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ON NEAR SKOLEM DIFFERENCE MEAN GRAPHS

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ABSTRACT

Let G be a (p,q)graph and $f: V(G) \rightarrow \{1,2,\dots,p+q-1,p+q+2\}$ be an injection. For each edge $e = uv$, the induced edge labeling f^* is defined as follows:

$$f^*(e) = \begin{cases} \dfrac{|f(u)-f(v)|}{2} & if\ |f(u)-f(v)|\ is\ even \\ \dfrac{|f(u)-f(v)|+1}{2} & if\ |f(u)-f(v)|\ is\ odd \end{cases}$$

Then f is called Near Skolem difference mean labeling if $f^*(e)$ are all distinct and are from $\{1,2,3,\dots.q\}$. A graph that admits a Near Skolem difference mean labeling is called a Near Skolem difference mean graph. In this paper, we investigate some properties of Near Skolem Difference Mean labelling and show that the path $P_n$and the star $K_{1,n}$ are Near Skolem Difference mean graphs whereas the graphs $K_{m,n}$, W_n and $T_n \odot K_1$ are Near Skolem Difference Mean only for certain cases.

Key words: *Near Skolem difference mean labeling, Near Skolem difference mean graphs.*

1. INTRODUCTION

In this paper, we consider only finite, simple connected and undirected graph. The vertex set and the edge set of G are denoted by $V(G)$ and $E(G)$ respectively. Terms and notations not defined here are used in the sense of Harary [1].

A graph labeling is an assignment of integers to the vertices or edges or both vertices and edges subject to certain conditions. A brief account of comprehensive bibliography of papers on graph labelling is found in Gallian survey [2]. The notion of Skolem difference mean labeling was due to Murugan and Subramanian [3]. It motivates us to define near skolem difference mean labelling.

In this paper, Near Skolem difference mean labelling has been defined and it has been established that the path P_n and the star $K_{1,n}$ are Near Skolem Difference mean graphs whereas the complete bi-partite graph $K_{m,n}$, the wheel graph W_n and the graph $(T_n \odot K_1)$ are Near Skolem difference mean graphs only for certain cases. Some properties on Near Skolem difference mean labelling have also been derived. The following definition is used in the subsequent section.

Definition 1.1: Let $u_1, u_2, \dots, u_n$ be a path of length n. Let $v_i, 1 \leq i \leq n-1$, be the new vertex joined to u_i and u_{i+1}. The resulting graph is called T_n. Let x_i be the vertex which is joined to $u_i, 1 \leq i \leq n$. Let y_i be the vertex which is joined to $v_i, 1 \leq i \leq n-1$. The resulting graph is $T_n \odot K_1$.

Result 1.2: [5]C_p is Near Skolem Difference Mean for $p \geq 3$.

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2. MAIN RESULT

Definition 2.1 A graph $G = (V, E)$ with p vertices and q edges is said to have Nearly Skolem Difference Mean labeling if it is possible to label the vertices x∈V with distinct elements $f(x)$ from $\{1,2,..,p+q-1,p+q+2\}$ in such a way that each edge e = uv , is labeled as $f^*(\mathrm{e}) = \frac{|f(u)-f(v)|}{2}$ if $|f(u)-f(v)|$ is even and $f^*(\mathrm{e}) = \frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd. The resulting labels of the edges are distinct and from $\{1, 2, \dots \dots, q\}$. A graph that admits a Near skolem difference mean labeling is called a Near skolem difference mean graph.

Theorem 2.2: *If G is Near skolem diference mean graph, then $p \geq q-2$.*

Proof: Consider the graph $G = (V, E)$ with p vertices and q edges.

Suppose, G is Near skolem difference mean graph.

Assume, $p < q-2$.

Let $f: V(G) \to \{1,2,.,p+q-1,p+q+2\}$ be a Near skolem difference mean labeling of G.
Then $f^*(E(G)) = \{1,2,\dots,q\}$.

