D.F.Anderson and P.S.Livingston [1]. Their motivation was to give a better illustration of the zero divisor structure of the ring. D.F.Anderson and P.S.Livingston, and others e.g., [4, 5, 6], investigate the interplay between the graph theoretic properties of  $\Gamma(R)$  and the ring theoretic properties of  $R$ . Through this paper, we consider the commutative ring  $R$  by  $Z_n$  and zero divisor graph  $\Gamma(R)$  by  $\Gamma(Z_n)$ . Let  $\Gamma(Z_n)$  be a graph. A bijection  $f : E(\Gamma(Z_n)) \rightarrow Z^+$ , where  $Z^+$  is a set of positive integers is called an edge mapping of the graph  $\Gamma(Z_n)$ . Now, we define,  $F(v) = \sum \{f(e) : e \text{ is incident on } v \text{ on } V(\Gamma(Z_n))\}$ . Then,  $F$  is called the edge sum mapping of the edge mapping  $f$ .  $f : E(\Gamma(Z_n)) \rightarrow N^+$  such that  $f$  and its corresponding edge sum mapping  $F$  on  $V(\Gamma(Z_n))$  satisfy the following conditions: (i)  $F$  is into mapping to  $Z^+$ . That is,  $F(v) \in Z^+$ , for every  $v \in E(\Gamma(Z_n))$ . (ii) If  $e_1, e_2, \dots, e_n \in E(\Gamma(Z_n))$  such that  $f(e_1) + f(e_2) + \dots + f(e_n) \in Z^+$ , then  $e_1, e_2, \dots, e_n$  are incident on a vertex in  $\Gamma(Z_n)$ . The edge sum labeling was introduced by Paulraj Joseph et al., [3].

2. INTEGRAL EDGE SUM OF  $\Gamma(Z_n)$

In this section, we evaluate the integral edge sum of  $\Gamma(Z_n)$  and study their properties. The integral edge sum graph is defined as follows,

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**Definition 2.1:** Let  $\Gamma(Z_n)$  be a graph. A bijection  $f : E \rightarrow S$  where S is a set of integers is called as integral edge function of the graph  $\Gamma(Z_n)$ . Define  $f(v) = \sum \{f(e) : e \text{ is incident on } v\}$  on V. Then F is called the integral edge sum function of the integral edge function f.

$\Gamma(Z_n)$  is said to be an integral edge sum graph if there exists an integral edge function  $f : E \rightarrow S$  such that f and its corresponding integral edge sum function F on V satisfy the following conditions:

- i) F is into S. That is,  $f(v) \in S$  for every  $v \in V$ .
- ii) If  $e_1, e_2, \dots, e_n \in E$  such that  $f(e_1) + f(e_2) + \dots + f(e_n) \in S$ , then  $e_1, e_2, \dots, e_n$  are incident on a vertex.

**Definition 2.2:** If e is an edge of  $\Gamma(Z_n)$ , then the graph obtained by subdividing ‘e’ exactly once is denoted as G(e).

**Definition 2.3:** Let  $\Gamma(Z_n)$  be an integral edge sum graph. The integral edge function  $f : E \rightarrow S$  and its corresponding integral edge sum function F which make G an integral edge sum graph are called optimal integral edge function and optimal integral edge sum function respectively.

For example, Let  $V = \{v_1, v_2, \dots, v_{p-1}\}$  be the vertex set and  $E = \{vv_i : 1 \leq i \leq p-1\}$  be the edge set of the graph  $\Gamma(Z_{2p})$ , where p is any prime number, greater than two. Let  $S = \left\{0, 1, 2, \dots, (p-3), \frac{-(p-3)(p-2)}{2}\right\}$ . The integral edge function  $f : E \rightarrow S$  is defined as  $f(vv_i) = i$  for  $1 \leq i \leq (p-3)$  and  $f(vv_2) = \frac{-(p-3)(p-2)}{2}$ .

