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On ωI -continuous functions in Ideal topological spaces

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ABSTRACT

In this paper, ωI -closed sets and, ωI –open sets are used to define and investigate a new class of functions and relationship between this new class and other classes of functions are established.

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Key words: *ideal, local function, semi-I-open set, ω-closed, ωI-closed sets and ωI -continuity.*

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1. Introduction and preliminaries:

Topological ideal plays an important role in topology for several years. In 1992, Jankovic and Hamlett [6] introduced the notion of I-open sets in topological spaces .El-monsef et.al [6] investigated I-open sets and I-continuous functions introduced and investigated the notion of ωI-closed sets. Quite recently, Hatir and Noiri [7] have introduced the notion of semi-I-open sets and semi-I-continuous function to obtain a decomposition of continuity via ideals.

In this paper, by using ωI-closed sets due to N. chandramathi et. al [3], we introduce the notion of ωI -continuous functions in ideal topological spaces and obtain several properties of ωI -continuity and the relationship between this function and other related functions .

An ideal I on a topological space (X,τ) is a Collection of subsets of X which satisfies

(i) $A\in I$ and $B\subset A$ implies $B\in I$ and

(ii) $A\in I$ and $B\in I$ implies $A\bigcup B\in I$.

Given a topological space $(X,\tau,)$ with an ideal I on X and if $\wp$(X) is the set of all subsets of X, a set operator $(.)^*$: $\wp(X)\rightarrow\wp(X)$, called a local function [7] of A with respect to τ and I is defined as follows: for $A\subset X$, $A^*(I,\tau)=\{x\in X / U\bigcap A\notin I$ for every $U\in\tau(x)\}$ where $\tau(x) = U \in\tau \mid x\in U$. We will make use of the basic facts about the local functions without mentioning it explicitly. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(I,\tau)$, called the * – topology, finer than τ and is defined by $cl^*(A)=A\bigcup A^*(I,\tau)$.When there is no chance for confusion we simply write A^* instead $A^*(I,\tau)$and τ^* or $\tau^*(I)$for $A^*(I,\tau)$. X* is often a proper subset of X. The hypothesis X = X* is equivalent to the hypothesis $\tau\cap I=\phi$ For every ideal topological space (X,τ,I), there exists a topology $\tau^*(I)$, finer than τ, generated by $\beta(I,\tau) = \{U\setminus I : U\in\tau$ and $I \in I\}$, but in general $\beta(I,\tau)$ is not always a topology[10] .If is I an ideal on X, Then (X,τ,I) is called ideal space. By an ideal space we always mean an ideal topological space (X,τ,I) with no separation properties assumed. If $A\subset X$, $cl(A)$ and $\text{int}(A)$ will respectively denote the closure and interior of A in $(X,\tau,)$ and $cl^*(A)$and $\text{int}^*(A)$ will respectively denote the closure and interior of A in (X,τ,I)

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Definition: 1.1

A subset A of a space (X,τ) is called

(i) Semi open [7] if A ⊆ cl (int (A)) and semi closed if int(cl (A)) ⊆ A.

(ii) Pre-open [7] if A ⊆ int (cl (A)) and pre-closed if cl (int (A)) ⊆ A.

(iii) ω-closed if [7] Cl(A) ⊆ U whenever A ⊆ U and U is semi-open in X, ω-open if

X – A is ω – Closed.

A subset A of a space (X,τ,I) is called

(iv) Semi - I -open [7] if A $\subseteq$ Cl^*(Int (A)) semi–I-Closed if int (cl^* (A)) ⊆ A

(v) α-I-open [7] if A ⊂ Int (Cl^*(Int (A).

2. ωI – closed sets:

Definition: 2.1 A subset A of an ideal topological space (X,τ,I) is called ωI – closed[3] if $cl^*(A) \subseteq U$ whenever A ⊆ U and U is semi- I- open in (X,τ,I) . The complement of ωI -closed set is called ωI- open if X – A is ωI – closed. We denote the family of all ωI – closed sets by ωIC(X,τ,I) .

