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SYNCRONIZATION BEHAVIOUR OF THE MAGNETIC BINARY PROBLEM WITH VARIABLE MASS

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ABSTRACT

This article deals with the complete synchronization and anti synchronization behavior of the magnetic binary problem when the charged particle has the variable mass, both primaries are the oblate bodies and transferring the origin of the coordinate system to the small primary. Here we have designed a non linear controller based on the Lyapunov stability theory. Numerical simulations are performed to plot time series analysis graphs of the master system and the slave system which further illustrate the effectiveness of the proposed control technique. For validation of results by numerical simulations we used the mathematica when the primaries are Sun and Earth.
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Key words: *Magnetic binary problem; complete synchronization; Lyapunov stability theory; Jean's Law; variable mass.*

1. INTRODUCTION

Pec-ora and Carroll introduced a method to synchronize two identical chaotic systems with deferent initial conditions [19] and they demonstrated that chaotic synchronization could be achieved by driving or replacing one of the variables of a chaotic system with a variable of another similar chaotic device, the active control scheme proposed by E. W. Bai and K. E. Lonngren [2] has been received and successfully implemented in almost all the field of nonlinear sciences for synchronization for different systems with various techniques.

The synchronization problem via nonlinear control scheme is studied by Amir Abbas Emadzadeh, and Mohammad Haeri [1], M. Mossa Al-sawalha, M.S.M. Noorani in [11] and Mohd. Arif [15], [16] and [17] Chen and Han [5], Chen [6], Ju H. Park [9].

Jeans [10] has studied the two-body problem with variable mass. Omarov [18] has also discussed the restricted problem of perturbed motion of two bodies with variable mass. Shrivastava and Ishwar [20] have studied the circular restricted three body problem with variable mass with the assumption that the mass of the infinitesimal body varies with respect to time. Singh *et al.* [8] has discussed the non-linear stability of equilibrium points in the restricted three body with variable mass.

So many cases of the magnetic binaries problem have been studied by A. Mavragnais [12], [13] and [14], Bhatnagar *et al.* [3] and Bhatnagar and Mohd. Arif [4].

In this article we have discussed the complete synchronization and anti synchronization behavior of the magnetic binary problem by taking into consideration the primaries as the oblate bodies when the charged particle has the variable mass and transferring the origin of the coordinate system to the small primary, here we have designed a nonlinear controller based on the Lyapunov stability in both cases. The system under consideration is chaotic for some values of parameter involved in the system. Here two systems (master and slave) are synchronized and start with deferent initial conditions. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time.

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2. EQUATION OF MOTION

The equation of motion in the rotating coordinate system including the effect of the gravitational forces of the primaries on the charged particle P with variable mass m written as: (Mohd. Arif [17])

$$\xi'' - 2\,\eta'\left[1 - \frac{\sqrt{\gamma}}{m_0}\left\{\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right\}\right] = \frac{\beta^2\,\xi}{4} - \frac{\beta\,\eta\,\gamma^{\frac{3}{2}}}{m_0}\left(\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right) - \frac{1}{m_0}\frac{\partial U}{\partial \xi} \tag{1}$$

$$\eta'' + 2\,\xi'\left[1 - \frac{\sqrt{\gamma}}{m_0}\left\{\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right\}\right] = \frac{\beta^2\eta}{4} + \frac{\beta\,\xi\,\gamma^{\frac{3}{2}}}{m_0}\left(\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right) - \frac{1}{m_0}\frac{\partial U}{\partial \eta} \tag{2}$$

Where

$$U = -\frac{m_0}{2}(\xi^2+\eta^2) - (\xi^2+\eta^2)\left\{\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right\}\sqrt{\gamma} - \xi\,\gamma\left\{\frac{\mu}{r_1^3} - \frac{\lambda(1-\mu)}{r_2^3}\right\} - m_0\sqrt[3]{\gamma}\left(\frac{(1-\mu)}{r_1} + \frac{\mu}{r_2}\right)$$

$$r_1^2 = (\xi-\mu)^2 + \eta^2,\quad r_2^2 = (\xi+1-\mu)^2 + \eta^2$$

Therefore, instead of dealing with the full equations of planar magnetic-binaries problem it makes more sense to work with a system of equations that describe the motion of the charged particle P in the vicinity of the secondary mass, this type of system was derived by Hill [7] By making some assumptions and transferring the origin of the coordinate system to the small primary the equations of motion (1) and (2), become