The corresponding integral edge sum function is,

$$\begin{aligned} f(v) &= 0 \\ f(v_i) &= i \text{ for } 1 \leq i \leq (p-3) \\ f(v_{p-2}) &= \frac{-(p-3)(p-2)}{2} \\ f(v_{p-1}) &= 0 \end{aligned}$$

Therefore, F is into S. As all the edges are incident on the second condition becomes trivial. Hence  $\Gamma(Z_{2p})$  is an integral edge graph for every natural number  $p-1$ .

For example, In  $\Gamma(Z_{2p})(e)$ ,  $V = \{v_1, v_2, \dots, v_{p-2}, u, v, w\}$  is the vertex set and  $E = \{uv, uw\} \cup \{vv_i : 1 \leq i \leq (p-2)\}$  is the edge set of  $\Gamma(Z_{2p})$ .

$S = \left\{0, 1, 2, \dots, (p-3), \frac{(p-1)(p-2)}{2}\right\}$ . The integral edge sum function  $f : E \rightarrow S$  is defined as  $f(uv) = 0$

$$\begin{aligned} f(uw) &= \frac{(p-1)(p-2)}{2} \\ f(vv_i) &= i, \quad 1 \leq i \leq (p-2). \end{aligned}$$

The corresponding integral edge sum function is as follows:

$$\begin{aligned} f(u) = f(v) = f(w) &= \frac{(p-1)(p-2)}{2} \\ f(v_i) &= i, \quad 1 \leq i \leq (p-2). \end{aligned}$$

Except uw all the other edges are incident on v. The value  $f(uw) = \frac{(p-1)(p-2)}{2}$  can be got by summing either  $f(vv_1), f(vv_2), \dots$ , and  $f(vv_n)$  (or)  $f(uv), f(vv_1), \dots, f(vv_n)$ . In both the cases the corresponding edges are incident on v. Hence,  $\Gamma(Z_{2p})(e)$  is an integral edge sum graph.

In both the examples 0 is one among the labels. But not all the integral edge sum graphs admit 0 as a label.

**Definition 2.4:** An integral edge sum graph  $\Gamma(Z_n)$  is called a zero integral sum graph if there exists an optimal integral edge function  $f : E \rightarrow S$  with  $0 \in S$ . We have proved that  $\Gamma(Z_{2p})$  or  $\Gamma(Z_{2p})(e)$ , where p is any prime number, are zero integral edge sum graphs. In the following theorem, we prove that there is no other zero integral edge sum graph.

**Theorem 2.5:** Let G be an integral edge sum graph without isolated vertices. Then G is a zero integral edge sum graph  $\Gamma(Z_{2p})$  or  $\Gamma(Z_{2p})(e)$  for some prime number 'p'.

**Proof:** Let  $f : E \rightarrow S$  be an optimal integral edge function and F be its corresponding optimal integral edge sum function. Let  $e = uv$  be such that  $f(e) = 0$ . If  $e_1$  is any other edge, then  $f(e_1) = f(e_1) + f(e) \in S$  and therefore e and  $e_1$  are incident on a vertex. That is, all the other edges are adjacent to e.

If w is any other then w and v, then  $F(w) = F(w) + f(0) \in S$ . Therefore if  $w_1, w_2, \dots, w_r$  are the vertices adjacent to w, then  $ww_1, ww_2, \dots, ww_r$  and e are adjacent on a vertex. The only possibility is that w is a pendent vertex adjacent on a vertex. The only possibility is that w is a pendant vertex adjacent to either u or v.

Suppose  $\deg u > 1$ . Let  $u_1, u_2, \dots, u_m$  be the pendent vertices adjacent to u apart from v and  $v_1, v_2, \dots, v_{p-1}$  be the pendent vertices adjacent to v apart from u. Then  $F(u) = f(uu_1) + f(uu_2) + \dots + f(uu_m) + f(uv) \in S$ . Let  $F(u) = f(e_1)$ . If  $e_1 = uu_i$  for some i (say  $uu_1$ ), then  $F(u) = f(uu_1) = f(uu_1) + f(uu_2) + \dots + f(uu_m)$ . That is,  $f(uu_2) + \dots + f(uu_m) = 0$ . Hence  $F(v) = F(v) + f(uu_2) + \dots + f(uu_m) \in S$ . That is,  $vv_1, vv_2, \dots, vv_{p-1}, uu_1, uu_2, \dots, uu_m$  and  $uv$  are incident on a vertex. That is positive only if  $m = 0$ .