Theorem: 2.1 Every open set is ωI –open.

Proof: Let U be an open set. We need to show that U is ωI –open. For this we show that X-U is ωI –closed. Let X-U $\subset G$ where G is semi-I-Open in X. Since X-U is closed. So by [8, Theorem2.3] or $cl^*(X-U) \subseteq cl(X-U) \subseteq G$ .this proves that X-U is ωI –closed or U is ωI –open.

Theorem: 2.2 A set A is ωI –open iff $F \subseteq \operatorname{int}^*(A)$ whenever F is semi –I-closed and $F \subseteq A$ **Proof:** Suppose that $F \subseteq \operatorname{int}^*(A)$, where F is semi-I-closed and $F \subseteq A$.Let $A^c \subseteq U$, where U is semi-I-open .Then $U^c \subseteq A$ and U^c is semi-I-closed. Therefore, $U^c \subseteq \operatorname{int}^*(A)$ .Since $U^c \subseteq \operatorname{int}^*(A)$, we have $(\operatorname{int}^*(A))^c \subseteq U$,i.e $cl^*(A^c) \subseteq U$, since $cl^*(A^c) = (\operatorname{int}^*(A))^c$ .Thus A^c is ωI –closed, i.e. A is ωI –open.

Theorem: 2.3 If A is an ωI –open set of (X,τ,I) such that $\operatorname{int}^*(A) \subseteq B \subseteq A$ then B is also an ωI –open set of (X,τ,I)

Proof: Let U be a semi-I-closed set of (X,τ,I)such that $U \subset B$ .Then, $U \subset A$ .since A is ωI –open set, we have $F \subseteq \operatorname{int}^*(A)$ .but $\operatorname{int}^*(A) \subseteq \operatorname{int}^*(B)$, implies $F \subseteq \operatorname{int}^*(B)$ .Therefore by theorem 2.2, B is also an ωI –open set of (X,τ,I) .

Theorem: 2.4 Let (X,τ,I) be an ideal space and A a non empty subset of X .Then A is ωI –closed if and only if $A \bigcup (X - cl^*(A))$ is ωI –closed.

Proof: Suppose A is ω I –closed. Let U be a semi-I-open set such that $A \bigcup (X - cl^*(A)) \subset U$. Then $X - U \subset X - (A \bigcup (X - cl^*(A)) = cl^*(A) - A.$

Since A is ωI –closed by [4, theorem 2.10] $X - U = \phi$ and hence $X = U$ .Thus X is the only set containing $A \bigcup (X - cl^*(A))$ .This gives, $[A \bigcup (X - cl^*(A))]^* \subset X$.This proves, $A \bigcup (X - cl^*(A))$ is ωI –closed.

Conversely let F be any semi-I-closed set such that $F \subset cl^*(A) - A$. Since $cl^*(A) - A = X - (A \bigcup (X - cl^*(A))$. This gives $A \bigcup (X - cl^*(A) \subset X - F$ and $X - F$ is semi-I-open. By hypothesis $[A \bigcup (X - cl^*(A))]^* = X - F$

and hence $F \subset X - cl^*(A)$ .since $F \subset cl^*(A) - A$ it proves that $F = \phi$ and hence $cl^*(A) \subset X - F$ and $X - F$ is semi –I-closed. This proves that A is ωI –closed.

Theorem: 2.5 Let (X,τ,I) be an ideal space and $A \subseteq X$ .Then $A \bigcup (X - cl^*(A))$ is ωI –closed if and only if $cl^*(A) - A$ is ωI –open.

