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SOME GRAPH LABELINGS IN THE FRAMEWORK OF TRIPLICATION

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ABSTRACT

In this paper, we investigate the existence of Z_3-edge magic labeling, total Z_3-edge magic labeling and n – edge magic labeling for the extended triplicate graph of comb by presenting algorithms.

Keywords: *Graph labeling, Z_3-edge magic labeling, n-Edge Magic Labeling.*

1 INTRODUCTION

Graph theory has various applications in the field of computer programming and networking, marketing and communications, business administration and so on. Some major research topics in graph theory are Graph coloring, Spanning trees, Planar graphs, Networks and Graph labeling.

In 1967, Rosa introduced the concept of graph labeling [9]. Labeled graphs serve as useful models for broad range of applications such as coding theory, X-ray, crystallography, radar, astronomy, circuit design, communication networks and models and so on.

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of mapping is the set of vertices(edges) then the labeling is called a vertex(an edge)labeling.

Sedlacek has introduced the concept of A-magic graph [10]. A graph is called a *A-magic graph* if the edges have distinct non-negative labels which satisfies the condition that the sum of the labels of the edges incident to a particular vertex is the same for all vertices.

Chou and Lee investigated the concept of Z_3-magic graphs [3]. A graph G admits Z_3-magic labeling, if there exists a function *f* from E to {1, 2} such that the sum of the labels on the edges incident at each vertex $v \in V$ is a constant.

Jayapriya and Thirusangu introduced and proved the existence of 0-Edge magic labeling for some class of graphs [5]. Motivated by 0-Edge Magic Labeling Neelam Kumari, and Seema Mehra introduced n-Edge Magic Labeling and proved the existence for certain graphs such as P_t , C_t , $S_{m,t}$ (i.e. double star) etc., [8]. A graph G(V, E) is said to admit n - edge magic labeling if there exists a function f from V to {-1, n+1} such that the induced function f* from E to {n} defined as $f^*(v_iv_j) = \{f(v_i) + f(v_j)\} = n$, a constant for all edges $v_iv_j \in E$ and v_j is adjacent with v_i.

In 2011, Bala and Thirusangu introduced the concept of extended triplicate graph of a path and proved the existence of E-Cordial and Z_3 –magic labelings [2].

Let $V = \{v_1, v_2,\ldots,v_{n+1}\}$ and $E = \{e_1, e_2, \ldots, e_n\}$ be the vertex and Edge set of a path P_n. For every $v_i \in V$, construct an ordered triple $\{v_i , v_i' ,v_i''\}$ where $1\le i \le n+1$ and for every edge $v_iv_j\in E$, construct four edges v_iv_j', $v_j' v_i''$, v_jv_i' and $v_i' v_j''$ where j = i +1, then the graph with this vertex set and edge set is called a Triplicate Graph of a path P_n. It is dentoted by $TG(P_n)$. Clearly the Triplicate graph $TG(P_n)$ is disconnected. Let $V_1 = \{v_1, v_2 \ldots,v_{3n+1}\}$ and $E_1 = \{e_1, e_2 \ldots, e_{4n}\}$ be the vertex and edge set of $TG(P_n)$. If n is odd, include a new edge (v_{n+1} , v_1) and if n is even, include a new edge (v_n ,v_1) in the edge set of $TG(P_n)$. This graph is called the Extended Triplicate of the path P_n and it is denoted by $ETG(P_n)$.

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In 2016, Sutha, Thirusangu and Bala proved some E-Cordial labelings on Extended Duplicate Graph of Comb Graph [11]. Let P_m, $m \geq 3$ be a path graph with m vertices and *m-1* edges. Comb graph is defined as $P_m \odot mK_1$. It is denoted as $(Comb)_m$. Denote the vertex set and edge set as $V_1 = \{(v_i \cup u_i)/1 \leq i \leq m\}$ and $E_1 = \{(v_i v_j \cup v_i u_i)/1 \leq i \leq m\}$. Clearly it has 2m vertices and 2m-1 edges.

Motivated by the study, in this paper we introduce a new graph called extended triplicate graph of comb and prove the existence of Z_3 – edge magic labeling, total Z_3-edge magic labeling and *n*-edge magic labeling for the extended triplicate graph of a comb..

Throughout this work, graph G = (V, E), we mean a simple, finite, connected and undirected graph with p vertices and q edges.

2. STRUCTURE OF THE EXTENDED TRIPLICATE GRAPH OF COMB

In this section, we discuss about a new class of graph called the extended triplicate graph of comb and provide the structure of the extended triplicate graph of comb by presenting algorithm.