If  $e_1 = e = uv$ , then  $F(u) = 0$  and  $F(u) + F(v) \in S$ . Again this is a possibility only if v is not adjacent to any other than u. Here again  $m = 0$ .

Let us consider the last case. Let  $e_1 = vv_i$  for some i. assuming  $e_1 = vv_1$ , we get the following:  $F(v) = f(vv_1) + f(vv_2) + \dots + f(vv_{p-1}) = f(uu_1) + f(uu_2) + \dots + f(uu_m) + f(vv_2) + f(vv_3) + \dots + f(vv_{p-1})$  is an element of S. This is not possible if  $p - 1 \geq 2$ , where p is any prime greater than 2.

Hence we have the following:

- (i) e is adjacent to all other edges of  $\Gamma(Z_n)$ .
- (ii) Any vertex other than u and v is a pendent vertex adjacent to either u or v.
- (iii) If  $\deg u > 1$ , then  $\deg v = 1$ . Thus G is either  $\Gamma(Z_{2p})$  or  $\Gamma(Z_{2p})(e)$  for some  $p - 1$ .

In all the other integral edge sum graphs zero is not among the edge labels. In a similar way it is easily seen that zero sum of edge labels is also not possible in the nonzero integral edge sum graphs. That is, if f is an optimal integral edge sum graph  $\Gamma(Z_{2n})$ , then  $f(e_1) + f(e_2) + \dots + f(e_n)$  is not zero for any sub collection  $\{e_1, e_2, \dots, e_n\}$  of the edge set  $\Gamma(Z_{2n})$ .

The optimal edge function and its corresponding optimal edge sum function of an edge sum graph and the optimal integral edge function and its corresponding optimal integral edge sum of a nonzero integral edge sum graph act almost similarly. We state the following observations concerning non zero integral edge sum of  $\Gamma(Z_n)$  without proof, as the result and its proof are similar to that of edge sum graphs in each case.

**Observation 1:** Let  $\Gamma(Z_n)$  be a nonzero integral edge sum graph with integral edge function  $f : E \rightarrow S$  and the integral edge sum function F of f. Let  $l_1, l_2, \dots, l_m$  where  $m > l$  be a collection of edges incident on a vertex w (say).

Let  $ww_i = l_i$  for  $l \leq i \leq m$ . If there exists an edge  $e = uv$  such that  $f(l_1) + f(l_2) + \dots + f(l_m) = f(e)$ , then one of the following holds:

- (i)  $\{u, v\}$  forms a  $\Gamma(Z_9)$  component in  $\Gamma(Z_n)$ .
- (ii)  $\{u, v, w\}$  is either  $\Gamma(Z_4)$  or  $\Gamma(Z_9)$  with one of  $u, v$  as a pendent vertex in  $\Gamma(Z_n)$ .

**Observation 2:**  $\Gamma(Z_n)$  be a nonzero integral edge sum graph with integral edge function  $f : E \rightarrow S$  and the integral edge sum function  $F$  of  $f$ . let  $w$  be a non pendent vertex and  $e = uv \in E$  be such that  $F(w) = f(e)$ . Then one of the following holds:

- (i)  $\{u, v\}$  forms a  $\Gamma(Z_9)$  component in  $\Gamma(Z_n)$ .
- (ii)  $\{u, v, w\}$  is either  $\Gamma(Z_9)$  with one of  $u, v$  as a pendent vertex in  $\Gamma(Z_n)$ .

**Remark:** Let  $f : E \rightarrow S$  be an optimal edge function and  $F$  be its corresponding optimal edge sum graph  $\Gamma(Z_n)$ . If  $u$  is a non pendent vertex of  $\Gamma(Z_n)$ , then there exist vertices  $v, w$  such that  $F(u) = f(vw)$ . If  $vw$  is not a  $\Gamma(Z_9)$  component then either one of them is adjacent to  $u$ . Let  $v$  be the adjacent to  $u$ . Then  $F(v) \geq f(u) + f(vw) > F(u) > f(e)$  for every edge incident on  $u$ . Hence if  $F(v) = f(e_1)$ , then  $e_1$  is incident on  $v$  and  $F(u) = f(e_2)$ , then  $e_2$  is not incident on  $u$ . But this is not the case in integral edge sum graphs. We will see it in proving complete graph in  $\Gamma(Z_n)$  is integral edge sum graph.