Proof: Let $A \bigcup (X - cl^*(A))$ be ωI –closed. We show that $X - (cl^*(A) - A)$ is ωI –closed. Let U be a semi-I-open set containing $X - (cl^*(A) - A)$ Then $X - U \subseteq (cl^*(A) - A)$ .By theorem [4, theorem 2.10,] $X - U = \phi$. Therefore, X is the only semi-I-open set which contains $X - (cl^*(A) - A)$ and hence $(X - (cl^*(A) - A))^* \subseteq X$. this proves $X - (cl^*(A) - A)$ is ωI –closed or $cl^*(A) - A$ is ωI –open.

Conversely, let $cl^*(A) - A$ is ωI –open. Then $X - (cl^*(A) - A) = A \cup (X - cl^*(A))$ is ωI –closed.

Corollary: 2.1 Let (X,τ,I) be an ideal space and $A \subseteq X$ .Then A is ωI –closed if and only if $cl^*(A) - A$ is ωI –open.

Theorem: 2.6 For a subset $A \subseteq X$ the following are equivalent:

(i) A is ωI -closed

(ii) $cl^*(A) - A$ contains no non empty semi-I-closed set

(iii) $cl^*(A) - A$ is ωI-open.

Proof: (i) $\Leftrightarrow$ (ii) by [4, theorem2.10], and (i) $\Leftrightarrow$ (iii) by corollary above.

Theorem: 2.7 Let (X,τ,I) be an ideal space .Then every subset of X is ωI –closed if and only if every semi –I-open set is *-closed

Proof: Suppose every subset of X is ωI –closed .Let U be semi-I-open set then U is ωI –closed and $cl^*(A) \subseteq U$ implies $U^* \subseteq U$ .Hence U is *-closed.

Conversely, suppose that every semi –I-open set is *-closed. Let U be a non empty subset of X contained in a semi-I-open set U. Then $A^* \subseteq U^*$ implies $A^* \subseteq U$ .This proves that A is ωI –closed.

3. ωI -continuous functions:

Definition: 3.1 A function f: $(X,\tau,I) \to (Y,\sigma)$ is called ωI -continuous if for every closed set V of (Y,σ), $f^{-1}(V) \in \omega IC(X,\tau,I)$

Definition: 3.2 An ideal topological space (X,τ,I) is said to be T-dense [14] if every subset of X is *-dense in itself.

Remark: 3.1 Every continuous function is ωI -continuous but not conversely as seen from the following example.

Example: 3.1 Let $X = Y = \{a,b,c,d\}$, $\tau = \sigma = \{X,\phi,\{a,b\}\}$ $I = \{\phi,\{a\}\}$.

Define $f:(X,\tau,I) \to (Y,\sigma)$ by $f(a) = a, f(b) = c, f(c) = b, f(d) = d$ .Then f is ωI -continuous but not continuous. Since $U = \{a,b\}$ is ωI -open in Y but $f^{-1}(U) = \{a,c\}$ is not open in X.

Definition: 3.3 A space is (X,τ,I) called a T_ω^* space if every ωI -closed set in it is closed.

Theorem: 3.1 Let (X,τ,I) be an ideal topological space, (Z,η) be topological space and (Y,σ,J) be a T_ω^* space. Then the composition $g\circ f:(X,\tau,I)\to(Z,\eta)$ of the ωI -continuous maps f: $(X,\tau,I)\to(Y,\sigma,J)$ and g: $(Y,\sigma,J)\to(Z,\eta)$ is ωI -continuous.

Proof: Let F be any closed set of (Z,η) . Then $g^{-1}(F)$ is closed in closed in (Y,σ,J) . Since g is ωI -continuous and (Y,σ,J) is a T_ω^* space. Since $g^{-1}(F)$ is closed in (Y,σ,J) and f is ωI -continuous, $f^{-1}(g^{-1}(F))$ is ωI -closed in (X,τ,I) . But $f^{-1}(g^{-1}(F))=(g\circ f)^{-1}(F)$ and so $g\circ f$ is ωI -continuous.

