International Journal of Mathematical Archive-9(2), 2018, 14-22
Available online through www.ijma.info ISSN 2229 – 5046

A PAIR OF GENERALIZED (σ,τ) n −DERIVATIONS IN PRIME NEAR RINGS

ASMA ALI*, INZAMAM UL HUQUE AND FARHAT ALI

Department of Mathematics, Aligarh Muslim University, Aligarh-202002, India.

(Received On: 23-12-17; Revised & Accepted On: 15-01-18)

ABSTRACT

The purpose of this paper is to study generalized (σ,τ) n −derivations satisfying certain identities on semigroup ideals of a prime near ring. Some well known results characterizing commutativity of 3-prime near rings by n-derivations have been generalized using semigroup ideals.

Keywords: *3-prime near-rings, (σ,τ) n-derivations, generalized (σ,τ) n-derivations, semigroup ideals.*

2010 Mathematics Subject Classification: *16N60, 16W25, 16Y30.*

1. INTRODUCTION

Throughout the paper, N denotes a zero-symmetric left near ring with multiplicative centre Z; and for any pair of elements $x,y\in N$, $[x,y]=xy-yx$ and $(x,y)=x+y-x-y$ stand for the commutator and additive commutator respectively. Let σ and τ be mappings on N. For any $x,y\in N$, set the symbol $[x,y]_{\sigma,\tau}$ will denote $x\sigma(y)-\tau(y)x$, while the symbol $(x\circ y)_{\sigma,\tau}$ will denote $x\sigma(y)+\tau(y)x$. A near ring N is called zero-symmetric if $0x=0$, for all $x\in N$ (recall that left distributivity yields that $x0=0$). The near ring N is said to be 3-prime if $xNy=\{0\}$ for $x,y\in N$ implies that $x=0$ or $y=0$. A near ring N is called 2-torsion free if $(N,+)$ has no element of order 2. A nonempty subset U of N is called a semigroup right (resp. semigroup left) ideal if $UN\subseteq U$ (resp. $NU\subseteq U$); and if U is both a semigroup right ideal and a semigroup left ideal, it is called a semigroup ideal. Let $n\geq 2$ be a fixed positive integer and $N^n=N\times N\times\ldots\times N_{\{n-times\}}$. A map $\Delta: N^n\rightarrow N$ is said to be permuting on a near ring N if the relation $\Delta(x_1,x_2,\ldots,x_n)=\Delta(x_{\pi(1)},x_{\pi(2)},\ldots,x_{\pi(n)})$ holds for all $x_i\in N$, $i=1,2,\ldots,n$ and for every permutation $\pi\in S_n$, where S_n is the permutation group on $\{1,2,\ldots,n\}$. An additive mapping $f: N\rightarrow N$ is said to be a right (resp. left) generalized derivation with associated derivation d if $F(xy)=F(x)y+xd(y)$ (resp. $F(xy)=d(x)y+xF(y)$), for all $x,y\in N$ and F is said to be a generalized derivation with associated derivation d on N if it is both a right generalized derivation and a left generalized derivation on N with associated derivation d.

Ozturk et al. [6] and Park et al. [8] studied bi-derivations and tri-derivations in near rings. A mapping $d: N\times N\rightarrow N$ is said to be symmetric (permuting) on a near ring N if $d(x,y)=d(y,x)$ for all $x,y\in N$. A symmetric bi-additive mapping $d: N\times N\rightarrow N$ (additive in each argument) is said to be a symmetric bi-derivation on N if $d(xy,z)=d(x,z)y+xd(y,z)$ holds for all $x,y,z\in N$. A permuting tri-additive map $\Delta: N\times N\times N\rightarrow N$ is said to be a permuting tri-derivation on N if

$$\Delta(xw,y,z)=\Delta(x,y,z)w+x\Delta(w,y,z)$$

is fulfilled for all $w,x,y,z\in N$.

Very recently Park [7] defined n-derivation in rings. Let σ, τ be endomorphisms on N. An additive mapping $d: N\rightarrow N$ is said to be a (σ,τ) derivation if $d(xy)=\sigma(x)d(y)+d(x)\tau(y)$, (or equivalently $d(xy)=d(x)\tau(y)+\sigma(x)d(y)$) for all $x,y\in N$. The notions of symmetric bi-(σ,τ) derivation and tri-(σ,τ) derivation have already been introduced and studied in near-rings by Ceven [3] and Ozturk [5], respectively. Motivated by the concept of tri-derivation in rings, Park [7] introduced the notion of n-derivation in rings. An n-additive (i.e. additive in each argument) mapping $d: N\times N\ldots\times N\rightarrow N$ is called (σ,τ)-n-derivation of N if there exist automorphisms $\sigma,\tau: N\rightarrow N$ such that the equations

Corresponding Author: Asma Ali*,
Department of Mathematics, Aligarh Muslim University, Aligarh-202002, India.

$$d(x_1x_1',x_2,\dots,x_n) = d(x_1,x_2,\dots,x_n)\sigma(x_1') + \tau(x_1)d(x_1',x_2,\dots,x_n)$$
$$d(x_1,x_2x_2',\dots,x_n) = d(x_1,x_2,\dots,x_n)\sigma(x_2') + \tau(x_2)d(x_1,x_2',\dots,x_n)$$
$$\vdots$$
$$d(x_1,x_2,\dots,x_nx_n') = d(x_1,x_2,\dots,x_n)\sigma(x_n') + \tau(x_n)d(x_1,x_2,\dots,x_n')$$
hold for all $x_1,x_1',x_2,x_2',\dots,x_n,x_n' \in N$.

Majeed and Adhab [4] defined Generalized (σ,τ) n-derivation in near rings. An n-additive mapping $F: N \times N \dots \times N \to N$ is called a right generalized (σ,τ) n-derivation associated with (σ,τ) n-derivation d on N if the relations
$$F(x_1x_1',x_2,\dots,x_n) = F(x_1,x_2,\dots,x_n)\sigma(x_1') + \tau(x_1)d(x_1',x_2,\dots,x_n)$$
$$F(x_1,x_2x_2',\dots,x_n) = F(x_1,x_2,\dots,x_n)\sigma(x_2') + \tau(x_2)d(x_1,x_2',\dots,x_n)$$
$$\vdots$$
$$F(x_1,x_2,\dots,x_nx_n') = F(x_1,x_2,\dots,x_n)\sigma(x_n') + \tau(x_n)d(x_1,x_2,\dots,x_n')$$
hold for all $x_1,x_1',x_2,x_2',\dots,x_n,x_n' \in N$.

An n-additive mapping $F: N \times N \dots \times N \to N$ is called a left generalized (σ,τ) n-derivation associated with (σ,τ) n-derivation d on N if the relations
$$F(x_1x_1',x_2,\dots,x_n) = d(x_1,x_2,\dots,x_n)\sigma(x_1') + \tau(x_1)F(x_1',x_2,\dots,x_n)$$
$$F(x_1,x_2\,x_2',\dots,x_n) = d(x_1,x_2,\dots,x_n)\sigma(x_2') + \tau(x_2)F(x_1,x_2',\dots,x_n)$$
$$\vdots$$
$$F(x_1,x_2,\dots,x_nx_n') = d(x_1,x_2,\dots,x_n)\sigma(x_n') + \tau(x_n)F(x_1,x_2,\dots,x_n')$$
hold for all $x_1,x_1',x_2,x_2',\dots,x_n,x_n' \in N$.