$$\alpha'' - 2\,\zeta'\left[\omega - \frac{\sqrt{\gamma}}{m_0}f\right] = \frac{\beta^2\,(\alpha+\mu-1)}{4} - \frac{\beta\,\zeta\,\gamma^{\frac{3}{2}}}{m_0}f - \frac{1}{m_0}\frac{\partial U}{\partial \alpha} \tag{3}$$

$$\zeta'' + 2\,\alpha'\left[\omega - \frac{\sqrt{\gamma}}{m_0}f\right] = \frac{\beta^2\zeta}{4} + \frac{\beta\,(\alpha+\mu-1)\,\gamma^{\frac{3}{2}}}{m_0}f - \frac{1}{m_0}\frac{\partial U}{\partial \zeta} \tag{4}$$

Where $f = \left(\frac{1}{\rho_1^3} + \frac{\lambda}{\rho_2^3} + \frac{I_1\,\gamma}{2(1-\mu)\,\rho_1^5} + \frac{\lambda\,I_2\gamma}{2\mu\,\rho_2^5}\right)$

$$U = \frac{m_0\,\omega^2}{2}(\alpha^2+\zeta^2) + (\alpha^2+\zeta^2)\,\omega\sqrt{\gamma}\,f + \alpha\,\omega\left\{\frac{\mu\,\gamma}{\rho_1^3} + \frac{\mu\,I_1\,\gamma^2}{2(1-\mu)\,\rho_1^5} - \frac{\lambda(1-\mu)\gamma}{\rho_2^3} - \frac{\lambda(1-\mu)I_2\gamma^2}{2\mu\rho_2^5}\right\}$$
$$+m_0\left(\frac{(1-\mu)\,\gamma^{\frac{3}{2}}}{\rho_1} + \frac{I_1\,\gamma^{\frac{5}{2}}}{2\,\rho_1^3} + \frac{\mu\,\gamma^{\frac{3}{2}}}{\rho_2} + \frac{I_2\,\gamma^{\frac{5}{2}}}{2\,\rho_2^3}\right) \tag{5}$$

$\rho_1^2 = ((\alpha-\sqrt{\gamma})^2 + \zeta^2)$, $\rho_2^2 = (\alpha^2+\zeta^2)$, $\lambda = \frac{M_2}{M_1}$ (M_1, M_2 are the magnetic moments of the primaries which lies perpendicular to the plane of the motion).

$I_1 = (1-\mu)\left(\frac{R_e^2-R_p^2}{5}\right)$, $I_2 = \mu\left(\frac{R_e^2-R_p^2}{5}\right)$, $R_e =$ equatorial radii and $R_p =$ polar radii of the primary.

$\omega = 1 + \frac{3I_1}{2(1-\mu)} + \frac{3I_2}{2\mu}$.

The equations of the transformation corresponding to a translation along the ξ-axis are given by

$$\alpha = \xi + 1 - \mu,$$
$$\eta = \zeta.$$

This transformation locates the smaller primary at the origin of the new system and the bigger primary is at $\alpha = 1$, $\zeta = 0$.

3. COMPLETE SYNCRONIZATION

Let

$$\alpha = \alpha_1,\quad \alpha' = \alpha_2,\quad \zeta = \alpha_3,\quad \zeta' = \alpha_4$$

Then the equation (3) and (4) can be written as:

$$\alpha_1' = \alpha_2 \tag{6}$$

$$\alpha_2' = 2\alpha_4\,\omega + \alpha_1\left(\frac{\beta^2}{4} - \omega^2\right) + A_1 \tag{7}$$

$$\alpha_3' = \alpha_4 \tag{8}$$

$$\alpha_4' = -2\alpha_2\omega + \alpha_3\left(\frac{\beta^2}{4} - \omega^2\right) + A_2 \tag{9}$$

