

ANALYTIC MEAN PRIME LABELING OF SOME CYCLE RELATED GRAPHS

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ABSTRACT

Analytic mean prime labeling of a graph is the labeling of the vertices with  $\{0, 1, 2, \dots, p-1\}$  and the edges with mean of the absolute difference of the squares of the labels of the incident vertices if square difference is even or mean of the absolute difference of the squares of the labels of the incident vertices and one if square difference is odd. The greatest common incidence number of a vertex (**gcin**) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the **gcin** of each vertex of degree greater than one is one, then the graph admits analytic mean prime labeling. Here we identify some cycle related graphs for analytic mean prime labeling.

**Keywords:** Graph labeling, square difference, greatest common incidence number, prime labeling, analytic mean, cycles.

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1. INTRODUCTION

All graphs in this paper are simple, finite, connected and undirected. The symbol  $V(G)$  and  $E(G)$  denotes the vertex set and edge set of a graph  $G$ . The graph whose cardinality of the vertex set is called the order of  $G$ , denoted by  $p$  and the cardinality of the edge set is called the size of the graph  $G$ , denoted by  $q$ . A graph with  $p$  vertices and  $q$  edges is called a  $(p, q)$ - graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2], [3] and [4]. Some basic concepts are taken from [1] and [2]. The analytic mean labeling was defined by T Tharmaraj and P Sarasia in [5]. In this paper we introduced the concept of analytic mean prime labeling and proved that some cycle related graphs admit analytic mean prime labeling.

**Definition 1.1:** Let  $G$  be a graph with  $p$  vertices and  $q$  edges. The greatest common incidence number (**gcin**) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

2. MAIN RESULTS

**Definition 2.1:** Let  $G = (V, E)$  be a graph with  $p$  vertices and  $q$  edges. Define a bijection  $f : V(G) \rightarrow \{0, 1, 2, 3, \dots, p-1\}$  by  $f(v_i) = i-1$ ,  $1 \leq i \leq p$ . Define a 1-1 mapping  $f_{amp}^* : E(G) \rightarrow$  set of natural numbers  $N$  by

$$f_{amp}^*(uv) = \frac{|f(u)^2 - f(v)^2|}{2}, \text{ if } f(u)^2 - f(v)^2 \text{ is even.}$$
$$f_{amp}^*(uv) = \frac{|f(u)^2 - f(v)^2| + 1}{2}, \text{ if } f(u)^2 - f(v)^2 \text{ is odd.}$$

The induced function  $f_{amp}^*$  is said to be analytic mean prime labeling, if the **gcin** of each vertex of degree at least 2, is 1.

**Definition 2.2:** A graph which admits analytic mean prime labeling is called analytic mean prime graph.

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**Theorem 2.1:** Cycle  $C_n$  admits analytic mean prime labeling, if  $n$  is even.

**Proof:** Let  $G = C_n$  and let  $v_1, v_2, \dots, v_n$  are the vertices of  $G$

Here  $|V(G)| = n$  and  $|E(G)| = n$ .

Define a function  $f : V \rightarrow \{0, 1, 2, 3, \dots, n-1\}$  by  
 $f(v_i) = i-1, i = 1, 2, \dots, n$

Clearly  $f$  is a bijection.

For the vertex labeling  $f$ , the induced edge labeling  $f_{amp}^*$  is defined as follows

$$\begin{aligned} f_{amp}^*(v_i v_{i+1}) &= i, & i = 1, 2, \dots, n-1 \\ f_{amp}^*(v_1 v_n) &= \frac{n(n-2)}{2} + 1 \end{aligned}$$

Clearly  $f_{amp}^*$  is an injection.

$$\begin{aligned} \text{gcin of } (v_1) &= \text{gcd of } \{f_{amp}^*(v_1 v_2), f_{amp}^*(v_1 v_n)\} \\ &= \text{gcd of } \left\{1, \frac{n(n-2)}{2} + 1\right\} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{gcin of } (v_{i+1}) &= \text{gcd of } \{f_{amp}^*(v_i v_{i+1}), f_{amp}^*(v_{i+1} v_{i+2})\} \\ &= \text{gcd of } \{i, i+1\} \\ &= 1, \end{aligned} \quad i = 1, 2, \dots, n-2$$

$$\begin{aligned} \text{gcin of } (v_n) &= \text{gcd of } \{f_{amp}^*(v_{n-1} v_n), f_{amp}^*(v_1 v_n)\} \\ &= \text{gcd of } \left\{n-1, \frac{n(n-2)}{2} + 1\right\} \\ &= 1 \end{aligned}$$

So, **gcin** of each vertex of degree greater than one is 1.