Theorem: 3.2 Let f: $(X,\tau,I)\to(Y,\sigma)$ and g: $(Y,\sigma,J)\to$(Z,μ) be two functions where I and J are ideals on X and Y ,respectively. Then $g\circ f$ is ωI -continuous if f is ωI -continuous and g is continuous

Proof: Let $w\in$ μ. Then $(g\circ f)^{-1}=(f^{-1}\circ g^{-1})(w)=f^{-1}g^{-1}(w)$ since $g^{-1}(w)$ is closed ad g is ωI -continuous. Now since f is ωI -continuous. So $f^{-1}g^{-1}(w)$ is ωI -closed. Hence $g\circ f$ is ωI -continuous.

Definition: 3.4[1] If (X,τ,I) *is an* ideal topological space and A is a subset of X, we denote $\tau|_{A.}$ the relative topology on A and $I_A=\{A\cap I: I\in I\}$ is obviously an ideal on A.

Theorem: 3.3 Let f: $(X,\tau,I)\to(Y,\sigma)$ be a ωI -continuous function and let A be $*$ - closed, then the restriction $f|_A:(A,\tau|_A,I|_A)\to(Y,\sigma)$ is ωI -continuous.

Proof: Let V be any closed subset of (Y,σ) since f is ωI -continuous we have $f^{-1}(V)$ is ωI –closed. Also, $(f|_A)^{-1}=f^{-1}(V)\cap A$ .Since A is * -closed, then by [4, theorem 2.6] $f^{-1}(V)\cap A\in\omega IC(X,\tau)$ .On the other hand $(f|_A)^{-1}=A\cap f^{-1}(V)$ and $(f|_A)^{-1}\in\omega IC\ (A,\tau|_A,I|_A)$ .This shows that $f|_A:(A,\tau|_A,I|_A)$ is ωI -continuous.

Theorem: 3.4 Let f: $(X,\tau,I)\to(Y,\sigma,J)$ be a function and let $\{U_\alpha:\alpha\in\Delta\}$ be an open cover of a T-dense space X. If the restriction function $f|_{U\alpha}$ is ωI -continuous for each $\alpha\in\Delta$, then f is ωI -continuous.

Proof: Let V be an arbitrary open set in (Y,σ,J) .Then for each $\alpha\in\Delta$, we have $\left[(f|U_\alpha)^{-1}(V)\right]=\left(f^{-1}(V)\cap U_\alpha\right)$ .Because $f|_{U\alpha}$ is ωI -continuous, therefore $f^{-1}(V)\cap U_\alpha$ is ωI -open in X for each $\alpha\in\Delta$. Since for each $\alpha\in\Delta$, U_α is open in X, $f^{-1}(V)=\bigcup_{\alpha\in\Delta}\left(f^{-1}(V)\cap U_\alpha\right)$ is ωI -open .by [4, theorem 2.5] $\left[(f|U_\alpha)^{-1}(V)\right]$ is ωI -open. Then by proposition $f^{-1}(V)$ is ωI -open. Hence f is ωI -continuous.

Definition: 3.5 Let x be a point of (X,τ,I) and W be a subset of (X,τ,I) .Then W is called an ωI –neighborhood of x in (X,τ,I) if there exists an ωI –open set U of (X,τ,I) such that $x\in U\subseteq W$.

Theorem: 3.5 Let (X,τ,I) be a T-dense. Then for a function f: $(X,\tau,I)\to(Y,\sigma)$, the following are equivalent .

(i) The function f is ωI -continuous.

(ii) The inverse image of each open set is ωI -open.

(iii) For each point x in (X,τ,I) and each open set V in (Y,σ) with $f(x)\in V$, there is an ωI - Open set U in (X,τ,I) such that $x\in U, f(U)\subseteq V$.

(iv) The inverse image of each closed set in (Y,σ) is ωI -closed.