**Definition 2.6:** Let  $V = \{v_1, v_2, \dots, v_{p-1}\}$  be the vertex set of  $\Gamma(Z_{p^2})$  for  $p \geq 5$  is any prime number. The integral edge function matrix  $A = (a_{i,j})$  of order  $p-1$  of the integral edge function  $f$  of  $\Gamma(Z_{p^2})$  is defined as  $a_{i,j} = f(v_i v_j)$  if  $i \neq j$  and is 0 if  $i = j$ . The integral edge function matrix is a symmetric matrix.

**Theorem 2.7:**  $\Gamma(Z_{p^2})$  for  $p \geq 5$  is an integral edge sum graph, where  $p$  is a prime number.

**Proof:** Let  $V = \{v_1, v_2, \dots, v_{p-1}\}$  be the vertex set of  $\Gamma(Z_{p^2})$  for  $p \geq 5$  is any prime number and  $E = \{v_i v_j : 1 \leq i \leq p-2 \text{ and } i+1 \leq j \leq p-1\}$  be the edge set of  $\Gamma(Z_{p^2})$ . For defining the integral edge function matrix we need the following definitions.

Let  $B = (b_{i,j})$  be a  $(p-5) \times (p-1)$  matrix defined as  $b_0 = 20$ ,  $b_{i,j} = b_{j,i}$  for  $1 \leq i, j \leq p-5$ ,  $b_{i,i+1} = b_{i-1} = \sum \{b_{i-1,j} : 1 \leq i \leq p-5\}$  and  $b_{i,j} = \sum \{2^{j-i} b_{i-1} : 1 \leq i \leq p-5 \text{ and } (i+2) \leq j \leq p-1\}$ .

$$\begin{aligned} \text{Let } d_{p-1} &= [(b_{1,p-1} - b_{1,2}) + (b_{2,p-1} - b_{2,3}) + \dots + (b_{p-5,p-1} - b_{p-5,p-4})] \\ d_{p-2} &= [(b_{1,p-2} - b_{1,2}) + (b_{2,p-2} - b_{2,3}) + \dots + (b_{p-5,p-2} - b_{p-5,p-4})] \\ d_{p-3} &= [(b_{1,p-3} - b_{1,2}) + (b_{2,p-3} - b_{2,3}) + \dots + (b_{p-5,p-3} - b_{p-5,p-4})] \\ d_{p-4} &= [(b_{1,p-4} - b_{1,2}) + (b_{2,p-4} - b_{2,3}) + \dots + (b_{p-5,p-4} - b_{p-5,p-4})] \end{aligned}$$

Since  $b_{i,i+1} = b_{i-1}$  for  $1 \leq i \leq p-5$ , we get that

$$\begin{aligned} d_j &= (b_{1,j} + b_{2,j} + \dots + b_{p-5,j}) - (b_{1,2}) + (b_{2,3} + \dots + b_{p-5,p-4}) \\ &= \sum \{b_{i,j} : 1 \leq i \leq p-5\} - \sum \{b_{i-1} : 1 \leq i \leq p-5\} \\ &= \sum \{b_{i,j} : 1 \leq i \leq p-5\} - B_{p-6} \end{aligned}$$

Where,  $B_j = \sum \{b_i : 1 \leq i \leq j\}$  for  $p-4 \leq j \leq p-1$ .

Let  $h_0$  be a content satisfying  $h_0 > (d_{p-1} - d_{p-2} + 2 - b_0)$ ;  $h_0 \equiv 0 \pmod{20}$  and  $h_0 > B_{p-6}$ .