Hence  $C_n$ , admits analytic mean prime labeling.

**Theorem 2.2:** Let  $G$  be the graph obtained by joining an edge to each vertex of cycle  $C_n$ .  $G$  is called crown graph.  $G$  admits analytic mean prime labeling.

**Proof:** Let  $G$  be the graph and let  $v_1, v_2, \dots, v_{2n}$  are the vertices of  $G$

Here  $|V(G)| = 2n$  and  $|E(G)| = 2n$ .

Define a function  $f : V \rightarrow \{0, 1, 2, 3, \dots, 2n-1\}$  by  
 $f(v_i) = i-1, i = 1, 2, \dots, 2n$

Clearly  $f$  is a bijection.

For the vertex labeling  $f$ , the induced edge labeling  $f_{amp}^*$  is defined as follows

$$\begin{aligned} f_{amp}^*(v_{2i-1} v_{2i}) &= 2i-1, & i = 1, 2, \dots, n. \\ f_{amp}^*(v_{2i} v_{2i+2}) &= 4i, & i = 1, 2, \dots, n-1. \\ f_{amp}^*(v_2 v_{2n}) &= 2n^2-2n. \end{aligned}$$

Clearly  $f_{amp}^*$  is an injection.

$$\begin{aligned} \text{gcin of } (v_2) &= \text{gcd of } \{f_{amp}^*(v_1 v_2), f_{amp}^*(v_2 v_4)\} \\ &= \text{gcd of } \{1, 4\} = 1. \end{aligned}$$

$$\begin{aligned} \text{gcin of } (v_{2i+2}) &= \text{gcd of } \{f_{amp}^*(v_{2i} v_{2i+2}), f_{amp}^*(v_{2i+1} v_{2i+2})\} \\ &= \text{gcd of } \{4i, 2i+1\} \\ &= \text{gcd of } \{2i, 2i+1\} = 1, \end{aligned} \quad i = 1, 2, \dots, n-1.$$

So, **gcin** of each vertex of degree greater than one is 1.

Hence  $G$ , admits analytic mean prime labeling.

**Theorem 2.3:** Two copies of cycle  $C_n$  ( $n > 3$ ) sharing a common edge admits analytic mean prime labeling.

**Proof:** Let  $G$  be the graph and let  $v_1, v_2, \dots, v_{2n-2}$  are the vertices of  $G$

Here  $|V(G)| = 2n-2$  and  $|E(G)| = 2n-1$ .

Define a function  $f : V \rightarrow \{0,1,2,3,\dots,2n-3\}$  by  
 $f(v_i) = i-1, i = 1,2,\dots,2n-2$ .

Clearly  $f$  is a bijection.

For the vertex labeling  $f$ , the induced edge labeling  $f_{amp}^*$  is defined as follows

$$\begin{aligned} f_{amp}^*(v_i v_{i+1}) &= i, & i = 1, 2, \dots, 2n-3 \\ f_{amp}^*(v_1 v_{2n-2}) &= 2n^2-6n+5. \end{aligned}$$

**Case (i):**  $n$  is even

$$f_{amp}^*\left(\frac{v_n}{2} \frac{v_{3n-2}}{2}\right) = \frac{2n^2-5n+4}{2}.$$

**Case (ii):**  $n$  is odd

$$f_{amp}^*\left(\frac{v_{n+1}}{2} \frac{v_{3n-1}}{2}\right) = n^2-2n+1.$$

Clearly  $f_{amp}^*$  is an injection.

$$\begin{aligned} \text{gcin of } (v_1) &= \text{gcd of } \{f_{amp}^*(v_1 v_2), f_{amp}^*(v_1 v_{2n-2})\} \\ &= \text{gcd of } \{1, 2n^2-6n+5\} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{gcin of } (v_{i+1}) &= \text{gcd of } \{f_{amp}^*(v_i v_{i+1}), f_{amp}^*(v_{i+1} v_{i+2})\} \\ &= 1, \end{aligned}$$

$$i = 1, 2, \dots, 2n-4$$

$$\begin{aligned} \text{gcin of } (v_{2n-2}) &= \text{gcd of } \{f_{amp}^*(v_{2n-3} v_{2n-2}), f_{amp}^*(v_1 v_{2n-2})\} \\ &= \text{gcd of } \{2n-3, 2n^2-6n+5\} \\ &= \text{gcd of } \{n-1, 2n-3\} \\ &= \text{gcd of } \{n-2, n-1\} = 1. \end{aligned}$$

So,  $\text{gcin}$  of each vertex of degree greater than one is 1.