A mapping $F: N^n \to N$ is called a generalized (σ,τ) n-derivation associated with (σ,τ) n-derivation on N if F is both a right generalized and a left generalized (σ,τ) n-derivation with associated (σ,τ) n-derivation d on N.

2. PRELIMINARY RESULTS

We begin with several Lemmas, most of which have been proved elsewhere.

Lemma 2.1: [1, Lemmas 1.2] Let N be 3-prime near ring.
 (i) If $z \in Z\backslash\{0\}$, then z is not a zero divisor.
 (ii) If $Z\backslash\{0\}$ and x is an element of N for which $xz \in Z$, then $x \in Z$.

Lemma 2.2: [1, Lemmas 1.3 and Lemma 1.4] Let N be 3-prime near ring and U be a nonzero semigroup ideal of N.
(i) If $x,y \in N$ and $xUy = \{0\}$, then $x = 0$ or $y = 0$.
(ii) If $x \in N$ and $xU = \{0\}$ or $Ux = \{0\}$, then $x = 0$.

Lemma 2.3: [1, Lemma 1.5] If N is a 3-prime near ring and Z contains a nonzero semigroup left ideal or a nonzero semigroup right ideal, then N is a commutative ring.

Lemma 2.4: [4, Lemma 2.9] If N is a 3-prime near ring admitting a generalized (σ,τ) n-derivation F associated with a (σ,τ) n-derivation d of N, then
$$(d(x_1,x_2,\dots,x_n)\sigma(x_1') + \tau(x_1)F(x_1',x_2,\dots,x_n))y = d(x_1,x_2,\dots,x_n)\sigma(x_1')y + \tau(x_1)F(x_1',x_2,\dots,x_n)y,$$
$$(d(x_1,x_2,\dots,x_n)\sigma(x_2') + \tau(x_2)F(x_1,x_2',\dots,x_n))y = d(x_1,x_2,\dots,x_n)\sigma(x_2')y + \tau(x_2)F(x_1,x_2',\dots,x_n)y;$$
$$\vdots$$
$$(d(x_1,x_2,\dots,x_n)\sigma(x_n') + \tau(x_n)F(x_1,x_2,\dots,x_n'))y = d(x_1,x_2,\dots,x_n)\sigma(x_n')y + \tau(x_n)F(x_1,x_2,\dots,x_n')y$$
for all $x_1,x_1',x_2,x_2',\dots,x_n,x_n',y \in N$.

Lemma 2.5: [2, Lemma 4] Let N be an arbitrary near ring. Let S and T be non-empty subsets of N such that $st = -ts$ for all $s \in S$ and $t \in T$. If $a,b \in S$ and c is an element of T for which $-c \in T$, then $(ab)c = c(ab)$.

Lemma 2.6: Let N be a 3-prime near ring and $U_1,U_2,\dots,U_n$ be nonzero semigroup ideals of N. Let σ,τ be mappings on N such that $\tau(U_i) = U_i$ for $i = 1,2,\dots,n$. If d is a nonzero (σ,τ) n-derivation on N, then $d(U_1,U_2,\dots,U_n) \neq \{0\}$.

Proof: Let
$$d(u_1,u_2,\dots,u_n) = 0 \text{ for all } u_1 \in U_1, u_2 \in U_2,\dots,u_n \in U_n. \tag{2.1}$$

Replacing u_1 by u_1r_1, where $r_1 \in N$ in (2.1), we get
$$d(u_1,u_2,\dots,u_n)\sigma(r_1) + \tau(u_1)d(r_1,u_2,\dots,u_n) = 0$$
and using (2.1), we get $\tau(u_1)d(r_1,u_2,\dots,u_n) = 0$. Since $\tau(U_i) = U_i$ for $i = 1,2,\dots,n$,
$$U_1d(r_1,u_2,\dots,u_n) = 0.$$

Using Lemma 2.2(ii), we obtain

$$d(r_1, u_2, \ldots, u_n) = 0. \tag{2.2}$$

Replacing u_2 by $u_2 r_2$, where $r_2 \in N$ in (2.2), we find

$$d(r_1, u_2, \ldots, u_n)\sigma(r_2) + \tau(u_2)d(r_1, r_2, \ldots, u_n) = 0$$

and using (2.2), we get $\tau(u_2)d(r_1, r_2, \ldots, u_n) = 0$ i.e. $U_2 d(r_1, r_2, \ldots, u_n) = 0$.

Another application of by Lemma 2.2(ii) yields that $d(r_1, r_2, \ldots, u_n) = 0$. Proceeding as above inductively, we conclude that $d(r_1, r_2, \ldots, r_n) = 0$ for all $r_1, r_2, \ldots, r_n \in N$, a contradiction which completes the proof.

Lemma 2.7: Let N be a 3-prime near ring and $U_1, U_2, \ldots, U_n$ be nonzero semigroup ideals of N. Let σ, τ be mappings on N such that $\sigma(U_1) = U_1, \tau(U_i) = U_i$ for $i = 1,2, \ldots, n$. If d is a nonzero (σ,τ) n-derivation on N such that $d(U_1, U_2, \ldots U_n)a = \{0\}$ or $ad(U_1, U_2, \ldots U_n) = \{0\}$, then $a = 0$.

Proof: Suppose that $d(U_1, U_2, \ldots U_n)a = \{0\}$. Then

$$d(u_1, u_2, \ldots u_n)a = 0 \text{ for all } u_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n. \tag{2.3}$$

Replacing u_1 by $u_1 u_1'$ for $u_1' \in U_1$ in (2.3), we get

$$(d(u_1, u_2, \ldots u_n)\sigma(u_1') + \tau(u_1)d(u_1', u_2, \ldots u_n))a = 0.$$

Using Lemma 2.4 and (2.3), we find

$$d(u_1, u_2, \ldots u_n)\sigma(u_1')a = 0.$$

Since $\sigma(U_1) = U_1$, we have

$$d(u_1', u_2, \ldots u_n)U_1 a = 0.$$

Using Lemma 2.2 (i) and Lemma 2.6, we obtain a=0.

Lemma 2.8: Let N be a 3-prime near ring and $U_1, U_2, \ldots, U_n$ be nonzero semigroup ideals of N. Let σ, τ be mappings on N such that $\sigma(U_1) = U_1; \tau(U_1) = U_1$ and $U_1 \cap Z \setminus \{0\}$. If d is a (σ,τ) n-derivation on N, then $d(Z, N, N, \ldots, N) \subseteq Z$.

Proof: Suppose that $z \in U_1 \cap Z$. Then

$$d(zx_1, x_2, \ldots, x_n) = d(x_1 z, x_2, \ldots, x_n) \text{ for all } x_1, x_2, \ldots, x_n \in N,$$
$$d(z, x_2, \ldots, x_n)\sigma(x_1) + \tau(z)d(x_1, x_2, \ldots, x_n) = \tau(x_1)d(z, x_2, \ldots, x\_n) + d(x_1, x_2, \ldots, x_n)\sigma(z)$$

By hypothesis

$$d(z, x_2, \ldots, x_n)x_1 = x_1 d(z, x_2, \ldots, x_n) \text{ for all } x_1, x_2, \ldots, x_n \in N \text{ and } z \in Z.$$

Hence $d(Z, N, N, \ldots, N) \subseteq Z$.