Where

$$A_1 = -\frac{1}{m_0}\left(\beta\,\alpha_3\,\gamma^{\frac{3}{2}} + 2\omega\,(\alpha_1+\mu-1)\sqrt{\gamma}\right)f$$
$$+\frac{\omega\sqrt{\gamma}}{m_0}\left((\alpha_1+\mu-1)^2 + \alpha_3^{\,2}\right)\left\{\frac{3\,(\alpha_1-\sqrt{\gamma})}{\rho_1^5} + \frac{5\,I_1\,\gamma(\alpha_1-\sqrt{\gamma})}{2(1-\mu)\rho_1^7} + \frac{3\,\lambda\alpha_3}{\rho_2^5} + \frac{5\,\lambda\,\gamma I_2\alpha_3}{2\mu\rho_2^7}\right\}$$
$$-\frac{\gamma\omega}{m_0}\left\{\frac{\mu}{\rho_1^3} + \frac{\gamma\mu\,I_1}{2(1-\mu)\rho_2^5} - \frac{\lambda(1-\mu)}{\rho_2^3} - \frac{\lambda\,\gamma(1-\mu)I_2}{2\mu\rho_2^5}\right\} + \frac{\gamma\omega\,(\alpha_1+\mu-1)}{m_0}$$
$$+\left\{\frac{-3\,\mu\,(\alpha_1-\sqrt{\gamma})}{\rho_1^5} - \frac{5\,\gamma I_1\,\mu(\alpha_1-\sqrt{\gamma})}{2(1-\mu)\rho_2^7} + \frac{3\,\lambda(1-\mu)\alpha_3}{\rho_2^5} + \frac{5\,\gamma I_2\,\lambda(1-\mu)\alpha_3}{2\mu\rho_2^7}\right\}$$
$$-\gamma^{\frac{3}{2}}\left\{\frac{(1-\mu)(\alpha_1-\sqrt{\gamma})}{\rho_1^3} + \frac{\mu\alpha_3}{\rho_2^3}\right\}$$
$$-\gamma^{\frac{3}{2}}\left\{\frac{3\,\gamma\,I_1\,(\alpha_1-\sqrt{\gamma}))}{2\,\rho_2^5} + \frac{3\,\gamma\,I_2\,\alpha_3}{2\,\rho_2^5}\right\} - \frac{2\,\alpha_4\sqrt{\gamma}}{m_0}f + \frac{\beta^2}{4}(\mu-1).$$

$$A_2 = \frac{1}{m_0}\left(\beta\, \alpha_1\, \gamma^{\frac{3}{2}} + 2\omega\sqrt{\gamma}\alpha_2 - 2\,\alpha_3\,\sqrt{\gamma}\right) f + \frac{\omega\,\alpha_3\sqrt{\gamma}}{m_0}\left((\alpha_1+\mu-1)^{\,2} + {\alpha_3}^2\right)$$
$$\left\{\frac{3}{\rho_1^5} + \frac{5\,\gamma I_1}{2(1-\mu)\rho_1^7} + \frac{3\,\lambda}{\rho_2^5} + \frac{5\,\gamma I_2\,\lambda}{2\mu\rho_2^7}\right\} + \frac{\gamma\,\omega(\alpha_1+\mu-1)\alpha_3}{m_0}\left\{\frac{3\,\mu}{\rho_1^5} + \frac{5\,\gamma I_1\,\mu}{2(1-\mu)\rho_1^7} - \frac{3\,\lambda(1-\mu)}{\rho_2^5} - \frac{5\,\gamma I_2\,\lambda(1-\mu)}{2\mu\rho_2^7}\right\}$$
$$+\gamma^{\frac{3}{2}}\,\alpha_3\left\{\frac{(1-\mu)}{\rho_1^3} + \frac{3\,\gamma\, I_1}{2\,\rho_2^5} + \frac{\mu}{\rho_2^3} + \frac{3\,\gamma\, I_2}{2\,\rho_2^5}\right\}.$$

$\rho_1^2 = ((\alpha_1 - \sqrt{\gamma})^2 + {\alpha_3}^2)$, $\rho_2^2 = ({\alpha_1}^2 + {\alpha_3}^2)$.
The system (6, 7, 8 and 9) is the master system. The state orbits of this system are shown in Figure (1) this figure shows that the system is chaotic.