Algorithm 2.1:
Input: Comb graph
procedure (structure of TG $(Comb)_m$)
for *i* = 1 to *m*
$X \leftarrow \{ v_i, v_i', v_i'', u_i, u_i', u_i''\}$
end for
for*i* = 1 to *m-1* do
$E_1 \leftarrow (u_i u_{i+1}' \cup u_i' u_{i+1}'')$
end for
for*i* = 2 to *m* **do**
$E_2 \leftarrow (u_i' u_{i-1}'' \cup u_i u_{i-1}')$
end for
for*i* = 1 to *m* **do**
$E_3 \leftarrow (u_i v_i' \cup v_i u_i' \cup v_i' u_i'' \cup u_i' v_i'')$
end for
$Y \leftarrow E_1 \cup E_2 \cup E_3$
end procedure
Output: $TG(Comb)_m$

From the structure of $TG(Comb)_m$, it is clear that the triplicate graph of a comb is disconnected with *6m* vertices and *8m-4edges.*

To make it as a connected graph , include a new edge $v_1 v_m$ to the edge set of $TG(Comb)_m$ if $m \equiv 0 \pmod 2$ and if $m \equiv 1 \pmod 2$, include a new edge $v_1 \ u_m$ in the edge set of $TG(Comb)_m$. This new graph is called the Extended Triplicate graph of Comb and it is denoted by ETG $(Comb)_m$. By the construction, the extended triplicate graph of comb has *6m* vertices and *8m-3* edges.

Illustration 2.1: The structure of $ETG(Comb)_5$ is shown below in figure 1:

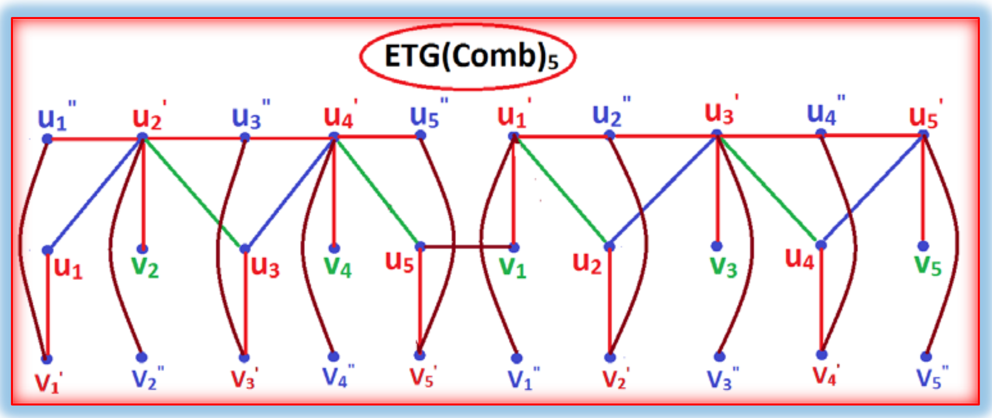


FIGURE 1

3. Z_3–EDGE MAGIC LABELING and TOTAL Z_3-EDGE MAGIC LABELING

In this section we prove the existence of Z_3 - edge magic labeling and total Z_3 - edge magic labeling for $ETG(Comb)_m$ by presenting algorithms.

Algorithm 3.1:

Input: Extended triplicate graph of Comb

Procedure Z_3 - edge magic labeling for *ETG(Comb)*$_m$
for $i = 1$ to m **do**
  $V \leftarrow \{ v_i, v_i', v_i'', u_i, u_i', u_i'' \}$
end for
for $i = 1$ to m **do**
  if $i \equiv 1\ (mod\ 2)$
    $u_i \leftarrow u_i'' \leftarrow u_i' \leftarrow 2$
    $v_i \leftarrow v_i'' \leftarrow v_i' \leftarrow 1$
  else
    $u_i \leftarrow u_i'' \leftarrow u_i' \leftarrow 1$
    $v_i \leftarrow v_i'' \leftarrow v_i' \leftarrow 2$
  end if
end for
end procedure

Output: labeled vertices of *ETG(Comb)*$_m$

Theorem 3.1: $ETG(Comb)_m$ admits Z_3-edge magic labeling.

Proof: From the construction of the extended triplicate graph of a comb, we know that $ETG(Comb)_m$ has $6m$ vertices and $8m-3$ edges.

The vertices are labeled by defining a function $f : V \rightarrow \{1, 2\}$ as given in algorithm 3.1

In order to obtain the labels for the edges, define the induced map $f^*: E \rightarrow \{0, 1, 2\}$ such that for any vertex v_i, $f^*(v_iv_j) = \{ f(v_i) + f(v_j) \}(mod\ 3)$ where v_j is adjacent with v_i.