Let $e = uv$ be any edge of G such that, $f(u) < f(v)$.
Then $1 \leq f(u) < f(v) \leq p+q+2$,----------------(1)

Now, two cases arise:

Case-(i): $\frac{|f(v)-f(u)|+1}{2} = q$.

This implies $f(v) - f(u) = 2q-1$.

Hence, $f(v) = 2q - 1 + f(u)$ ---------------- (2)

Now, $p < q-2$

This implies $p+2 < q$
$\Rightarrow p+q+2 < 2q$
$\Rightarrow f(v) \leq p+q+2 < 2q.$ [From (1)]
$\Rightarrow 2q-1+f(u) < 2q.$ [Using (2)]
$\Rightarrow f(u) < 1.$
This is not possible.

Case-(ii): $\frac{|f(v)-f(u)|}{2} = q$.
This implies $f(v) = 2q + f(u)$------------------(3)

Now, $p < q-2$.
$\Rightarrow p+2 < q$
$\Rightarrow p+q+2 < 2q$
$\Rightarrow f(v) < 2q$ [From (1)]
$\Rightarrow 2q + f(u) < 2q$ [Using (2)]
$\Rightarrow f(v) < 0.$
This is also not possible.

Hence from both cases, it can be concluded that if G is Near skolem difference mean then
$p \geq q-2$.

Theorem 2.3: *The path P_n is a Near skolem difference mean graph for every $n \geq 2$.*

Proof: Let $V(P_n) = \{v_i / 1 \leq i \leq n\}$ and
$E(P_n) = \{v_i v_{i+1} / 1 \leq i \leq n-1\}$
Then $|V(P_n)| = n$ and $|E(P_n)| = n-1$

Define $f: V(P_n) \to \{1,2,\dots\dots\dots\dots\dots,2n-2,2n+1\}$ as follows:

***Case-(i):n* is odd**

$f(v_{2i+1}) = 3 + 2i\,, \qquad 0 \le i \le \frac{n-1}{2}$

$f(v_2) \quad = 2n + 1$

$f(v_{2i}) \quad = 2n + 2 - 2i\,, \qquad 2 \le i \le \frac{n-1}{2}$

***Case-(ii):n* is even**

$f(v_{2i+1}) = 3 + 2i, \qquad 0 \le i \le \frac{n-2}{2}$

$f(v_2) \quad = 2n + 1$

$f(v_{2i}) \quad = 2n + 2 - 2i, \qquad 2 \le i \le \frac{n}{2}$

In both cases, let f^* be the induced edge labeling of f. Then,
$f^*(v_i v_{i+1}) = n - i, \qquad 1 \le i \le n - 1$
Clearly the induced edge labels $f^*(E(G)) = \{1,2, \dots \dots \dots .. n - 1\}$ are all distinct.
Hence, the graph P_n admits Near skolem difference mean labeling for $n \ge 2$.

Example 2.4: *The Near skolem difference mean labeling patterns of the paths P_7 and P_8 are as shown in fig 1 and fig 2 respectively.*

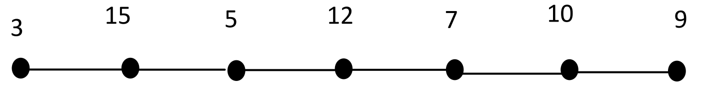

Figure-1

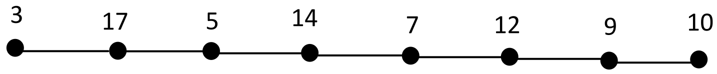

Figure-2

Theorem 2.5: *The graph $K_{1,n}$ is Near skolem difference mean graph for every n ≥ 1.*

Proof: Let $G = K_{1,n}$

Let $V(G) = \{u, u_i / 1 \le i \le n\}$ and
$E(G) = \{uu_i / 1 \le i \le n\}$.
Then $|V(G)| = n + 1$ and $|E(G)| = n$

Define $f: V(G) \to \{1, 2, \dots, 2n, 2n + 3\}$ as follows:
$f(u) = 2n$
$f(u_i) = 2i - 1, \qquad 1 \le i \le n.$

Let f^* be the induced edge labeling of f. Then,
$f^*(u\, u_i) = n + 1 - i, \qquad 1 \le i \le n.$

The induced edge labels are all distinct and are $f^*(E\,(G)) = \{1, 2, \dots, n\}$.