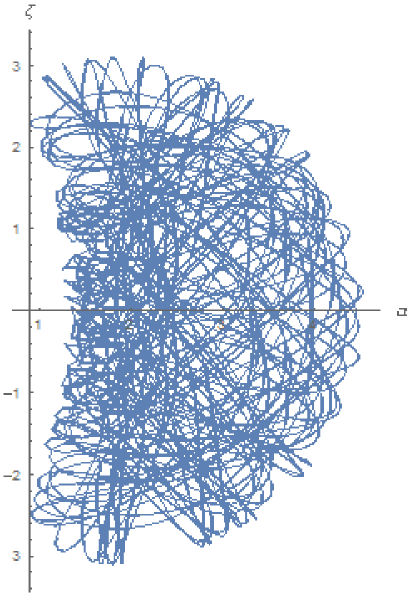


Figure-1

Corresponding to master system (6,7,8 and 9), the identical slave system is defined as:

$$\alpha'_{21} = \alpha_{22} + v_1 \tag{10}$$
$$\alpha'_{22} = 2\alpha_{24}\,\omega + \alpha_{21}\left(\frac{\beta^2}{4} - \omega^2\right) + B_1 + v_2 \tag{11}$$
$$\alpha'_{23} = \alpha_{24} + v_3 \tag{12}$$
$$\boldsymbol{\alpha'_{24} = -2\alpha_{22}\omega + \alpha_{23}\left(\frac{\beta^2}{4} - \omega^2\right) + B_2 + v_4} \tag{13}$$

where $v_i(t)$; i =1 ,2,3,4 are control functions to be determined.

Let $e_i = \alpha_{2i} - B_1 = -\frac{1}{m_0}\left(\beta\,\alpha_{23}\,\gamma^{\frac{3}{2}} + 2\omega\,(\alpha_{21}+\mu-1)\sqrt{\gamma}\right) f$
$$+\frac{\omega\sqrt{\gamma}}{m_0}\left((\alpha_{21}+\mu-1)^{\,2} + {\alpha_{23}}^2\right)\left\{\frac{3\,(\alpha_{21}-\sqrt{\gamma})}{\rho_1^5} + \frac{5\,I_1\,\gamma(\alpha_{21}-\sqrt{\gamma})}{2(1-\mu)\rho_1^7} + \frac{3\,\lambda\alpha_{23}}{\rho_2^5} + \frac{5\,\lambda\,\gamma I_2\alpha_{23}}{2\mu\rho_2^7}\right\}$$
$$-\frac{\gamma\omega}{m_0}\left\{\frac{\mu}{\rho_1^3} + \frac{\gamma\mu\, I_1}{2(1-\mu)\rho_2^5} - \frac{\lambda(1-\mu)}{\rho_2^3} - \frac{\lambda\,\gamma(1-\mu)I_2}{2\mu\rho_2^5}\right\} + \frac{\gamma\omega\,(\alpha_{21}+\mu-1)}{m_0}$$
$$+\left\{\frac{-3\,\mu\,(\alpha_{21}-\sqrt{\gamma})}{\rho_1^5} - \frac{5\,\gamma I_1\,\mu(\alpha_{21}-\sqrt{\gamma})}{2(1-\mu)\rho_2^7} + \frac{3\,\lambda(1-\mu)\alpha_{23}}{\rho_2^5} + \frac{5\,\gamma I_2\,\lambda(1-\mu)\alpha_{23}}{2\mu\rho_2^7}\right\} - \gamma^{\frac{3}{2}}\left\{\frac{(1-\mu)(\alpha_{21}-\sqrt{\gamma})}{\rho_1^3} + \frac{\mu\alpha_{23}}{\rho_2^3}\right\}$$
$$-\gamma^{\frac{3}{2}}\left\{\frac{3\,\gamma\, I_1\,(\alpha_{21}-\sqrt{\gamma}))}{2\,\rho_2^5} + \frac{3\,\gamma\, I_2\,\alpha_{23}}{2\,\rho_2^5}\right\} - \frac{2\,\alpha_{24}\sqrt{\gamma}}{m_0}\, f + \frac{\beta^2}{4}(\mu-1).$$