(v) For each x in (X,τ,I), the inverse of every neighborhood of f(x) is an ωI -neighborhood of x.

(vi) For each x in (X,τ,I) and each neighborhood N of f(x), there is an ωI -neighborhood W of x such that $f(W)\subseteq N$.

(vii) For each subset A of (X,τ,I), $f(\omega I-cl(A))\subseteq cl(f(A))$

(viii) For each subset B of (Y,σ), $\omega I-cl(f^{-1}(B))\subseteq f^{-1}(cl(B))$.

Proof: The implications $(i)\Leftrightarrow(ii)$: This follows from Theorem 3.1.

$(i)\Leftrightarrow(iii)$ Suppose that (iii) holds and let V be an open set in (Y,σ) and let $x\in f^{-1}(V)$ .Then $f(x)\in V$ and thus there exists an ωI -open set U, such that $x\in U_x$ and $f(U_x)\subseteq V$, Now, $x\in U_x\subseteq f^{-1}(V)$. Hence $f^{-1}(V)=\bigcup_{x\in f^{-1}(V)}U_x$ and so by theorem 2.6 [4] , $f^{-1}(V)$ is ωI -open in and (X,τ,I) therefore, f is ωI -continuous.

Conversely, suppose that (i) holds and let $f(x)\in V$. since f is ωI -continuous, $f^{-1}(V)$ is ωI -open in X. By putting $U=f^{-1}(V)$, we have $x\in U$ and

$(ii)\Leftrightarrow(iv)$ This result follows from the fact that if A is a subset of (Y,σ) , then $f^{-1}(A^c)=(f^{-1}(A))^c$.

$(ii)\rightarrow(v)$: For x in (X,τ,I) let N be a neighborhood of f(x) .Then there exists an open set U in (Y,σ) such that $f(x)\in U\subseteq N$. Consequently, $f^{-1}(U)$ is an ωI -open set in (X,τ,I) and $f^{-1}(f(x))\in f^{-1}(U)\subseteq f^{-1}(N)$. that is $x\in f^{-1}(U)\subseteq f^{-1}(N)$. Thus $f^{-1}(N)$ is an neighborhood of x.

$(v)\rightarrow(vi)$ .Let $x\in X$ and N be a neighborhood of f(x).Then by assumption, $W=f^{-1}(N)$ is an ωI -neighborhood of x and $f(W)=f(f^{-1}(N))\subseteq N$.

$(vi)\rightarrow(iii)$. For x in (X,τ,I),Let V be an open set containing f(x).Then V is a neighborhood of f(x), so by assumption, there exists an ωI - neighborhood W of x such that $f(W)\subseteq V$. Hence there exists an ωI - open set U in (X,τ,I) such that $x\in U\subseteq W$ and so $f(U)\subseteq f(W)\subseteq V$.

$(vii)\Leftrightarrow(iv)$: Suppose that (iv) holds and let A be a subset of (X,τ,I) .Since $A\subseteq f^{-1}(f(A))$, we have $A\subseteq f^{-1}(cl(f(A))$. Since $cl(f(A)$ is a closed set in (Y,σ), by assumption $f^{-1}(cl(f(A)))$ is an ωI -closed set containing A. Consequently, $\omega I-cl(A)\subseteq f^{-1}(cl(f(A)))$. Thus $f(\omega I-cl(A))\subseteq f(f^{-1}(cl(f(A))))\subseteq cl(f(A))$.

Conversely, suppose that (vii) holds for any subset A of (X,τ,I) . Let F be a closed subset of (Y,σ) .Then by assumption, i.e. $f(\omega I-cl(f^{-1}(F))\subseteq cl(f(f^{-1}(F)))\subseteq cl(F)=F$. $(\omega I-cl(f^{-1}(F))\subseteq f^{-1}(F)$ and so $f^{-1}(F)$ is ωI -closed.