Thus for all $v_iv_j \in V$, the induced function yields a constant ‘0’.

Hence the extended triplicate graph of comb $ETG(Comb)_m$ admits Z_3-edge magic labeling.

Illustration 3.1: ETG $(Comb)_4$ and its Z_3-edge magic labeling is shown in figure 2.

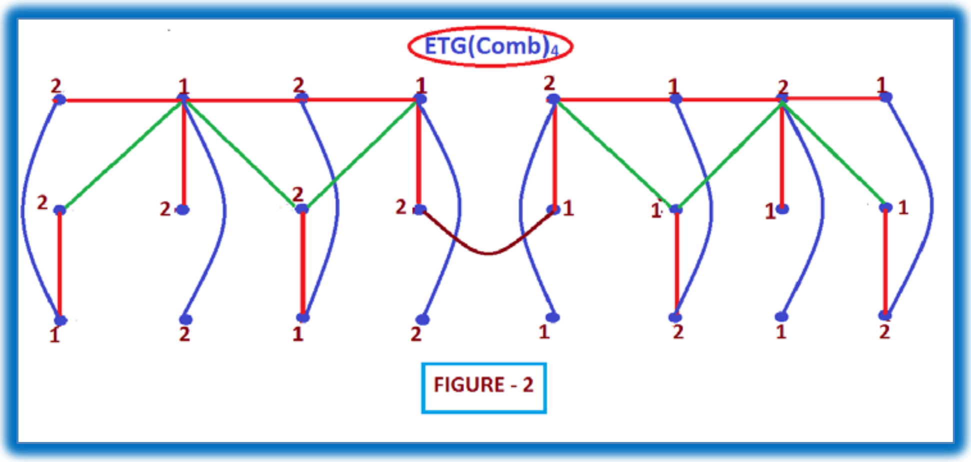


FIGURE - 2

Algorithm 3.2:

Input: Extended triplicate graph of comb

Procedure (total Z_3 - edge magic labeling for $ETG(Comb)_m$)
for $i = 1$ to m **do**
  $V \leftarrow \{ v_i, v_i', v_i'', u_i, u_i', u_i''\}$
end for
for $i = 1$ to m **do**
  $u_i \leftarrow u_i'' \leftarrow u_i' \leftarrow v_i \leftarrow 2$
  $v_i'' \leftarrow v_i' \leftarrow 1$
end for
for $i = 1$ to *m-1* **do**
  $u_i u_{i+1}' \leftarrow u_i' u_{i+1}'' \leftarrow 1$
end for
for $i = 2$ to m **do**
  $u_i u_{i-1}' \leftarrow u_i' u_{i-1}'' \leftarrow 1$
end for
for $i = 1$ to m **do**
  $u_i v_i' \leftarrow v_i' u_i'' \leftarrow u_i' v_i'' \leftarrow 2$
  $v_i u_i' \leftarrow 1$
end for
if m $\equiv 1 (mod\ 2)$
  $v_1 u_m \leftarrow 1$
else
  $v_1 v_m \leftarrow 1$
end if
end procedure

Output: *labeled vertices and edges of $ETG(Comb)_m$*

Theorem 3.2: $ETG(Comb)_m$) admits total Z_3-edge magic labeling.

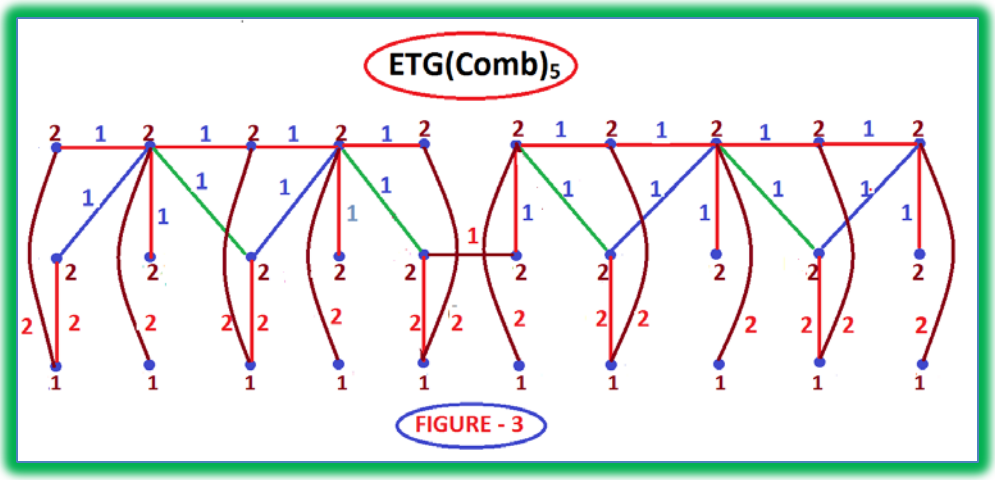
Proof: From the construction of the extended triplicate graph of a comb, we know that $ETG(Comb)_m$ has *6m* vertices and *8m-3* edges. The vertices and edges are labeled by defining a function f: V∪E→ {1, 2} as given in algorithm 3.2.