Hence  $G$ , admits analytic mean prime labeling.

**Theorem 2.4:** Let  $G$  be the graph obtained by joining path  $P_m$  to any one vertex of a cycle  $C_n$ .  $G$  is called tadpole Graph.  $G$  admits analytic mean prime labeling, if  $n > 4$  and  $m < n$ .

**Proof:** Let  $G$  be the graph and let  $v_1, v_2, \dots, v_{n+m-1}$  are the vertices of  $G$

Here  $|V(G)| = n+m-1$  and  $|E(G)| = n+m-1$ .

Define a function  $f : V \rightarrow \{0,1,2,3,\dots,n+m-2\}$  by  
 $f(v_i) = i-1, i = 1,2, \dots, n+m-1$ .

Clearly  $f$  is a bijection.

For the vertex labeling  $f$ , the induced edge labeling  $f_{amp}^*$  is defined as follows

$$\begin{aligned} f_{amp}^*(v_i v_{i+1}) &= i, & i = 1, 2, \dots, n+m-2 \\ f_{msqsp}^*(v_1 v_n) &= \frac{(n-1)^2}{2}, & \text{if } n \text{ is odd} \\ &= \frac{(n-1)^2+1}{2}, & \text{if } n \text{ is even} \end{aligned}$$

Clearly  $f_{amp}^*$  is an injection.

$$\begin{aligned} \text{gcin of } (v_1) &= \text{gcd of } \{f_{amp}^*(v_1 v_2), f_{amp}^*(v_1 v_n)\} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{gcin of } (v_{i+1}) &= \text{gcd of } \{f_{amp}^*(v_i v_{i+1}), f_{amp}^*(v_{i+1} v_{i+2})\} \\ &= 1, \end{aligned}$$

$$i = 1, 2, \dots, n+m-3$$

So,  $\text{gcin}$  of each vertex of degree greater than one is 1.

Hence  $G$ , admits analytic mean prime labeling.

**Theorem 2.5:** Let G be the graph obtained by joining two copies of path  $P_n$  to two consecutive vertices of cycle  $C_n$ . G admits analytic mean prime labeling, if  $n > 3$ .

**Proof:** Let G and let  $v_1, v_2, \dots, v_{3n-2}$  are the vertices of G

Here  $|V(G)| = 3n-2$  and  $|E(G)| = 3n-2$ .

Define a function  $f: V \rightarrow \{0, 1, 2, 3, \dots, 3n-3\}$  by  
 $f(v_i) = i-1, i = 1, 2, \dots, 3n-2$ .

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling  $f_{amp}^*$  is defined as follows

$$\begin{aligned} f_{amp}^*(v_i v_{i+1}) &= i, & i = 1, 2, \dots, 3n-3 \\ f_{amp}^*(v_n v_{2n-1}) &= \frac{3n^2-6n+3}{2}, \text{ if } n \text{ is odd.} \\ &= \frac{3n^2-6n+4}{2}, \text{ if } n \text{ is even.} \end{aligned}$$

Clearly  $f_{amp}^*$  is an injection.

$$\begin{aligned} \text{gcin of } (v_{i+1}) &= \text{gcd of } \{f_{amp}^*(v_i v_{i+1}), f_{amp}^*(v_{i+1} v_{i+2})\} \\ &= 1, & i = 1, 2, \dots, 3n-4 \end{aligned}$$

So, **gcin** of each vertex of degree greater than one is 1.

Hence G, admits analytic mean prime labeling.

**Theorem 2.6:** Let G be the graph obtained by joining two copies of cycle  $C_n$  by an edge. G is called dum bell graph. G admits analytic mean prime labeling, when n is odd and  $n > 5$ .