$(vii)\Leftrightarrow(viii)$: Suppose that (vii) holds and B be any subset of (Y,σ) .Then replacing A by in $f^{-1}(B)$ in (vii), we obtain $f(\omega I-cl(f^{-1}(B))\subseteq cl(f(f^{-1}(B))\subseteq cl(B)$ i.e $(\omega I-cl(f^{-1}(B))\subseteq(f^{-1}cl(B))$.

Conversely, suppose that (viii) holds. Let B=f(A) where A is a subset of (X,τ,I). Then we have, $(\omega I-cl(A))\subseteq(\omega I-cl(f^{-1}(B))\subseteq f^{-1}(cl(f(A)))$ and so $f(\omega I-cl(A))\subseteq(cl(f(A)))$.

Theorem: 3.6 If $(X,\tau,I)$is a T-dense space and a function f: $(X,\tau,I)\to(Y,\sigma,J)$ is ωI -continuous, then the graph function $g:X\to X\times Y$ defined by $g(x)=(x,f(x))$ for each $x\in X$ is ωI -continuous.

Proof: Let f be ωI -continuous. Now let $x\in X$ and let W be any open set in $X\times Y$ containing $g(x)=(x,f(x))$ .Then there exists a basic open set $U\times V$ such that $g(x)\subset U\times V\subset W$. Since f is ωI -continuous, there exists a ωI -open set U_1 in X such that $x\in U_1\subset X$ and $f(U_1)\subset V$. Then by [4,Theorem 2.5]s $U_1\cap U\in\omega IO(X,\tau)$ and $U_1\cap U\subset U$,then $g(U_1\cap U)\subset U\times V\subset W$. This shows that g is ωI -continuous.

Theorem: 3.7 A function f: $(X,\tau)\to(Y,\sigma,J)$ is ωI -continuous, if the graph function $g:X\to X\times Y$ defined by $g(x)=(x,f(x))$ for each $x\in X$ is ωI -continuous.

Proof: Suppose that g is ωI -continuous and let V be open set in Y containing f(x).Then $X\times V$ is open set in $X\times Y$ and by ωI -continuous of g, there exists a ωI -open set U containing x such that $g(U)\subset X\times V$. Therefore, we obtain $f(U)\subset V$. This shows that f is ωI -continuous.

Theorem: 3.8 Let $\{X_\alpha:\alpha\in\Delta\}$ be any family of ideal topological spaces. If f: $(X,\tau,I)\to(\prod_{\alpha\in\Delta}X_\alpha,\sigma)$ is a ωI -continuous function, then $P_\alpha\circ f:X\to X_\alpha$ is ωI -continuous for each $\alpha\in\Delta$, where P_α is the projection of $\prod X_\alpha$ onto X_α.

Proof: We will consider a fixed $\alpha_\circ\in\Delta$.Let $G_{\alpha_\circ}$ be an open set of $X_{\alpha_\circ}$. Then $(P_{\alpha_\circ})^{-1}(G_{\alpha_\circ})$ is open in $\prod X_\alpha$. Since f is ωI -continuous, $f^{-1}((P_{\alpha_\circ})^{-1}(G_{\alpha_\circ}))=(P_{\alpha_\circ}\circ f)^{-1}(G_{\alpha_\circ})$ is ωI -open in X. Thus, $P_\alpha\circ f$ is ωI -continuous.

Theorem: 3.9 For any bijection f: $(X,\tau)\to(Y,\sigma,I)$, the following statements are equivalent:

(i) $f^{-1}:(Y,\sigma,I)\to(X,\tau)$ is ωI -continuous.

(ii) f (U) is ωI -open in Y for every open set U in X.

(ii) f (U) is ωI -closed in Y for every closed set U in X.

Proof: The proof is trivial.

Definition: 3.7 A collection $\{A_\alpha:\alpha\varepsilon\nabla\}$ of ωI – open set in an ideal topological Space X is called a ωI -open cover of a subset B of X is $B\subset\bigcup\{A_\alpha:\alpha\varepsilon\nabla\}$ holds.