Define the induced function $f^* : E \rightarrow \{ 0, 1, 2\}$ such that for any vertex v_i, $f^*(v_i v_j) = \{ f(v_i) + f(v_j) + f(v_i v_j)\} (mod\ 3)$ where v_j is adjacent with v_i.

Thus, for all $v_i v_j \in E$, the induced function yields the constant '2'.

Hence the extended triplicate graph of comb $ETG(Comb)_m$ admits total Z_3- edge magic labeling.

Illustration 3.2: $ETG(Comb)_5$ and its total Z_3-edge magic labeling is shown in figure 3.



4 n - EDGE MAGIC LABELING

In this section we prove the existence of *n* - edge magic labeling for $ETG(Comb)_m$ by presenting algorithms.

Algorithm 4.1:

Input: Extended triplicate graph of ETG(Comb)$_m$
procedure (*n* - edge magic labeling for ETG(Comb)$_m$)
for *i* = 1 to *m* **do**
    $V \leftarrow \{ v_i, v_i', v_i'', u_i, u_i', u_i'' \}$
end for
for *i* = 1 to *m* **do**

    if i $\equiv 1(mod\ 2)$
        $u_i \leftarrow u_i'' \leftarrow u_i' \leftarrow n+1$
        $v_i \leftarrow v_i'' \leftarrow v_i' \leftarrow -1$
    else
        $u_i \leftarrow u_i'' \leftarrow u_i' \leftarrow -1$
        $v_i \leftarrow v_i'' \leftarrow v_i' \leftarrow n+1$
    end if
end for
end procedure

Output: Labeled vertices of ETG(Comb)$_m$

Theorem 4.1: ETG(Comb)$_m$ admits n-edge magic labeling.

Proof: We know that, the extended triplicate graph of a comb has *6m* vertices and *8m-3* edges. The vertices are labeled by defining a function f : V → {-1, n+1} as given in algorithm 4.1.

In order to obtain the labels for the edges, define the induced map f* : E→{ n } such that for any vertex v_i, $f^*(v_iv_j) = \{f(v_i) + f(v_j)\}(mod\ 3)$ where v_j is adjacent with v_i.

Thus for all $v_iv_j \in E$, the induced function yields a constant '*n*'.

Hence the extended triplicate graph of comb ETG(Comb)$_m$ admits *n*-edge magic labeling.

Illustration 4.1: ETG(Comb)$_4$ and its n-edge magic labeling is shown in figure 4.

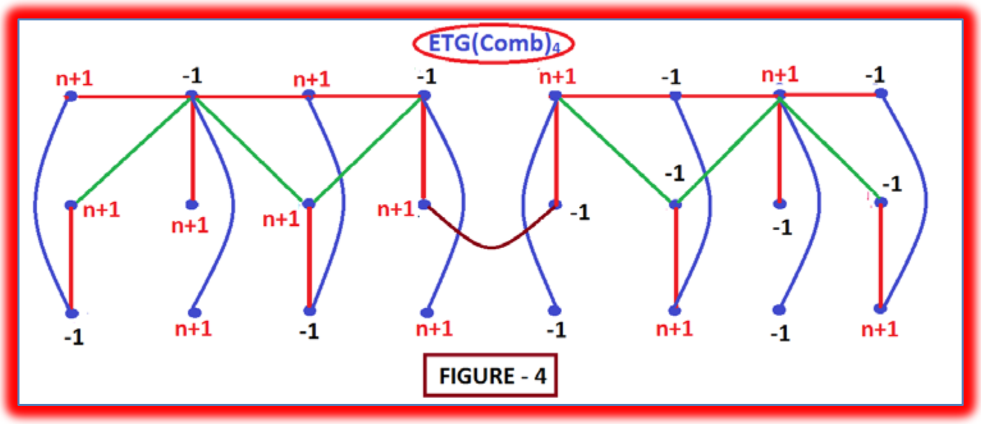


FIGURE - 4

CONCLUSION

In this paper, we have proved the existence of Z_3-edge magic labeling, total Z_3-edge magic labeling and *n* – edge magic labeling for the extended triplicate graph of twig by presenting algorithms.

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