Let  $H = (h_{i,j})$  be a  $(p-5) \times (p-1)$  matrix defined as  $h_{i,j} = h_0$  for  $2 \leq j \leq p-1$  and  $h_{i,j} = h_{i-1} = \sum \{h_{i-1,k} : 1 \leq k \leq p-1\}$  for  $2 \leq i \leq p-5$  and  $i+1 \leq j \leq p-1$  where  $h_{j,i} = h_{i,j}$  for  $1 \leq i \leq p-5$  and  $1 \leq j \leq p-5$ . Let  $x_i = b_i + h_i$  for  $0 \leq i \leq p-6$ .

The integral edge function matrix  $A = (a_{i,j})$  of order  $p-1$  is defined as,

$$a_{i,j} = b_{i,j} + h_{i,j} \text{ for } 1 \leq i \leq p-5 \text{ and } i+1 \leq j \leq p-1.$$

$$a_{p-4,p-3} = x_{p-5} = \sum \{a_{p-5,j} : 1 \leq j \leq p-1\}$$

$$a_{p-4,p-2} = -y$$

$$a_{p-4,p-1} = x_{p-5} + a$$

$$a_{p-3,p-2} = x_{p-4} = \sum \{a_{p-4,j} : 1 \leq j \leq p-1\}$$

$$a_{p-3,p-1} = -z$$

$$a_{p-2,p-1} = x_{p-3} = \sum \{a_{p-3,j} : 1 \leq j \leq p-1\}$$

Where a, z, y are defined as follows.

$$\text{Let } X_i = \sum \{x_j : 0 \leq j \leq i\}$$

$$y = 3X_{p-5} - 2x_{p-4} + d_{p-1} + d_{p-3} + d_{p-4} + 4$$

$$z = 2y - X_{p-5} + d_{p-1} - d_{p-2} + d_{p-4} + 4 \text{ and}$$

$$2a = 5(y - X_{p-5}) + d_{p-1} - 2d_{p-2} - d_{p-3} - 3d_{p-4}$$

$$\begin{aligned} \text{We have, } d_j &= \sum \{b_{i,j} : 1 \leq i \leq p-5\} - B_{p-6} \text{ for } p-4 \leq j \leq p-1 \\ &= \sum \{(b_{i,j} + h_{i,j}) : 1 \leq i \leq p-5\} - \sum \{(b_{i-1} + h_{i,j}) : 1 \leq i \leq p-5\} \\ &= \sum \{a_{i,j} : 1 \leq i \leq p-5\} - \sum \{(b_{i-1} + h_{i,j}) : 1 \leq i \leq p-5\} \\ &= \sum \{a_{i,j} : 1 \leq i \leq p-5\} - \sum \{x_{i-1} : 1 \leq i \leq p-5\} \\ &= \sum \{a_{i,j} : 1 \leq i \leq p-5\} - X_{p-6} \end{aligned}$$

$$\text{Let } S = \{a_{i,j} : 1 \leq i \leq p-2 \text{ and } i+1 \leq j \leq p-1\}.$$

The edge function  $f : E \rightarrow S$  is defined  $i+1 \leq j \leq p-1$ . The corresponding integral edge sum function F is as follows:

$$\begin{aligned} F(v_i) &= \sum \{f(v_i v_j) : 1 \leq j \leq p-1 \text{ and } j \neq i\} \text{ for } 1 \leq i \leq p-3. \\ &= \sum \{a_{i,j} : 1 \leq j \leq p-1 \text{ and } j \neq i\} \\ &= \sum \{(b_{i-1} + h_{i,j}) : 1 \leq j \leq p-1\} \\ &= b_i + h_i \\ &= x_i \end{aligned}$$

$$\begin{aligned} F(v_{p-2}) &= \sum \{f(v_{p-2} v_j) : 1 \leq j \leq p-1 \text{ and } j \neq p-2\} \\ &= \sum \{a_{p-2,j} : 1 \leq j \leq p-1 \text{ and } j \neq p-2\} \\ &= \sum \{a_{p-2,j} : 1 \leq j \leq p-5\} + a_{p-2,p-4} + X_{p-6} - y + x_{p-4} + x_{p-3} \\ &= x_{p-5} \\ &= a_{p-4,p-3} \\ &= f(v_{p-4} v_{p-3}) \text{ and} \end{aligned}$$