Lemma 2.9: Let N be a 3-prime near ring and $U_1, U_2, \ldots, U_n$ be nonzero semigroup ideals of N. Let σ, τ be mappings on N such that $\sigma(U_i) = U_i$, $\tau(U_i) = U_i$ for $i = 1,2, \ldots, n$. If F is a nonzero generalized (σ,τ) n-derivation associated with a (σ,τ) n-derivation d on N, then $F(U_1, U_2, \ldots, U_n) \neq \{0\}$.

Proof: Let

$$F(u_1, u_2, \ldots, u_n) = 0 \text{ for all } u_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n. \tag{2.4}$$

Replacing u_1 by $u_1 r_1$, where $r_1 \in N$ in (2.4) and using it, we get

$$\tau(u_1)d(r_1, u_2, \ldots, u_n) = 0 \text{ for all } u_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n \text{ and } r_1 \in N. \tag{2.5}$$

Since $\tau(U_i) = U_i$ for $i = 1,2, \ldots, n$, we get

$$U_1 d(r_1, u_2, \ldots, u_n) = 0 \text{ for all } u_2 \in U_2, \ldots, u_n \in U_n \text{ and } r_1 \in N.$$

Applying Lemma 2.2(ii), we find

$$d(r_1, u_2, \ldots, u_n) = 0 \text{ for all } u_2 \in U_2, \ldots, u_n \in U_n \text{ and } r_1 \in N. \tag{2.6}$$

Now replacing u_2 by $u_2 r_2$ in (2.6) for $r_2 \in N$, we get $\tau(u_2)d(r_1, r_2, \ldots, u_n) = 0$ and by hypothesis $U_2 d(r_1, r_2, \ldots, u_n) = \{0\}$. Again application of Lemma 2.2(ii) yields that $d(r_1, r_2, u_3, \ldots, u_n) = 0$ for all $u_3 \in U_3, \ldots, u_n \in U_n$ and $r_1, r_2 \in N$. Proceeding inductively, we get $d(r_1, r_2, \ldots, r_n) = 0$ for all $r_1, r_2, \ldots, r_n \in N$. i.e., d=0. Hence

$$F(r_1 u_1, u_2, \ldots, u_n) = F(r_1, u_2, \ldots, u_n)\sigma(u_1) = 0 \text{ for all } u_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n \text{ and } r_1 \in N$$

which implies that $F(r_1,u_2,\dots,u_n)U_1=0$. By Lemma 2.2 (ii), we find
$$F(r_1,u_2,\dots,u_n)=0 \text{ for all } u_2\in U_2,\dots,u_n\in U_n \text{ and } r_1\in N. \quad (2.7)$$

Replacing u_2 by r_2u_2 in (2.7), we get $F(r_1,r_2,\dots,u_n)U_2=\{0\}$ and Lemma 2.2 (ii) gives
$$F(r_1,r_2,\dots,u_n)=0 \text{ for all } r_1,r_2\in N.$$

Proceeding inductively, we obtain $F=0$ on N, a contradiction.

Lemma 2.10: Let N be a 3-prime near ring and $U_1,U_2,\dots,U_n$ be nonzero semigroup ideals of N. Let σ,τ be mappings on N such that $\sigma(U_i)=U_i$, $\tau(U_i)=U_i$ for $i=1,2,\dots,n$. If F is a nonzero generalized (σ,τ) n-derivation associated with a (σ,τ) n-derivation d on N such that $F(U_1,U_2,\dots U_n)a=\{0\}$ or $aF(U_1,U_2,\dots U_n)=\{0\}$, then $a=0$.

Proof: Suppose that $F(U_1,U_2,\dots U_n)a=\{0\}$. Then
$$F(u_1,u_2,\dots u_n)a=0 \text{ for all } u_1\in U_1,u_2\in U_2,\dots,u_n\in U_n. \quad (2.8)$$

Replacing u_1 by u_1u_1' where $u_1'\in U_1$ in (2.8) and using Lemma 2.4, we obtain
$$d(u_1,u_2,\dots u_n)\sigma(u_1')a=0 \text{ for all } u_1,u_1'\in U_1,u_2\in U_2,\dots,u_n\in U_n.$$

Since $\sigma(U_i)=U_i$ for $i=1,2,\dots,n$, we have
$$d(u_1,u_2,\dots u_n)U_1a=\{0\} \text{ for all } u_1\in U_1,u_2\in U_2,\dots,u_n\in U_n.$$

By Lemma 2.2 (i), either $a=0$ or $d(u_1,u_2,\dots,u_n)=0$. If $d(u_1,u_2,\dots u_n)=\{0\}$, then
$$F(u_1u_1',u_2,\dots u_n)a=F(u_1,u_2,\dots u_n)\sigma(u_1')a+\tau(u_1)d(u_1',u_2,\dots u_n)a=0$$
yields that $F(u_1,u_2,\dots u_n)U_1a=0$. Again using Lemma 2.2(i), we find either $a=0$ or $F(u_1,u_2,\dots,u_n)=0$. Later yields contradiction by Lemma 2.9, hence we get the result.

3. MAIN RESULTS

Theorem 3.1: Let N be a 3-prime near ring and $U_1,U_2,\dots,U_n$be nonzero semigroup ideals of N. Let σ,τ be mappings on N such that $\sigma(U_i)=U_i,\tau(U_i)=U_i$ for $i=1,2,\dots,n$. If F is a nonzero generalized (σ,τ) n-derivation associated with a (σ,τ) n-derivation d on N such that $F(U_1,U_2,\dots,U_n)\subseteq Z$, then N is a commutative ring.

Proof: If $d\neq 0$ for all $u_1,u_1'\in U_1,u_2\in U_2,\dots,u_n\in U_n$, we get
$$F(u_1u_1',u_2,\dots,u_n)=d(u_1,u_2,\dots,u_n)\sigma(u_1')+\tau(u_1)F(u_1',u_2,\dots,u_n)\in Z. \quad (3.1)$$

Now commuting (3.1) with the element $\tau(u_1)$, we have
$$\big(d(u_1,u_2,\dots,u_n)\sigma(u_1')+\tau(u_1)F(u_1',u_2,\dots,u_n)\big)\tau(u_1)$$
$$=\tau(u_1)(d(u_1,u_2,\dots,u_n)\sigma(u_1')+\tau(u_1)F(u_1',u_2,\dots,u_n)).$$

Using Lemma 2.4, we obtain
$$d(u_1,u_2,\dots,u_n)\sigma(u_1')\tau(u_1)+\tau(u_1)F(u_1',u_2,\dots,u_n)\tau(u_1)$$
$$=\tau(u_1)d(u_1,u_2,\dots,u_n)\sigma(u_1')+\tau(u_1)\tau(u_1)F(u_1',u_2,\dots,u_n).$$

This implies that,
$$d(u_1,u_2,\dots,u_n)\sigma(u_1')\tau(u_1)=\tau(u_1)d(u_1,u_2,\dots,u_n)\sigma(u_1'). \quad (3.2)$$