$$B_2 = \frac{1}{m_0}\left(\beta\,\alpha_{21}\,\gamma^{\frac{3}{2}} + 2\omega\sqrt{\gamma}\alpha_{22} - 2\,\alpha_{23}\,\sqrt{\gamma}\right) f + \frac{\omega\,\alpha_{23}\sqrt{\gamma}}{m_0}\left((\alpha_{21}+\mu-1)^{\,2} + {\alpha_{23}}^2\right)$$
$$\left\{\frac{3}{\rho_1^5} + \frac{5\,\gamma I_1}{2(1-\mu)\rho_1^7} + \frac{3\,\lambda}{\rho_2^5} + \frac{5\,\gamma I_2\,\lambda}{2\mu\rho_2^7}\right\} + \frac{\gamma\,\omega(\alpha_{21}+\mu-1)\alpha_{23}}{m_0}\left\{\frac{3\,\mu}{\rho_1^5} + \frac{5\,\gamma I_1\,\mu}{2(1-\mu)\rho_1^7} - \frac{3\,\lambda(1-\mu)}{\rho_2^5} - \frac{5\,\gamma I_2\,\lambda(1-\mu)}{2\mu\rho_2^7}\right\}$$
$$+\gamma^{\frac{3}{2}}\,\alpha_{23}\left\{\frac{(1-\mu)}{\rho_1^3} + \frac{3\,\gamma\, I_1}{2\,\rho_2^5} + \frac{\mu}{\rho_2^3} + \frac{3\,\gamma\, I_2}{2\,\rho_2^5}\right\}.$$

$\rho_1^2 = ((\alpha_{21} - \sqrt{\gamma})^2 + {\alpha_{23}}^2)$, $\rho_2^2 = ({\alpha_{21}}^2 + {\alpha_{23}}^2)$.
α_i; i = 1, 2, 3, 4 be the synchronization errors. From (6) to (13), we obtain the error dynamics as follows:

$$e'_1 = e_2 + v_1 \tag{14}$$
$$e'_2 = 2\omega e_4 + \left(\frac{\beta^2}{4} - \omega^2\right) e_1 + B_1 - A_1 + v_2 \tag{15}$$
$$e'_3 = e_4 + v_3 \tag{16}$$
$$e'_4 = -2\omega e_2 + \left(\frac{\beta^2}{4} - \omega^2\right) e_3 + B_2 - A_2 + v_4 \tag{17}$$

Lyapunov stability theory state that when controller satisfies the assumption with $V(e) = \frac{1}{2}\, e^t\, e$ a positive definite function and the first derivative of this function $V' < 0$ the chaos synchronization of two identical systems (master and slave) for different initial conditions is achieved.

The first derivative of $V(e)$ Will be

$$V' = e_1(e_2 + v_1) + e_2\left\{2\omega e_4 + \left(\frac{\beta^2}{4} - \omega^2\right)e_1 + B_1 - A_1 + v_2\right\} + e_3(e_4 + v_3)$$
$$+e_4\left\{-2\omega e_2 + \left(\frac{\beta^2}{4} - \omega^2\right)e_3 + B_2 - A_2 + v_4\right\}$$

Therefore, if we choose the controller v as follows,

$$v_1 = -e_1 - \frac{\beta^2}{4}\, e_2 - e_2 \tag{18}$$
$$v_2 = -e_2 - B_1 + A_1 + \omega^2 e_1 \tag{19}$$
$$v_3 = -e_3 - e_4 - \frac{\beta^2}{4}\, e_4 \tag{20}$$
$$v_4 = -e_4 - B_2 + A_2 + \omega^2 e_3 \tag{21}$$

Then

$$V' = -{e_1}^2 - {e_2}^2 - {e_3}^2 - {e_4}^2 < 0 \tag{22}$$

Hence the error state

$$\lim_{t\to\infty} \|e(t)\| = 0$$

which gives asymptotic stability of the system. This means that the controlled chaotic systems (master and slave) are synchronized for deferent initial conditions.

4. NUMERICAL SIMULATION

We select the parameters $= 3.0034609314 \times 10^{-6}$, γ =.0001, β =.1 and λ=1, with the different initial conditions for master and slave systems. Simulation results for uncoupled system are presented in figures. 2, 4, 6 and 8 and that of controlled system are shown in figures.3, 5, 7 and 9 respectively..

It can be seen from the figures that the chaos-synchronization errors converge to zero rapidly.