**Proof:** Let G be the graph and let  $v_1, v_2, \dots, v_{2n}$  are the vertices of G

Here  $|V(G)| = 2n$  and  $|E(G)| = 2n+1$ .

Define a function  $f: V \rightarrow \{0, 1, 2, 3, \dots, 2n-1\}$  by  
 $f(v_i) = i-1, i = 1, 2, \dots, 2n$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling  $f_{amp}^*$  is defined as follows

$$\begin{aligned} f_{amp}^*(v_i v_{i+1}) &= i, & i = 1, 2, \dots, 2n-1 \\ f_{amp}^*(v_1 v_n) &= \frac{(n-1)^2}{2}. \\ f_{amp}^*(v_{n+1} v_{2n}) &= \frac{3n^2-4n+1}{2}. \end{aligned}$$

Clearly  $f_{amp}^*$  is an injection.

$$\begin{aligned} \text{gcin of } (v_1) &= \text{gcd of } \{f_{amp}^*(v_1 v_2), f_{amp}^*(v_1 v_n)\} \\ &= 1 \\ \text{gcin of } (v_{i+1}) &= 1, & i = 1, 2, \dots, 2n-2 \\ \text{gcin of } (v_{2n}) &= \text{gcd of } \{f_{amp}^*(v_{n+1} v_{2n}), f_{amp}^*(v_{2n-1} v_{2n})\} \\ &= \text{gcd of } \{2n-1, \frac{3n^2-4n+1}{2}\} \\ &= 1 \end{aligned}$$

So, **gcin** of each vertex of degree greater than one is 1.

Hence G, admits analytic mean prime labeling.

**Theorem 2.7:** Dum bell graph admits analytic mean prime labeling, when n is even  $> 5$  and  $(n-2) \not\equiv 0 \pmod{6}$ .

**Proof:** Let G be the graph and let  $v_1, v_2, \dots, v_{2n}$  are the vertices of G

Here  $|V(G)| = 2n$  and  $|E(G)| = 2n+1$ .

Define a function  $f: V \rightarrow \{0,1,2,3,\dots,2n-1\}$  by  
 $f(v_i) = i-1, i = 1,2,\dots,2n$

Clearly  $f$  is a bijection.

For the vertex labeling  $f$ , the induced edge labeling  $f_{amp}^*$  is defined as follows

$$\begin{aligned} f_{amp}^*(v_i v_{i+1}) &= i, & i = 1,2,\dots,2n-1 \\ f_{amp}^*(v_1 v_n) &= \frac{n^2-2n+2}{2}. \\ f_{amp}^*(v_{n+1} v_{2n}) &= \frac{3n^2-4n+2}{2}. \end{aligned}$$

Clearly  $f_{amp}^*$  is an injection.

$$\begin{aligned} \text{gcin of } (v_1) &= \text{gcd of } \{f_{amp}^*(v_1 v_2), f_{amp}^*(v_1 v_n)\} \\ &= 1 \\ \text{gcin of } (v_{i+1}) &= 1, & i = 1,2,\dots,2n-2 \\ \text{gcin of } (v_{2n}) &= \text{gcd of } \{f_{amp}^*(v_{n+1} v_{2n}), f_{amp}^*(v_{2n-1} v_{2n})\} \\ &= \text{gcd of } \{2n-1, \frac{3n^2-4n+2}{2}\} \\ &= 1 \end{aligned}$$

So,  $\text{gcin}$  of each vertex of degree greater than one is 1.

Hence  $G$ , admits analytic mean prime labeling.

**Theorem 2.8:** Let  $G$  be the graph obtained by duplicating a vertex in cycle  $C_n$ .  $G$  admits analytic mean prime labeling, if  $n$  is odd.

**Proof:** Let  $G$  be the graph and let  $v_1, v_2,\dots,v_{n+1}$  are the vertices of  $G$

Here  $|V(G)| = n+1$  and  $|E(G)| = n+2$ .

Define a function  $f: V \rightarrow \{0,1,2,3,\dots,n\}$  by  
 $f(v_i) = i-1, i = 1,2,\dots,n+1$

Clearly  $f$  is a bijection.