Replacing u_1' by $u_1'x$ for $x\in N$ in (3.2) and using $\sigma(U_i)=U_i$ for $i=1,2,\dots,n$, we find
$$d(u_1,u_2,\dots,u_n)u_1'x\tau(u_1)=\tau(u_1)d(u_1,u_2,\dots,u_n)u_1'x.$$

Now replacing u_1' by $\sigma(u_1')$, we get
$$d(u_1,u_2,\dots,u_n)\sigma(u_1')x\tau(u_1)=\tau(u_1)d(u_1,u_2,\dots,u_n)\sigma(u_1')x.$$

Using (3.2), we obtain $d(u_1,u_2,\dots,u_n)\sigma(u_1')x\tau(u_1)=d(u_1,u_2,\dots,u_n)\sigma(u_1')\tau(u_1)x$,
$$d(u_1,u_2,\dots,u_n)\sigma(u_1')[x,\tau(u_1)]=0.$$

This implies that
$$d(u_1,u_2,\dots,u_n)U_1[x,u_1]=0.$$

Using Lemma 2.2 (i), we get $d(u_1,u_2,\dots,u_n)=0$ or $[x,u_1]=0$. Later gives $U_1\subseteq Z$. Now if $d(u_1,u_2,\dots,u_n)=0$ for all $u_i\in U_i$ for $i=1,2,\dots,n$, then
$$F(u_1u_1',u_2,\dots,u_n)=u_1F(u_1',u_2,\dots,u_n)\in Z.$$

An appeal to Lemma 2.1 and Lemma 2.9 gives $U_1 \subseteq Z$. Hence by Lemma 2.3, N is a commutative ring. Assume that $d = 0$. From (3.1), we have $F(u_1u_1', u_2, \dots, u_n) = \tau(u_1)F(u_1', u_2, \dots, u_n) \in Z$ and invoking Lemma 2.1 and Lemma 2.9, we get $U_1 \subseteq Z$ and N is a commutative ring by Lemma 2.3.

Theorem 3.2: Let N be a 3-prime near ring and $U_1, U_2, \dots, U_n$ be nonzero semigroup ideals of N. Let σ, τ be mappings on N such that $\sigma(U_i) = U_i$, $\tau(U_i) = U_i$ for $i = 1,2,\dots,n$. If F_1, F_2 are generalized (σ, τ) n-derivations associated with nonzero (σ, τ) n-derivations d_1, d_2 respectively such that $[F_1(U_1, U_2, \dots, U_n), F_2(U_1, U_2, \dots, U_n)] = \{0\}$, then $(N, +)$ is abelian.

Proof: Suppose that $x \in N$ is such that

$$[x, F_2(U_1, U_2, \dots, U_n)] = [x + x, F_2(U_1, U_2, \dots, U_n)] = 0.$$

For all $u_1, u_1' \in U_1$ such that $u_1 + u_1' \in U_1$,

$$[x + x, F_2(u_1 + u_1', u_2, \dots, u_n)] = 0.$$

This implies that

$$(x + x)F_2(u_1 + u_1', u_2, \dots, u_n) = F_2(u_1 + u_1', u_2, \dots, u_n)(x + x),$$

$$(x + x)F_2(u_1, u_2, \dots, u_n) + (x + x)F_2(u_1', u_2, \dots, u_n) = F_2(u_1 + u_1', u_2, \dots, u_n)x + F_2(u_1 + u_1', u_2, \dots, u_n)x,$$

$$F_2(u_1, u_2, \dots, u_n)(x + x) + F_2(u_1', u_2, \dots, u_n)(x + x) = xF_2(u_1 + u_1', u_2, \dots, u_n) + xF_2(u_1 + u_1', u_2, \dots, u_n),$$

$$F_2(u_1, u_2, \dots, u_n)x + F_2(u_1, u_2, \dots, u_n)x + F_2(u_1', u_2, \dots, u_n)x + F_2(u_1', u_2, \dots, u_n)x = xF_2(u_1, u_2, \dots, u_n) + xF_2(u_1', u_2, \dots, u_n) + xF_2(u_1, u_2, \dots, u_n) + xF_2(u_1', u_2, \dots, u_n).$$

This implies that

$$xF_2((u_1, u_1'), u_2, \dots, u_n) = 0,$$

where (u_1, u_1') is the additive commutator $(u_1 + u_1' - u_1 - u_1')$, $u_2 \in U_2, \dots, u_n \in U_n$.

If $r, s \in U_1$, we have $rs \in U_1$ and $rs + rs = r(s + s) \in U_1$ and since $[F_1(U_1, U_2, \dots, U_n), F_2(U_1, U_2, \dots, U_n)] = \{0\}$, taking $x = F_1(rs, u_2', \dots, u_n')$ where $r, s \in U_1, u_2' \in U_2, \dots, u_n' \in U_n$ gives

$$[F_1(rs, u_2', \dots, u_n'), F_2(U_1, U_2, \dots, U_n)] = \{0\} = [F_1(rs, u_2', \dots, u_n') + F_1(rs, u_2', \dots, u_n'), F_2(U_1, U_2, \dots, U_n)].$$

Arguing in the similar manner as above, we get

$$F_1(U_1^2, U_2, \dots, U_n)F_2(u_1 + u_1' - u_1 - u_1', u_2, \dots, u_n) = \{0\}.$$

Since U_1^2 is a semigroup ideal, Lemma 2.10 gives

$$F_2(u_1 + u_1' - u_1 - u_1', u_2, u_3, \dots, u_n) = 0 \tag{3.3}$$

for all u_1, $u_1' \in U_1$ such that $u_1 + u_1' \in U_1$. Now take $u_1 = rx'$ and $u_1' = ry'$ for $r \in U_1$ and $x', y' \in N$, so that u_1, u_1' and $u_1 + u_1' = rx' + ry' = r(x' + y') \in U_1$. It follows from relation (3.3) that

$$F_2(rx' + ry' - rx' - ry', u_2, u_3, \dots, u_n) = 0 \text{ for all } r \in U_1 \text{ and } x', y' \in N.$$

Replacing r by rw for $w \in U_1$ in above expression, we get

$$d_2(U_1, U_2, \dots, U_n)w(x' + y' - x' - y') = \{0\}$$

for all $w \in U_1$ and $x', y' \in N$ i.e., $d_2(U_1, U_2, \dots, U_n)U_1(x' + y' - x' - y') = \{0\}$ for all $x', y' \in N$ and by Lemma 2.2 (i) and Lemma 2.6, we get $x' + y' - x' - y' = 0$ for all $x', y' \in N$ and hence $(N, +)$ is abelian.

Theorem 3.3: Let N be a 3-prime near ring and $U_1, U_2, \dots, U_n$ be nonzero semigroup ideals of N. Let σ, τ be mappings on N such that $\sigma(U_i) = U_i$, $\tau(U_i) = U_i$ for $i = 1,2,\dots,n$. If F_1, F_2 are generalized (σ, τ) n-derivations associated with nonzero (σ, τ) n-derivations d_1, d_2 respectively such that $F_1(U_1, U_2, \dots, U_n) \circ F_2(U_1, U_2, \dots, U_n) = \{0\}$, then $(N, +)$ is abelian.