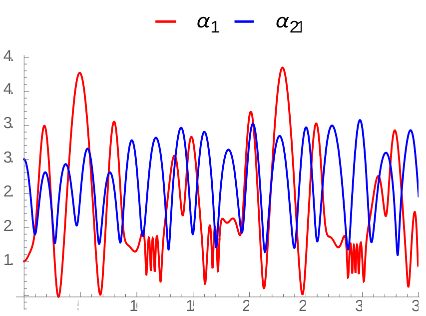


Figure-2

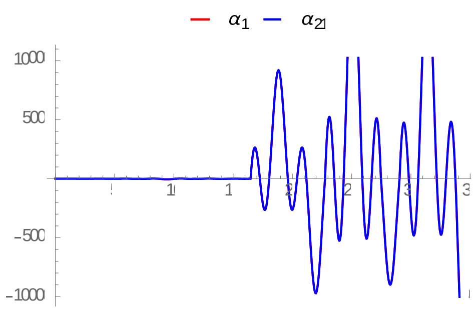


Figure-3

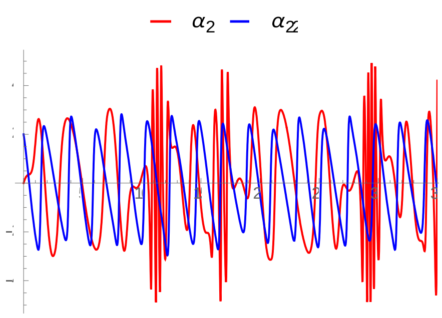
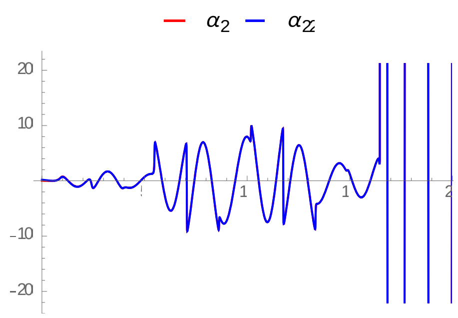


Figure-4



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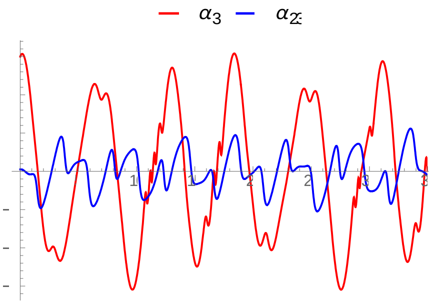

Figure-6

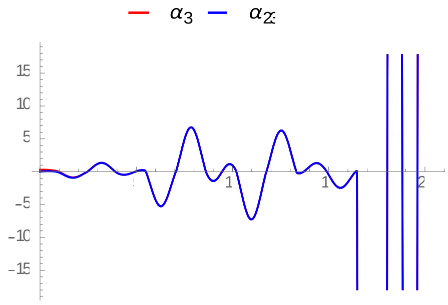

Figure-7

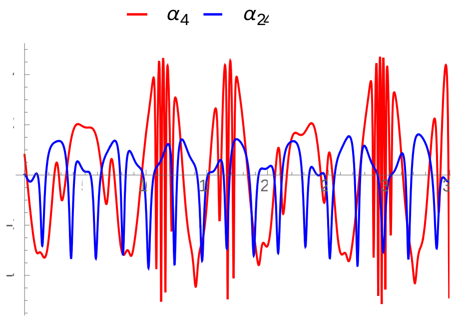

Figure-8

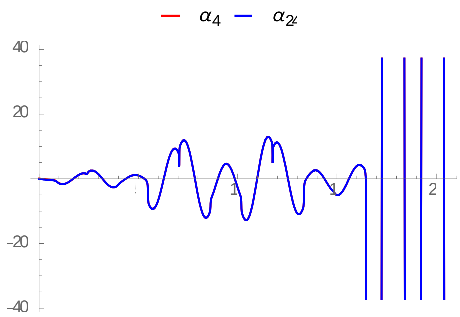

Figure-9

5. ANTI SYNCRONIZATION

To observe anti-synchronization between the master and the slave system, let $E_i = \alpha_{2i} + \alpha_i$; i = 1, 2, 3, 4 be the synchronization errors. Now from (6) to (13), we obtain the error dynamics as.

$$E_1' = E_2 + v_{11} \tag{23}$$

$$E_2' = 2\omega E_4 + \left(\frac{\beta^2}{4} - \omega^2\right) E_1 + B_1 + A_1 + v_{12} \tag{24}$$

$$E_3' = E_4 + u_{13} \tag{25}$$

$$E_4' = -2\omega E_2 + \left(\frac{\beta^2}{4} - \omega^2\right) E_3 + B_2 + A_2 + v_{14} \tag{26}$$