Hence, from the above labeling pattern, the graph $K_{1,n}$ admits Near skolem difference mean labelling for every n ≥ 1.

Example 2.6: *The following fig 3 shows the Near skolem difference mean labeling pattern of the star graph $K_{1,8}$.*

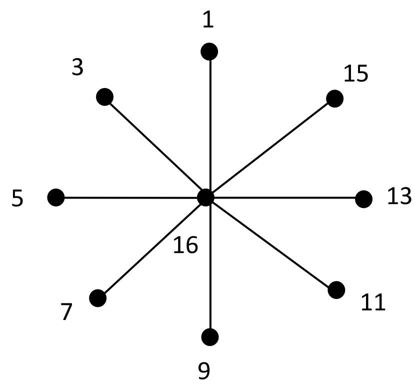

Figure-3

Theorem 2.7: *If G is a non- Near skolem difference $r - regular$ graph, then* $p > \frac{4}{r-2}, r \geq 3$.

Proof: Let G be a non –Near skolem difference mean graph. Then $p < q - 2$.

Also, for a $r - regular$ graph, $q = \frac{pr}{2}$.

Now, $p < q - 2$
i.e., $p < \frac{pr}{2} - 2$.
$2p < pr - 4$.
$2p - pr < -4$.
$-p(r-2) < -4$.
$p(r-2) > 4$.
Hence, $p > \frac{4}{r-2}$, $r \geq 3$.

Corollary 2.8: *For a $r - regular$ graph, if $r \geq 4$, then G is not Near skolem difference mean graph.*

Proof: Let G be a $r - regular$ graph, $r \geq 4$ and G is a Near skolem difference mean graph. Then, $q \leq p + 2$.

For a $r - regular$ graph, $q = \frac{pr}{2}$.

Therefore, $\frac{pr}{2} \leq p + 2$
$pr \leq 2p + 4$.
$p(r-2) \leq 4$.
$p \leq \frac{4}{r-2}$ ------------(1)
As G is $r - regular$, if $r \geq 4$, then $p \geq 5$.

From (1), it is not possible.

Hence, the $r - regular$ graph is non-Near skolem difference mean when$r \geq 4$.

Corollary 2.9: *Let $\delta \geq 3$. Then G is Near skolem difference mean if and only if $G \cong K_4$.*

Proof: If $\delta \geq 3$ $then$ $p \geq 4$.

Now, $q = \frac{1}{2}\sum deg\ v$.
$\geq \frac{3p}{2}$.
Then $2q \geq 3p$ -----------------(1)

If G is Near skolem difference mean graph, then $p \geq q - 2$---------------(2)

Combining (1) and (2), we get $p \leq 4$.

Now, $p \leq 4$ and $\delta \geq 3$ implies $p = 4$ and $\delta = 3$.

Hence $G \cong K_4$.

Conversely, if $G \cong K_4$, then from fig 4, G is Near Skolem Difference Mean.

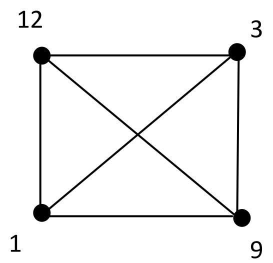

Figure-4

Corollary 2.10: *A r −regular graph is Near Skolem Difference Mean if and only if* $G \cong K_4$ *or* C_p *or* K_2.

Proof: Let G be a r −regular graph.

If G is Near Skolem Difference Mean graph then by Corollary 2.8 and Corollary 2.9, $G \cong K_4$ or C_p or K_2.

Conversely if $G \cong K_4$ or C_p or K_2, then from fig 4, Result 1.2 and fig 5, it follows that G is a Near Skolem Difference Mean Graph.

