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COMMON FIXED POINT THEOREM IN INTUITIONISTIC MENGER SPACES

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ABSTRACT

The main purpose of this paper is to proof some common fixed point theorem in Instuitionistic Menger Space .

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Key words: *Fixed point, Common Fixed point, Menger Space, instuitionistic Menger Space, Continuous mapping, Compatible and weakly compatible Self mapping.*

1. INTRODUCTION

There have been a number of generalization of metric space, one such generalization is Menger Space introduced in 1942 by Menger, who was used distribution function instead of non negative real numbers as values of the metric. The concept of Fuzzy sets was introduced initially by Zadeh [1] in 1965. Since then, to use this concept in topology and analysis, many authors have expansively developed the theory of fuzzy sets and applications. In metric fixed point theory, various mathematicians weakened the notation of compatibility by introducing the notation of weak commutatively, compatibility and weak compatibility and produced the number of fixed point theorems using these notations. Recently, many mathematicians formulated the definition of weakly computing, compatible and weakly compatible maps in instuitionistic fuzzy metric spaces and prove a number of fixed point theorem in instuitionistic fuzzy metric spaces.

In this paper, we prove the common fixed point theorem for six maps under the condition of weak compatibility and compatibility in instuitionistic Menger spaces.

2. PRELIMINARIES

Definition: 2.1 A binary operation $\star: [0,1] \times [0,1] \rightarrow [0,1]$ is continuous $\mathrm{t-norm}$ if $\star$ is satisfying the following condition:

(1) $\star$ is commutative and associative,
(2) $\star$ is continuous,
(3) $a \star 1 = a$ for all $a \in [0,1]$
(4) $a \star b \leq c \star d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0,1]$

Examples of $\mathrm{t-norm}$ are $a \star b = \min\{a, b\}$ and $a \star b = ab$.

Definition: 2.2 A binary operation $\nabla : [0,1] \times [0,1] \rightarrow [0,1]$ is continuous $\mathrm{t-conorm}$ if ∇ is satisfying the following condition:

(1) ∇ is commutative and associative,
(2) ∇ is continuous,
(3) $a \nabla 1 = a$ for all $a \in [0,1]$
(4) $a \nabla b \leq c \nabla d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0,1]$

Examples of $\mathrm{t-conorm}$ are $a \nabla b = \max\{a, b\}$ and $a \nabla b = \min\{1, a + b\}$.

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Definition: 2.3 A distribution function is a function $F: [-\infty, \infty] \to [0,1]$ which is left continuous on $\mathcal{R}$, non decreasing and $F(-\infty) = 0,\ f(\infty) = 1$.

We will denote by Δ the family of all distribution function on $[-\infty, \infty]$, $\mathcal{H}$ is a special element of Δ defined by,

$$\mathcal{H}(t) = \begin{cases} 0 & \text{if } t \leq 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$

If X is a nonempty set, $F: X \times X \to \Delta$ is called a probabilistic distance on X and $F(x, y)$ is usually denoted by F_{xy}.

Definition: 2.3 The ordered pair (X, F) is called the probabilistic metric space (shortly PM- space) if X is a nonempty set and F is a probabilistic distance satisfy the following conditions:for all $x, y, z \in X$ and $t, s > 0$,

(1) $F_{xy}(t) = 1 \Leftrightarrow x = y$;
(2) $F_{xy}(0) = 0$
(3) $F_{xy} = F_{yx}$
(4) $F_{xz}(t) = 1,\ F_{zy}(s) = 1 \Rightarrow F_{xy}(s + t) = 1$

The order triple $(X, F, \star)$ is called Menger space if (X, F) is a PM-space, $\star$ is a $t-$ norm and the following condition is also satisfies: for all $x, y, z \in X$ and $t, s > 0$,

(1) $F_{xy}(t + s) \geq F_{xz}(t) \star F_{zy}(s)$.

Definition: 2.4 Let $(X, F, \star)$ be a Menger space and $\star$ be a continuous $t-$ norm, then

(a) A sequence $\{x_n\}$ in X is said to be converges to a point x in X, iff for every $\epsilon > 0$ and $\lambda \in (0,1)$ there exists an integer $n_0 = n_0(\epsilon, \lambda)$ such that $F_{x_n x}(\epsilon) > 1 - \lambda$ for all $n \geq n_0$.