Proof: Assume that $x \in N$ is such that

$$x \circ F_2(U_1, U_2, \dots, U_n) = (x + x) \circ F_2(U_1, U_2, \dots, U_n) = 0.$$

For all $u_1, u_1' \in U_1$ such that $u_1 + u_1' \in U_1$,

$$(x + x) \circ F_2(u_1 + u_1', u_2, \dots, u_n) = 0,$$

$$(x + x)F_2(u_1 + u_1', u_2, \dots, u_n) + F_2(u_1 + u_1', u_2, \dots, u_n)(x + x) = 0,$$

$$(x+x)F_2(u_1,u_2,\dots,u_n) + (x+x)F_2(u_1',u_2,\dots,u_n) = -F_2(u_1+u_1',u_2,\dots,u_n)x - F_2(u_1+u_1',u_2,\dots,u_n)x,$$

$$xF_2(u_1,u_2,\dots,u_n) + xF_2(u_1,u_2,\dots,u_n) + xF_2(u_1',u_2,\dots,u_n) + xF_2(u_1',u_2,\dots,u_n) = -F_2(u_1+u_1',u_2,\dots,u_n)x - F_2(u_1+u_1',u_2,\dots,u_n)x,$$

$$xF_2(u_1,u_2,\dots,u_n) + xF_2(u_1,u_2,\dots,u_n) + xF_2(u_1',u_2,\dots,u_n) + xF_2(u_1',u_2,\dots,u_n) = xF_2(u_1+u_1',u_2,\dots,u_n) + xF_2(u_1+u_1',u_2,\dots,u_n),$$

$$zF_2(u_1,u_2,\dots,u_n) + xF_2(u_1,u_2,\dots,u_n) + xF_2(u_1',u_2,\dots,u_n) + xF_2(u_1',u_2,\dots,u_n) = xF_2(u_1,u_2,\dots,u_n) + xF_2(u_1',u_2,\dots,u_n) + xF_2(u_1,u_2,\dots,u_n + xF_2(u_1',u_2,\dots,u_n).$$

This implies that $xF_2((u_1,u_1'),u_2,\dots,u_n) = 0$, where (u_1,u_1') is the additive commutator $(u_1+u_1'-u_1-u_1'), u_2 \in U_2,\dots,u_n \in U_n$. Arguing in the similar manner as in Theorem 3.2, we get the required result.

Theorem 3.4: Let N be a 2-torsion free 3-prime near ring and $U_1,U_2,\dots,U_n$ be nonzero semigroup ideals of N. Let σ,τ be mappings on N such that $\sigma(U_i) = U_i, \tau(U_i) = U_i$ for $i = 1,2,\dots,n$. If F_1,F_2 are generalized (σ,τ) n-derivations associated with (σ,τ) n-derivations d_1,d_2 respectively with at least one of d_1,d_2 not zero such that

$$F_1(x_1,x_2,\dots,x_n)d_2(y_1,y_2,\dots,y_n) + F_2(x_1,x_2,\dots,x_n)d_1(y_1,y_2,\dots,y_n) = 0$$

for all $x_1,y_1 \in U_1; x_2,y_2 \in U_2,\dots; x_n,y_n \in U_n$, then $F_1 = 0$ or $F_2 = 0$.

Proof: By hypothesis for all $x_1,y_1 \in U_1; x_2,y_2 \in U_2,\dots,x_n,y_n \in U_n$

$$F_1(x_1,x_2,\dots,x_n)d_2(y_1,y_2,\dots,y_n) + F_2(x_1,x_2,\dots,x_n)d_1(y_1,y_2,\dots,y_n) = 0. \quad (3.4)$$

Replacing x_1 by u_1v_1 for $u_1,v_1 \in U_1$ in (3.4), we get

$$\{d_1(u_1,x_2,\dots,x_n)\sigma(v_1) + \tau(u_1)F_1(v_1,x_2,\dots,x_n)\}d_2(y_1,y_2,\dots,y_n) + \{\tau(u_1)F_2(v_1,x_2,\dots,x_n) + d_2(u_1,x_2,\dots,x_n)\sigma(v_1)\}d_1(y_1,y_2,\dots,y_n) = 0,$$

$$d_1(u_1,x_2,\dots,x_n)\sigma(v_1)d_2(y_1,y_2,\dots,y_n) + \tau(u_1)F_1(v_1,x_2,\dots,x_n)d_2(y_1,y_2,\dots,y_n) + \tau(u_1)F_2(v_1,x_2,\dots,x_n)d_1(y_1,y_2,\dots,y_n) + d_2(u_1,x_2,\dots,x_n)\sigma(v_1)d_1(y_1,y_2,\dots,y_n) = 0,$$

$$d_1(u_1,x_2,\dots,x_n)\sigma(v_1)d_2(y_1,y_2,\dots,y_n) + \tau(u_1)\{F_1(v_1,x_2,\dots,x_n)d_2(y_1,y_2,\dots,y_n) + F_2(v_1,x_2,\dots,x_n)d_1(y_1,y_2,\dots,y_n)\} + d_2(u_1,x_2,\dots,x_n)\sigma(v_1)d_1(y_1,y_2,\dots,y_n) = 0.$$

Using (3.4) middle summand is 0, we conclude that

$$d_1(u_1,x_2,\dots,x_n)\sigma(v_1)d_2(y_1,y_2,\dots,y_n) + d_2(u_1,x_2,\dots,x_n)\sigma(v_1)d_1(y_1,y_2,\dots,y_n) = 0.$$

Since $\sigma(U_i) = U_i$ for $i = 1,2,\dots,n$, we get

$$d_1(u_1,x_2,\dots,x_n)v_1d_2(y_1,y_2,\dots,y_n) + d_2(u_1,x_2,\dots,x_n)v_1d_1(y_1,y_2,\dots,y_n) = 0. \quad (3.5)$$

Replacing y_1 by y_1t for $t \in U_1$ in (3.5), we obtain

$$d_1(u_1,x_2,\dots,x_n)v_1\{d_2(y_1,y_2,\dots,y_n)\sigma(t) + \tau(y_1)d_2(t,y_2,\dots,y_n)\} + d_2(u_1,x_2,\dots,x_n)v_1\{\tau(y_1)d_1(t,y_2,\dots,y_n) + d_1(y_1,y_2,\dots,y_n)\sigma(t)\} = 0,$$

$$d_1(u_1,x_2,\dots,x_n)v_1d_2(y_1,y_2,\dots,y_n)\sigma(t) + d_1(u_1,x_2,\dots,x_n)v_1\tau(y_1)d_2(t,y_2,\dots,y_n) + d_2(u_1,x_2,\dots,x_n)v_1\tau(y_1)d_1(t,y_2,\dots,y_n) + d_2(u_1,x_2,\dots,x_n)v_1d_1(y_1,y_2,\dots,y_n)\sigma(t) = 0.$$

Using (3.5), we get

$$d_1(u_1,x_2,\dots,x_n)v_1d_2(y_1,y_2,\dots,y_n)t + d_2(u_1,x_2,\dots,x_n)v_1d_1(y_1,y_2,\dots,y_n)t = 0. \quad (3.6)$$

Replacing t by $td_1(w_1,w_2,\dots,w_n)$ for all $w_1 \in U_1, w_2 \in U_2,\dots,w_n \in U_n$ in (3.6), we have

$$d_1(u_1,x_2,\dots,x_n)v_1d_2(y_1,y_2,\dots,y_n)td_1(w_1,w_2,\dots,w_n) + d_2(u_1,x_2,\dots,x_n)v_1d_1(y_1,y_2,\dots,y_n)td_1(w_1,w_2,\dots,w_n) = 0.$$