Definition: 3.8 An ideal topological space (X,τ,I) is called ω I -compact if for every ω I -open cover $\{W_\alpha:\alpha\in\Delta\}$ of (X,τ,I) there exists a finite subset $\Delta_\circ$ of Δ such that $(X-\cup\{W_\alpha:\alpha\in\Delta_\circ\})\in I$.

Lemma: 3.1 [Newcomb, 1967]: For any function f: $(X,\tau,I)\to(Y,\tau,)$, $f(I)$ is ideal on Y.

Theorem: 3.10 The image of a ωI -compact space under a ωI -continuous surjective function is $f(I)$ -compact.

Proof: Let f: $(X,\tau,I)\to(Y,\tau,)$ be an ωI -continuous subjective function and $\{A_\alpha:\alpha\varepsilon\nabla\}$be an open cover of Y. Then $\{f^{-1}(A_\alpha):\alpha\varepsilon\nabla\}$is an ωI -open cover of X. From the assumption, there exists a finite subset ∇_0 of ∇ such that $X-\bigcup\{f^{-1}(A_\alpha):\alpha\in\nabla_0\}\in I$ .Therefore, $Y-\bigcup\{A_\alpha:\alpha\in\nabla_0\}\in f(I)$ which shows that $(Y,\sigma,f(I))$ is $f(I)$ -compact.

Definition: 3.9 An ideal topological space (X,τ,I) is said to be ωI – connected if X cannot be written as the disjoint union of two non – empty ωI – open sets. A subset of is ωI – connected if it is ωI – connected as a subspace.

Definition: 3.10 An ideal topological space (X,τ,I) is called ωI -normal if for every pair of disjoint ωI -closed sets A and B of (X,τ,I),there exist disjoint ωI -open sets $U,V \subseteq X$ such that $A \subseteq U$ and $B \subseteq V$

Theorem: 3.11 If f: $(X,\tau,I) \to (Y,\tau)$ is a ωI -continuous, closed injection and Y is normal, then X is ωI -normal.

Proof: Let F_1 and F_2 be disjoint ωI - closed subsets of X.Since f is closed and injective, are disjoint closed subsets of Y. Since Y is ωI - normal , $f(F_1)$ and $f(F_2)$ are separated by disjoint ωI -closed sets V_1 and V_2 respectively. Hence $F_1 \subset f^{-1}(V_1), F_2 \subset f^{-1}(V_2)$,and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$ .Since f is ωI -continuous and $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are ωI -open in (X,τ,I) , we have (X,τ,I) is ωI -normal.

Theorem: 3.12 An ωI –continuous image of ωI –connected space is connected.

Proof: Let f: $(X,\tau,I) \to (Y,\tau)$ be a ωI – continuous function of a ωI –connected space X onto a topological space Y. If possible .Let Y be disconnected. Let A and B form a disconnected set of Y. Then A and B are clopen and $Y = A \cup B$, where $A \cap B = \phi$ .since f is ωI –continuous, $X = f^{-1}(Y) == f^{-1}(A \cup B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are nonempty ωI –closed sets in X. Also, $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$ .Hence X is non ωI –connected, which is a contradiction. Therefore, Y is connected.

Theorem: 3.12 f: $(X,\tau,I) \to (Y,\sigma,J)$ and g: $(Y,\sigma,J) \to (Z,\eta)$ are functions. Then their composition $g \circ f : (X,\tau,I) \to (Z,\eta)$ is ωI - continuous if f is ωI - continuous and g is continuous.

Proof: Let W be any closed set in (Z,η) .Since g is continuous, $g^{-1}(W)$ is closed in (Y,σ) .Since f is ωI -continuous, then $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is ωI -closed in (X,τ,I) and hence $(g \circ f)$ is ωI -continuous.

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