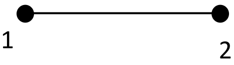

Figure-5

Corollary 2.11: *The complete graph* K_n *is Near skolem difference mean if and only if* $n \leq 4$.

Proof: It follows from Corollary 2.10.

Theorem 2.12: *The complete bipartite graph* $K_{m,n}$ *is Near skolem difference mean if and only if* $(m,n) = (1,n), (2,2), (2,3)$ and $(2,4)$.

Proof: Consider $K_{m,n}$ with $(m,n) = (1,n)$ and $(2,k), k \leq 4$.

When $m = 1$, it is a star which is a Nearly skolem difference mean graph [by Theorem 2.5].

When $m = 2$ and $n = 2,3,4$ the graphs are $K_{2,2}$, $K_{2,3}$ and $K_{2,4}$ which are Near skolem difference mean graphs as shown in the fig 6, fig 7 and fig 8.

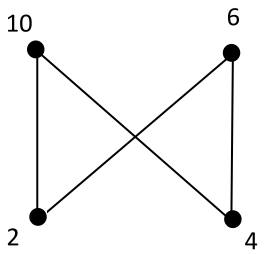

Figure-6

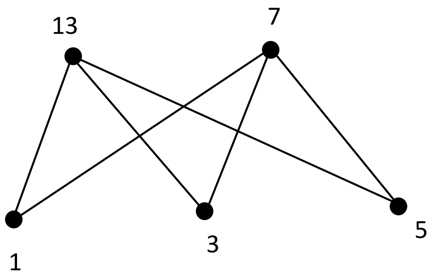

Figure-7

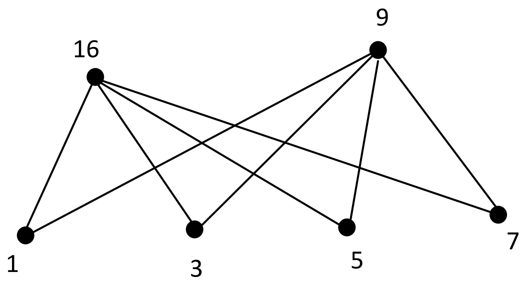

Figure-8

Conversely, suppose $K_{m,n}$ is Near Skolem Difference Mean graph.

Then $p \geq q - 2$.

That is $m + n \geq mn - 2$

Assume $m \leq n$.

Then solution for the inequality is $(m,n) = (1,n)$ or $(m,n) = (2,k), 2 \leq k \leq 4$.

Theorem 2.13: *The wheel graph* W_n *is not Near skolem difference mean, for* $n > 3$.

Proof: For $n = 3$, the graph is $W_3 = K_1 + C_3$. The Near skolem difference mean labeling of W_3 is given in the following fig 9.

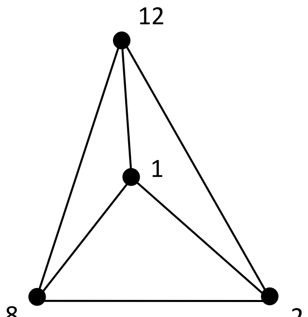

Figure-9

Suppose $W_n = K_1 + C_n$ is Near Skolem Difference Mean graph for $n > 3$.
Then $|V(W_n)| = n + 1$ and $|E(W_n)| = 2n$

Define $f: V(W_n) \to \{1,2, \ldots ,3n, 3n + 3\}$ as follows:

Let $e = uv$ be any edge of W_n, with$1 \leq f(u) < f(v) \leq 3n + 3$.

Suppose, $f^*(uv) = 2n$.
Then two cases arise:

Case-(i): $\frac{|f(v)-f(u)|}{2} = 2n$.
Then $f(v) = 4n + f(u)$
$\geq 4n + 1$

Case-(ii): $\frac{|f(v)-f(u)|+1}{2} = 2n$
Then $f(v) = 4n + 1 + f(u)$
$\geq 4n+2$

In both the cases, we have $f(v) \geq 4n + 1 \geq 3n + 4 \text{ as } n \geq 3$.