Using (3.5) in the above relation, we obtain

$$d_1(u_1,x_2,\dots,x_n)v_1\{-d_1(y_1,y_2,\dots,y_n)td_2(w_1,w_2,\dots,w_n)\} - d_1(u_1,x_2,\dots,x_n)v_1d_1(y_1,y_2,\dots,y_n)td_2(w_1,w_2,\dots,w_n) = 0$$

which implies that

$$2d_1(u_1,x_2,\dots,x_n)v_1d_1(y_1,y_2,\dots,y_n)td_2(w_1,w_2,\dots,w_n) = 0,$$

2-torsion freeness implies that

$$d_1(u_1,x_2,\dots,x_n)v_1d_1(y_1,y_2,\dots,y_n)td_2(w_1,w_2,\dots,w_n) = 0.$$

Then by Lemma 2.2 (i) and Lemma 2.6, one of d_1, d_2 must be zero. Assuming without loss $d_1 = 0$ in which case $d_2 \neq 0$, then (3.4) gives $F_1(x_1,x_2,\dots,x_n)d_2(y_1,y_2,\dots,y_n) = 0$. By using Lemma 2.7 and Lemma 2.9, we get $F_1 = 0$.

Theorem 3.5: Let N be a 3-prime near ring; $U_1, U_2, \dots, U_n$ be nonzero semigroup ideals of N and σ,τ be mappings on N such that $\sigma(U_1) = U_1, \tau(U_1) = U_1$. Let F_1, F_2 be generalized (σ,τ) n-derivations associated with (σ,τ) n-derivations d_1, d_2 respectively such that $F_1(U_1, U_2, \dots, U_n) \circ F_2(U_1, U_2, \dots, U_n) = \{0\}$. If $d_1(Z, N \dots, N) \neq \{0\}$; $d_2(Z, N \dots, N) \neq \{0\}$ and $U_1 \cap Z \neq \{0\}$, then N is a commutative ring. Moreover $F_1 = 0$ or $F_2 = 0$.

Proof: By hypothesis

$$F_1(x_1, x_2, \dots, x_n)F_2(y_1, y_2, \dots, y_n) + F_2(y_1, y_2, \dots, y_n)F_1(x_1, x_2, \dots, x_n) = 0. \tag{3.7}$$

for all $x_1, y_1 \in U_1$; $x_2, y_2 \in U_2, \dots, x_n, y_n \in U_n$. Suppose that $z_1 \in U_1 \cap Z$ such that $d_1(z_1, x_2, \dots, x_n) \neq 0$ for all $x_2 \in U_2, \dots, x_n \in U_n$.

Replacing z_1 by $z_1 x_1$ in (3.7), we get

$$\begin{aligned} &d_1(z_1, x_2, \dots, x_n)\sigma(x_1)F_2(y_1, y_2, \dots, y_n) + \tau(z_1)F_1(x_1, x_2, \dots, x_n)F_2(y_1, y_2, \dots, y_n) \\ &\quad + F_2(y_1, y_2, \dots, y_n)F_1(x_1, x_2, \dots, x_n)\sigma(z_1) + F_2(y_1, y_2, \dots, y_n)\tau(x_1)d_1(z_1, x_2, \dots, x_n) = 0. \end{aligned}$$

Since $z_1 \in U_1 \cap Z\backslash\{0\}$ and $\sigma(U_1) = U_1, \tau(U_1) = U_1$, we have

$$\begin{aligned} &d_1(z_1, x_2, \dots, x_n)x_1F_2(y_1, y_2, \dots, y_n) + z_1\{F_1(x_1, x_2, \dots, x_n)F_2(y_1, y_2, \dots, y_n) \\ &\quad + F_2(y_1, y_2, \dots, y_n)F_1(x_1, x_2, \dots, x_n)\} + F_2(y_1, y_2, \dots, y_n)x_1d_1(z_1, x_2, \dots, x_n) = 0. \end{aligned}$$

Using (3.7) the middle summand is 0, we conclude that

$$d_1(z_1, x_2, \dots, x_n)x_1F_2(y_1, y_2, \dots, y_n) + F_2(y_1, y_2, \dots, y_n)x_1d_1(z_1, x_2, \dots, x_n) = 0.$$

Since $d_1(z_1, x_2, \dots, x_n) \in Z \setminus \{0\}$, we have

$$x_1F_2(y_1, y_2, \dots, y_n) + F_2(y_1, y_2, \dots, y_n)x_1 = 0. \tag{3.8}$$

Choosing $z_2 \in Z$ such that $d_2(z_2, y_2, \dots, y_n) \neq 0$ and replacing y_1 by z_2y_1 in (3.8), we have

$$\begin{aligned} &x_1\{d_2(z_2, y_2, \dots, y_n)\sigma(y_1) + \tau(z_2)F_2(y_1, y_2, \dots, y_n)\} \\ &\quad + \{F_2(y_1, y_2, \dots, y_n)\sigma(z_2) + \tau(y_1)d_2(z_2, y_2, \dots, y_n)\}x_1 = 0, \\ &x_1d_2(z_2, y_2, \dots, y_n)y_1 + xz_2F_2(y_1, y_2, \dots, y_n) + F_2(y_1, y_2, \dots, y_n)z_2x_1 + y_1d_2(z_2, y_2, \dots, y_n)x_1 = 0. \end{aligned}$$

Using (3.8), we obtain $x_1d_2(z_2, y_2, \dots, y_n)y_1 + y_1d_2(z_2, y_2, \dots, y_n)x_1 = 0$ and since $d_2(z_2, y_2, \dots, y_n) \in Z \setminus \{0\}$, we have

$$x_1y_1 + y_1x_1 = 0 \text{ for all } x_1, y_1 \in U_1. \tag{3.9}$$

Applying Lemma 2.5 with $S = U_1$ and $T = U_1^2$ shows U_1^2 centralizes U_1^2 so that $U_1^2 \subseteq Z$ by Lemma 2.6 and hence N is a commutative ring by Lemma 2.3. Further, it follows that

$$\begin{aligned} F_1(x_1, x_2, \dots, x_n)F_2(y_1, y_2, \dots, y_n) &= F_2(y_1, y_2, \dots, y_n)F_1(x_1, x_2, \dots, x_n) \\ &= -F_1(y_1, y_2, \dots, y_n)F_2(x_1, x_2, \dots, x_n) \end{aligned}$$

Hence, $F_1(U_1, U_2, \dots, U_n)F_2(U_1, U_2, \dots, U_n) = \{0\}$. Therefore $F_1 = 0$ or $F_2 = 0$.

Theorem 3.6: Let N be a 3-prime near ring and $U_1, U_2, \dots, U_n$ be nonzero semigroup ideals of N. Let σ,τ be mappings on N such that $\sigma(U_i) = U_i$ for $i = 1,2, \dots, n$. If F is a generalized (σ,τ) n-derivation associated (σ,τ) n-derivation d on N and $a \in N$ such that $[F(U_1, U_2, \dots, U_n), a]_{\sigma,\tau} = \{0\}$, then either $\sigma(a) \in Z$ or $d(a, a, \dots, a) = 0$.