Now the first derivative of $V(e)$ Will be

$$V' = E_1(E_2 + v_{11}) + E_2\left\{2\omega E_4 + \left(\frac{\beta^2}{4} - \omega^2\right) E_1 + B_1 + A_1 + v_{12}\right\} + E_3(E_4 + v_{13}) + +E_4\left\{-2\omega E_2 + \left(\frac{\beta^2}{4} - \omega^2\right) E_3 + B_2 + A_2 + v_{14}\right\}$$

Therefore, if we choose the controller v as follows,

$$v_{11} = -E_1 - \frac{\beta^2}{4} E_2 - E_2 \tag{27}$$

$$v_{12} = -E_2 - B_1 - A_1 + \omega^2 E_1 \tag{28}$$

$$v_{13} = -E_3 - E_4 - \frac{\beta^2}{4} E_4 \tag{29}$$

$$v_{14} = -E_4 - B_2 - A_2 + \omega^2 E_3 \tag{30}$$

Then

$$V' = -{E_1}^2 - {E_2}^2 - {E_3}^2 - {E_4}^2 < 0 \tag{31}$$

Hence the error state

$$\lim_{t\to\infty} \|E(t)\| = 0$$

which gives asymptotic stability of the system. This means that the controlled chaotic systems (master and slave) are Anti synchronized for deferent initial conditions.

6. NUMERICAL SIMULATION

We select the parameters= $3.0034609314 \times 10^{-6}$, γ =.0001, β =.1 and λ=1 with the different initial conditions for master and slave systems and anti synchronization is achieved between the master and slave systems. Time Series Analysis graphs of the above are shown next to each via figures 10 to 13.

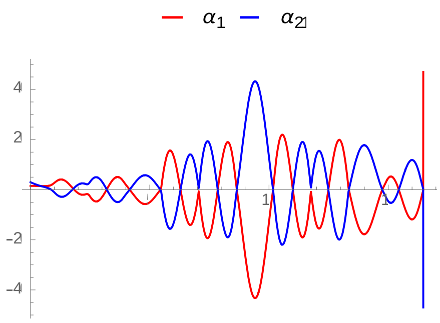

Figure-10

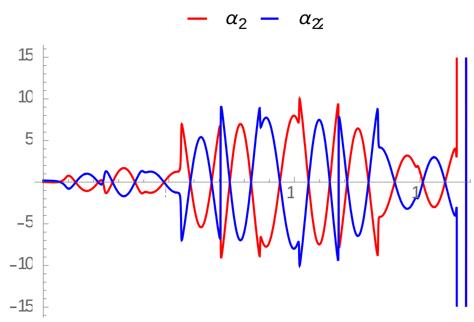

Figure-11

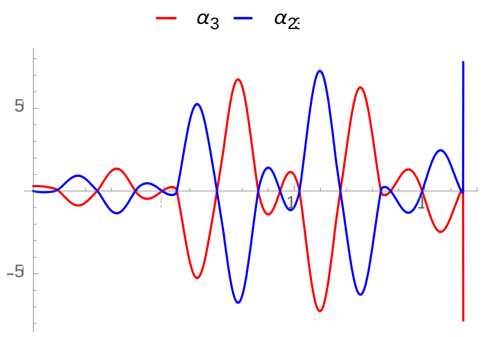

Figure-12

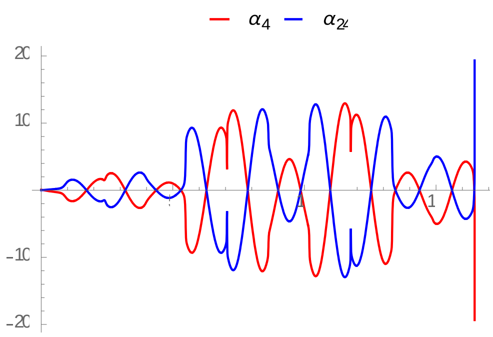

Figure-13

7. CONCLUSION

An investigation on complete synchronization and anti synchronization behavior of the magnetic binary problem when the charged particle has the variable mass, both primaries are the oblate bodies and transferring the origin of the coordinate system to the small primary via non linear controller based on the Lyapunov stability theory have been made. Here two systems (master and slave) are compete synchronized and start with deferent initial conditions. This problem may be treated as the design of control laws for chaotic slave system using the known information of the master system so as to ensure that the controlled receiver synchronizes with master system. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time. For validation of results by numerical simulations we used the mathematica when the primaries are Sun and Earth.

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