$$\begin{aligned}
 F(v_{p-1}) &= \sum \{f(v_{p-1}v_j) : 1 \leq j \leq p-2\} \\
 &= \sum \{a_{p-1,j} : 1 \leq j \leq p-2\} \\
 &= \sum \{a_{p-1,j} : 1 \leq j \leq p-5\} + a_{p-1,p-4} + a_{p-1,p-3} + a_{p-1,p-2} \\
 &= d_{p-1} + X_{p-6} + x_{p-5} + a - z + x_{p-3} = x_{p-5} \\
 &= a_{p-4,p-3} \\
 &= f(v_{p-4}v_{p-3})
 \end{aligned}$$

Hence, F is into S. Let  $S_1 = \{a_{i,j} : 1 \leq i \leq p-5 \text{ and } i+1 \leq j \leq p-1\}$  and

$S_2 = \{a_{i,j} : p-4 \leq i \leq p-2 \text{ and } i+1 \leq j \leq p-1\}$ . The elements of  $S_1$  satisfy the following properties:

- (i)  $a_{i,j} = b_{i,j} + h_{i,j}$
- (ii)  $b_{i,i+1} = \sum \{b_{i-1,j} : 1 \leq i \leq p-5\}$
- (iii)  $b_{i,j} > (b_{1,2} + b_{1,3} + \dots + b_{1,p-1}) + (b_{2,3} + b_{2,4} + \dots + b_{2,p-1}) + \dots$   
 $+ (b_{i-1,i} + b_{i-1,i+1} + \dots + b_{i-1,p-1}) + (b_{i,i+1} + b_{i,i+2} + \dots + b_{i,p-1})$
- (iv)  $h_{i,j} = h_0$  for  $2 \leq j \leq p-1$  and  $h_{i,j} = h_{i-1} = \sum \{h_{i-1,k} : 1 \leq k \leq p-1\}$  for  $2 \leq j \leq p-5$  and  $i+1 \leq j \leq p-1$ .
- (v) All the elements of  $S_1$  are congruent to  $0 \pmod{20}$ . Hence, no element of  $S_1$  except  $x_i$  for  $1 \leq j \leq p-6$  is a sum of two or more elements of S.

The elements of  $S_2$  satisfy the following properties:

- (i)  $a_{p-4,p-3} = x_{p-5} = \sum \{a_{p-5,j} : 1 \leq i \leq p-1\}$
- (ii)  $y = 3X_{p-5} - 2x_{p-5} + d_{p-1} + d_{p-3} + d_{p-4} + 4 \equiv 4 \pmod{20}$ .  
Hence,  $a_{p-3,p-2} = -y \equiv 10 \pmod{20}$ .
- (iii)  $a \equiv 10 \pmod{20}$  and hence  $x_{p-4,p-1} = x_{p-5} + a \equiv 10 \pmod{20}$ .
- (iv)  $a_{p-3,p-2} = x_{p-4} \equiv 6 \pmod{20}$ .
- (v)  $z \equiv 8 \pmod{20}$  and hence  $a_{p-3,p-1} = -z \equiv 12 \pmod{20}$ .
- (vi)  $x_{p-3} \equiv 8 \pmod{20}$ .
- (vii)  $x_{p-5} + a - z = 2$ .
- (viii)  $x_{p-4} - y = d_{p-1} - d_{p-2} + 2$ .
- (ix)  $x_0 = b_0 + h_0$   
 $> \{d_{p-1} - d_{p-2} + 2\}$   
 $= x_{p-4} - y$ .

Hence, no element of  $S_2$  except  $x_{p-3}, x_{p-4}$  and  $x_{p-5}$  are a sum of two or more elements of S. Thus, we get that if  $f(e_1) + f(e_2) + \dots + f(e_n) \in S$ , then  $e_1, e_2, \dots, e_n$  from the set of all edges incident on a vertex. Hence,  $\Gamma(Z_{p^2})$  is an integral edge sum graph for all prime  $p \geq 5$ .

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