Proof: For all $u_1 \in U_1$, $u_2 \in U_2, \dots, u_n \in U_n$ and $a \in N$, we have

$$F(u_1, u_2, \dots, u_n)\sigma(a) = \tau(a)F(u_1, u_2, \dots, u_n). \tag{3.10}$$

Replacing u_1 by au_1 in (3.10), we get

$$\begin{aligned} &\{d(a, u_2, \dots, u_n)\sigma(u_1) + \tau(a)F(u_1, u_2, \dots, u_n)\}\sigma(a) \\ &\quad = \tau(a)\{d(a, u_2, \dots, u_n)\sigma(u_1) + \tau(a)F(u_1, u_2, \dots, u_n)\}. \end{aligned}$$

Using Lemma 2.4 and (3.10), we get

$$\begin{aligned} &d(a, u_2, \dots, u_n)\sigma(u_1)\sigma(a) + \tau(a)F(u_1, u_2, \dots, u_n)\sigma(a) \\ &\quad = \tau(a)d(a, u_2, \dots, u_n)\sigma(u_1) + \tau(a)F(u_1, u_2, \dots, u_n)\sigma(a), \end{aligned}$$

$$d(a, u_2, \dots, u_n)u_1\sigma(a) = \tau(a)d(a, u_2, \dots, u_n)u_1. \tag{3.11}$$

Replacing u_1 by u_1v_1 for $v_1 \in U_1$ in (3.11) and using (3.11), we obtain

$$d(a, u_2, \dots, u_n)U_1[v_1, \sigma(a)] = 0 \text{ for all } v_1 \in U_1, u_2 \in U_2, \dots, u_n \in U_n.$$

By Lemma 2.2 (i), we get $d(a, u_2, \dots, u_n) = 0$ or $[v_1, \sigma(a)] = 0$. If $[v_1, \sigma(a)] = 0$, then

$$v_1\sigma(a) = \sigma(a)v_1 \text{ for all } v_1 \in U_1. \tag{3.12}$$

Replacing v_1 by $v_1 r$ for $r \in N$ in (3.12) and using (3.12), we obtain $U_1[r,\sigma(a)] = 0$. Hence $\sigma(a) \in Z$. If $d(a,u_2,\ldots,u_n) = 0$, then replacing u_2 by au_2, we get

$$d(a,a,\ldots,u_n)\sigma(u_2) + \tau(a)d(a,u_2,\ldots,u_n) = 0$$
$$d(a,a,u_3,\ldots,u_n)\sigma(u_2) = 0$$
$$d(a,a,u_3,\ldots,u_n)U_2 = \{0\}.$$

Using Lemma 2.2(ii), we get $d(a,a,u_3,\ldots,u_n) = 0$. Proceeding inductively, we conclude that $d(a,a,a,\ldots,a) = 0$.

Theorem 3.7: Let N be a 3-prime near ring and $U_1,U_2,\ldots,U_n$ be nonzero semigroup ideals of N. Let σ,τ be mappings on N such that σ is additive, $\sigma(U_i) = U_i$, $\tau(U_i) = U_i$ for $i = 1,2,\ldots,n$. If F_1,F_2 are nonzero generalized (σ,τ) $n-$derivations associated with (σ,τ) n- derivations d_1,d_2 on N respectively such that

$$F_1(x_1,x_2,\ldots,x_n)\sigma\,(F_2(y_1,y_2,\ldots,y_n)) + \tau(F_2(x_1,x_2,\ldots,x_n))F_1(y_1,y_2,\ldots,y_n) = 0$$

for all $x_1,y_1 \in U_1$; $x_2,y_2 \in U_2,\ldots,x_n,y_n \in U_n$, then $(N,+)$ is abelian.

Proof: By hypothesis, we have

$$F_1(x_1,x_2,\ldots,x_n)\sigma\left(F_2(y_1,y_2,\ldots,y_n)\right) + \tau\left(F_2(x_1,x_2,\ldots,x_n)\right)F_1(y_1,y_2,\ldots,y_n) = 0 \tag{3.13}$$

for all $x_1,y_1 \in U_1$, $x_2,y_2 \in U_2,\ldots,x_n,y_n \in U_n$.

If we choose $u_1,u_1' \in U_1$ such that $u_1+u_1' \in U_1$, then replacing y_1 by u_1+u_1' in (3.13), we get

$$F_1(x_1,x_2,\ldots,x_n)\sigma\,(F_2(u_1+u_1',y_2,\ldots,y_n)) + \tau\left(F_2(x_1,x_2,\ldots,x_n)\right)F_1(u_1+u_1',y_2,\ldots,y_n) = 0,$$

$$F_1(x_1,x_2,\ldots,x_n)\sigma\,\{F_2(u_1,y_2,\ldots,y_n) + F_2(u_1',y_2,\ldots,y_n)\} + \tau(F_2(x_1,x_2,\ldots,x_n))$$
$$F_1(u_1,y_2,\ldots,y_n) + \tau\left(F_2(x_1,x_2,\ldots,x_n)\right)F_1(u_1',y_2,\ldots,y_n) = 0,$$

$$F_1(x_1,x_2,\ldots,x_n)\sigma(F_2(u_1,y_2,\ldots,y_n)) + F_1(x_1,x_2,\ldots,x_n)\sigma\,(F_2(u_1',y_2,\ldots,y_n))$$
$$+\tau\left(F_2(x_1,x_2,\ldots,x_n)\right)F_1(u_1,y_2,\ldots,y_n) + \tau\left(F_2(x_1,x_2,\ldots,x_n)\right)F_1(u_1',y_2,\ldots,y_n) = 0.$$

Using (3.13), we get

$$F_1(x_1,x_2,\ldots,x_n)\sigma(F_2(u_1,y_2,\ldots,y_n)) + F_1(x_1,x_2,\ldots,x_n)\sigma\,(F_2(u_1',y_2,\ldots,y_n))$$
$$F_1(x_1,x_2,\ldots,x_n)\sigma\,(F_2(-u_1,y_2,\ldots,y_n)\,) + F_1(x_1,x_2,\ldots,x_n)\sigma\,(F_2(-u_1',y_2,\ldots,y_n)) = 0.$$
$$F_1(x_1,x_2,\ldots,x_n)\sigma\,(F_2(u_1+u_1'-u_1-u_1',y_2,\ldots,y_n)) = 0,$$
$$F_1(U_1,U_2,\ldots,U_n)\sigma\,(F_2((u_1,u_1'),y_2,\ldots,y_n)) = \{0\}$$

for all $u_1,u_1' \in U_1$ such that $u_1+u_1' \in U_1, y_2 \in U_2,\ldots,y_n \in U_n$. Using Lemma 2.10, we get $\sigma\,(F_2((u_1,u_1'),y_2,\ldots,y_n)) = 0$. By hypothesis, we conclude that $F_2((u_1,u_1'),y_2,\ldots,y_n) = 0$ for all u_1, $u_1' \in U_1$ such that $u_1+u_1' \in U_1$, $y_2 \in U_2,\ldots,y_n \in U_n$. Arguing in the similar manner as in Theorem 3.2, we get the result.

The following examples show that the 3-primeness hypothesis in Theorems 3.1, 3.2, 3.3, 3.6 and 3.7 cannot be omitted.