For the vertex labeling  $f$ , the induced edge labeling  $f_{amp}^*$  is defined as follows

$$\begin{aligned} f_{amp}^*(v_i v_{i+1}) &= i, & i = 1,2, \dots,n \\ f_{amp}^*(v_1 v_n) &= \frac{n^2-2n+1}{2}. \\ f_{amp}^*(v_{n+1} v_2) &= n^2-1. \end{aligned}$$

Clearly  $f_{amp}^*$  is an injection.

$$\begin{aligned} \text{gcin of } (v_1) &= \text{gcd of } \{f_{amp}^*(v_1 v_2), f_{amp}^*(v_1 v_n)\} \\ &= 1 \\ \text{gcin of } (v_{i+1}) &= 1, & i = 1,2, \dots,n-1 \\ \text{gcin of } (v_{n+1}) &= \text{gcd of } \{f_{amp}^*(v_{n+1} v_n), f_{amp}^*(v_{n+1} v_2)\} \\ &= \text{gcd of } \{n, n^2-1\} \\ &= 1 \end{aligned}$$

So,  $\text{gcin}$  of each vertex of degree greater than one is 1.

Hence  $G$ , admits analytic mean prime labeling.

**Theorem 2.9:** 2- tuple graph of cycle  $C_n$  admits analytic mean prime labeling.

**Proof:** Let  $G = T^2(C_n)$  and let  $v_1, v_2,\dots,v_{2n}$  are the vertices of  $G$

Here  $|V(G)| = 2n$  and  $|E(G)| = 3n$ .

Define a function  $f: V \rightarrow \{0,1,2,3,\dots,2n-1\}$  by  
 $f(v_i) = i-1, i = 1,2,\dots,2n$

Clearly  $f$  is a bijection.

For the vertex labeling  $f$ , the induced edge labeling  $f_{amp}^*$  is defined as follows

$$\begin{aligned} f_{amp}^*(v_{2i-1} v_{2i}) &= 2i-1, & i = 1, 2, \dots, n. \\ f_{amp}^*(v_{2i-1} v_{2i+1}) &= 4i-2, & i = 1, 2, \dots, n-1. \\ f_{amp}^*(v_{2i} v_{2i+2}) &= 4i, & i = 1, 2, \dots, n-1. \\ f_{amp}^*(v_1 v_{2n-1}) &= 2n^2-4n+2. \\ f_{amp}^*(v_2 v_{2n}) &= 2n^2-2n. \end{aligned}$$

Clearly  $f_{amp}^*$  is an injection.

$$\begin{aligned} \text{gcin of } (v_1) &= \text{gcd of } \{f_{amp}^*(v_1 v_2), f_{amp}^*(v_1 v_3)\} \\ &= \text{gcd of } \{1, 2\} = 1. \\ \text{gcin of } (v_2) &= \text{gcd of } \{f_{amp}^*(v_1 v_2), f_{amp}^*(v_2 v_4)\} \\ &= \text{gcd of } \{1, 4\} = 1. \\ \text{gcin of } (v_{2i+1}) &= \text{gcd of } \{f_{amp}^*(v_{2i-1} v_{2i+1}), f_{amp}^*(v_{2i+1} v_{2i+2})\} \\ &= \text{gcd of } \{4i-2, 2i+1\} \\ &= \text{gcd of } \{2i-1, 2i+1\} \\ &= \text{gcd of } \{2i-1, 2\} = 1, & i = 1, 2, \dots, n-1. \\ \text{gcin of } (v_{2i+2}) &= \text{gcd of } \{f_{amp}^*(v_{2i} v_{2i+2}), f_{amp}^*(v_{2i+1} v_{2i+2})\} \\ &= \text{gcd of } \{4i, 2i+1\} \\ &= \text{gcd of } \{2i, 2i+1\} = 1, & i = 1, 2, \dots, n-1. \end{aligned}$$

So,  $\text{gcin}$  of each vertex of degree greater than one is 1.

Hence  $T^2(P_n)$ , admits analytic mean prime labeling.

## REFERENCES

1. Apostol. Tom M, Introduction to Analytic Number Theory, Narosa, (1998).
2. F Harary, Graph Theory, Addison-Wesley, Reading, Mass, (1972).
3. Joseph A Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics (2016), #DS6, 1 – 408.
4. T K Mathew Varkey, Some Graph Theoretic Generations Associated with Graph Labeling, PhD Thesis, University of Kerala 2000.
5. T Thamaraj, P B Sarasija, Analytic Mean Graphs, International Journal of Mathematical Analysis, Vol. 8, No.12, 2014, 595-609.

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