But by definition, $f(v) \leq 3n + 3$.

This is not possible.

Hence, the wheel graph $W_n$is not a Near skolem difference mean graph when $n > 3$.

Theorem 2.14: *The graph* $T_n \odot K_1$*is Near skolem difference mean if and only if* $n \leq 4$.

Proof: Let $G = T_n \odot K_1$.

Let $n \leq 4$.

When $n = 2,3 \text{ and } 4$ then, from fig 4, fig 8, fig 9 and fig 10, $T_n \odot K_1$ is Near Skolem Difference Mean graph.
Hence, for $n \leq 4$, the graph $T_n \odot K_1$ is Near skolem difference mean.

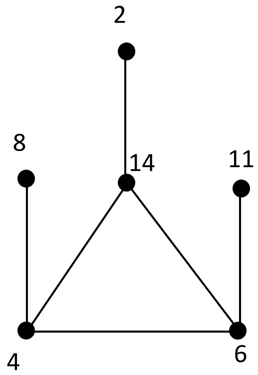

Figure-10

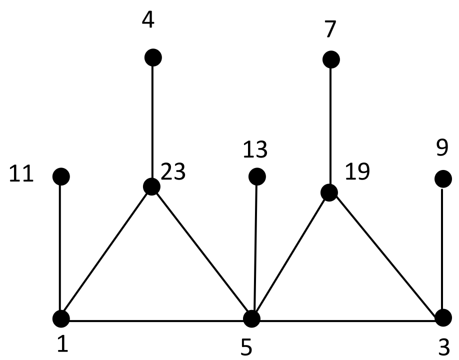

Figure-11

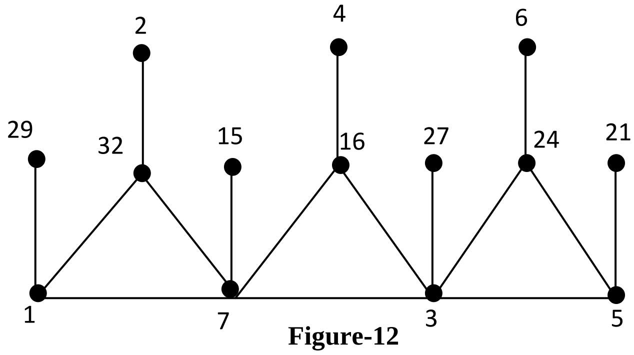

Figure-12

Conversely, let $T_n \odot K_1$ be a Near skolem difference mean graph.

Assume $n > 4$.

Let $V(G) = \{u_i, x_i, v_j, y_j / 1 \leq i \leq n, 1 \leq j \leq n-1\}$ and
$E(G) = \{u_i u_{i+1}, u_j v_j, v_j u_{j+1}, u_k x_k, v_j y_j / 1 \leq i \leq n-1, 1 \leq j \leq n-1, 1 \leq k \leq n\}$.

Then $|V(G)| = 4n - 2$ and $|E(G)| = 5n - 4$

Define$f: V(G) \rightarrow \{1,2,..,9n-7, 9n-4\}$.

Let $e = uv$ be any edge of G, with $1 \leq f(u) < f(v) \leq 9n - 4$.

Suppose$f^*(uv)$ $=5n-4$.

Now, two cases arise:

Case-(i): $\frac{|f(v)-f(u)|}{2} = 5n - 4$
This implies $f(v) - f(u) = 10n - 8$
$f(v) = 10n - 8 + f(u)$
$\geq 10n - 7$

Case-(ii): $\frac{|f(v)-f(u)|+1}{2} = 5n - 4$
This implies $|f(v) - f(u)| + 1 = 10n - 8$
$f(v) = 10n - 8 - 1 + f(u)$
$\geq 10n - 9 + 1$
$= 10n - 8$

In both the cases, we have $f(v) \geq 10n - 8 > 9n - 4 \ as \ n > 4$.

But by definition, $f(v) \leq 9n - 6$.

This is not possible.

Hence, $n \leq 4$.

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