Example 3.8: Let S be a left near ring. Let us consider

$$N = \left\{ \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} \middle| x,y,z \in S \right\}$$

Then N is a zero-symmetric left near ring with regard to matrix addition and matrix multiplication but not a 3-prime near ring. If we set $U = \left\{ \begin{pmatrix} 0 & 0 & y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \middle| y \in S \right\}$, then clearly U is a nonzero semigroup ideal of N. Define mappings $F_1,F_2,d_1,d_2 \colon N \times N \ldots \times N \to N$ by

$$F_1\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & z_1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & z_2 \\ 0 & 0 & 0 \end{pmatrix}\cdots\begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & z_n \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 & z_1z_2\ldots z_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$d_1\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & z_1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & z_2 \\ 0 & 0 & 0 \end{pmatrix}\cdots\begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & z_n \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 & x_1x_2\ldots x_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$F_2\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & z_1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & z_2 \\ 0 & 0 & 0 \end{pmatrix}\cdots\begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & z_n \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 & x_1x_2\ldots x_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$d_2\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & z_1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & z_2 \\ 0 & 0 & 0 \end{pmatrix}\cdots\begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & z_n \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 & z_1z_2\ldots z_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Define $\sigma,\tau: N \to N$ by $\sigma\begin{pmatrix}0 & x & y\\ 0 & 0 & z\\ 0 & 0 & 0\end{pmatrix} = \begin{pmatrix}0 & x & z\\ 0 & 0 & y\\ 0 & 0 & 0\end{pmatrix}$ and $\tau\begin{pmatrix}0 & x & y\\ 0 & 0 & z\\ 0 & 0 & 0\end{pmatrix} = \begin{pmatrix}0 & y & z\\ 0 & 0 & x\\ 0 & 0 & 0\end{pmatrix}$.

If we choose $U_1 = U_2 = \ldots = U_n = U$, then it is easy to see that F_1 and F_2 are nonzero generalized (σ,τ) n- derivations associated with (σ,τ) n- derivations d_1 and d_2 on N respectively such that the following situations hold:
(i) $[F_1(U_1,U_2,\ldots,U_n), F_2(U_1,U_2,\ldots,U_n)] = \{0\}$,
(ii) $F_1(U_1,U_2,\ldots,U_n) \circ F_2(U_1,U_2,\ldots,U_n) = \{0\}$,
(iii) $F_1(x_1,x_2,\ldots,x_n)\sigma(F_2(y_1,y_2,\ldots,y_n)) + \tau(F_2(x_1,x_2,\ldots,x_n))F_1(y_1,y_2,\ldots,y_n) = 0$
(iv) $[F_1(U_1,U_2,\ldots,U_n), a]_{\sigma,\tau} = \{0\}$ for all $x_1,y_1 \in U_1;\ x_2,y_2 \in U_2,\ldots;x_n,y_n \in U_n$; $\sigma(U_i) \neq U_i, \tau(U_i) \neq U_i$ for $i = 1,2,\ldots,n$, $\sigma(a) \notin Z$; $d_1(a,a,\ldots,a) \neq 0$ and $(N,+)$ is not abelian.

Example 3.9: Let S be a left near ring. Let us consider

$$N = \left\{\begin{pmatrix}0 & x & y\\ 0 & 0 & 0\\ 0 & z & 0\end{pmatrix} \middle| x,y,z \in S\right\}$$

Then N is a zero-symmetric left near ring with regard to matrix addition and matrix multiplication but not a 3-prime near ring. If we set $U = \left\{\begin{pmatrix}0 & x & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix} \middle| x \in S\right\}$, then clearly U is a nonzero semigroup ideal of N. Define mappings $F,d: N \times N \ldots \times N \to N$ by

$$F\left(\begin{pmatrix}0 & x_1 & y_1\\ 0 & 0 & 0\\ 0 & z_1 & 0\end{pmatrix}\begin{pmatrix}0 & x_2 & y_2\\ 0 & 0 & 0\\ 0 & z_2 & 0\end{pmatrix}\ldots\begin{pmatrix}0 & x_n & y_n\\ 0 & 0 & 0\\ 0 & z_n & 0\end{pmatrix}\right) = \begin{pmatrix}0 & y_1y_2\ldots y_n & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$d\left(\begin{pmatrix}0 & x_1 & y_1\\ 0 & 0 & 0\\ 0 & z_1 & 0\end{pmatrix}\begin{pmatrix}0 & x_2 & y_2\\ 0 & 0 & 0\\ 0 & z_2 & 0\end{pmatrix}\ldots\begin{pmatrix}0 & x_n & y_n\\ 0 & 0 & 0\\ 0 & z_n & 0\end{pmatrix}\right) = \begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & z_1z_2\ldots z_n & 0\end{pmatrix}$$

Define $\sigma,\tau: N \to N$ by $\sigma\begin{pmatrix}0 & x & y\\ 0 & 0 & 0\\ 0 & z & 0\end{pmatrix} = \begin{pmatrix}0 & z & y\\ 0 & 0 & 0\\ 0 & x & 0\end{pmatrix}$ and $\tau\begin{pmatrix}0 & x & y\\ 0 & 0 & 0\\ 0 & z & 0\end{pmatrix} = \begin{pmatrix}0 & y & 0\\ 0 & 0 & 0\\ 0 & x & 0\end{pmatrix}$

If we choose $U_1 = U_2 = \ldots = U_n = U$, then it is easy to see that F ia a nonzero generalized (σ,τ) n- derivation associated with (σ,τ) n- derivation d on N satisfying $F(U_1,U_2,\ldots,U_n) \subseteq Z$; $\sigma(U_i) \neq U_i$, $\tau(U_i) \neq U_i$ for $i = 1,2,\ldots,n$ and N is not a commutative ring.

REFERENCES

1. Bell H. E., *On derivations in near rings II*, Kluwer Academic Publ. Math. Appl. Dordr. 426, (1997), 191-197.
2. Bell H. E. and Argac N., *Derivations, product of derivations and commutativity in near rings*, Algebra Colloq. 8 (4), (2001), 399-407.
3. Ceven Y. and Ozturk M.A., *Some properties of symmetric bi-(σ,τ)-derivations in near- rings*, Commun. Korean Math. Soc., 22 (4) (2007), 487-491.
4. Majeed A. R. H. and Adhab E. F. *On Generalized $(\sigma,\tau) - n$-Derivations in Prime Near Rings*, Gen. Math. Notes, 31 (2), (2015), 66-87.
5. Ozturk M.A. and Yazarli H., *A note on permuting tri-derivation in near ring*, Gazi Uni. J. Science, 24 (4), (2011), 723-729.
6. Ozturk, M. A. and Jun, Y. B., *On trace of symmetric bi-derivations in near rings*, Inter. J. Pure and Appl. Math., 17 (2004), 95-102.
7. Park, K. H., *On prime and semiprime rings with symmetric n-derivations*, J. Chungcheong Math. Soc., 22 (2009), 451-458.
8. Park, K. H. and Jung, Y. S., *On permuting 3-derivations and commutativity in prime near rings*, Commun. Korean Math. Soc., 25 (2010), 1-9.

Source of support: Nil, Conflict of interest: None Declared.