(b) A sequence $\{x_n\}$ in X is said to be Cauchy sequence converges to a point x in X, iff for every $\epsilon > 0$ and $\lambda \in (0,1)$ there exists an integer $n_0 = n_0(\epsilon, \lambda)$ such that $F_{x_n\, x_{n+p}}(\epsilon) > 1 - \lambda$ for all $n \geq n_0$ and $p > 0$.

(c) A Menger space in which every Cauchy sequence is convergent is said to be complete.

Definition: 2.5 Self map A and B of a Menger space $(X, F, \star)$ are said to be weakly compatible if they commute at their coincidence points that is if $Ax = Bx$ for some $x \in X$ then $ABx = BAx$.

Definition: 2.6 Self map A and B of a Menger space $(X, F, \star)$ are said to be compatible if $F_{ABx_n BAx_n}(t) \to 1$ for all $t > 0$, whenever $\{x_n\}$ is sequence in X such that $Ax_n,\ Bx_n \to x$ for some $x \in X$ as $n \to \infty$.

Definition: 2.7 A 5- tuple $(X, M, N, \star, \nabla)$ is said to be intuitionistic Menger space, if X is arbitrary set, $\star$ is a continuous $t-$ norm, ∇ is a continuous $t-$ conorm and M, N are PM-space on $X \times X \times [0, \infty)$ satisfying the following conditions:

(1) $M_{x,y}(t) + N_{x,y}(t) \leq 1$. For all $x, y \in X$ and $t > 0$,
(2) $M_{x,y}(0) = 0$ for all $x, y \in X$
(3) $M_{x,y}(t) = 1$, for all $x, y \in X$, and $t > 0$ *if and only if* $x = y$.
(4) $M_{x,y}(t) = M_{y,x}(t)$ For all $x, y \in X$ and $t > 0$
(5) $M_{x,y}(t) \star M_{y,z}(s) \leq M_{x,z}(t + s)$ For all $x, y \in X$ and $s, t > 0$
(6) For all $x, y \in X$, $M_{x,y}(\cdot) : [0, \infty) \to [0,1]$ is left continuous.
(7) $\lim_{t\to\infty} M_{x,y}(t) = 1$, for all $x, y \in X$ and $t > 0$,
(8) $N_{x,y}(0) = 1$, for all $x, y \in X$,
(9) $N_{x,y}(t) = 0$, for all $x, y \in X$, for all $x, y \in X$, and $t > 0$ *if and only if* $x = y$.
(10) $N_{x,y}(t) = N_{y,x}(t)$ For all $x, y \in X$ and $t > 0$
(11) $N_{x,y}(t) \nabla N_{y,z}(s) \geq N_{x,z}(t + s)$ for all $x, y \in X$ and $s, t > 0$
(12) For all $x, y \in X$, $N_{x,y}(\cdot) : [0, \infty) \to [0,1]$ is right continuous.
(13) $\lim_{t\to\infty} N_{x,y}(t) = 0$, for all $x, y \in X$ and $t > 0$,

Then (M, N) is called intuionistic Menger space on X. The function $M_{x,y}(t)$ and $N_{x,y}(t)$ denote the degree of nearness and degree of non nearness between x and y with respect to t.

Lemma: 2.8 Let $\{x_n\}$ be a sequence in a Menger space $(X, F, \star)$ with continuous $t-$ norm $\star$ and $t \star t \geq t$. If there exists a constant $k \in (0,1)$ such that,

$$F_{x_n,x_{n+1}}(kt) \geq F_{x_{n-1},x_n}(t)$$

for all $t > 0$ *and* $n = 1, 2, \ldots..$ then $\{x_n\}$ is a Cauchy sequence in X.

3. MAIN RESULT

Theorem: 3.1 Let A, B, S, T, P and Q be self mapping of an intuitionistic Menger space, X into itself, with continuous, $t-$ norm, $\star$ and continuous $t-$ conorm ∇ defined by $t \star t \geq t$ and $(1-t)\nabla(1-t) \leq (1-t)$ for all $t \in [0,1]$, satisfying the following condition:

(1) $P(X) \subset ST(X)$, $Q(X) \subset AB(X)$,
(2) There exists a constant $k \in (0,1)$ such that,

$$M^2_{Px,Qy}(kt) \star [M_{ABx,Px}(kt).M_{STy,Qy}(kt)] \geq \min\left\{\begin{matrix} M_{ABx,STy}(t), M_{Px,ABy}(t), M_{Qy,STy}(t) \\ , M_{Px,STy}(t), M_{Qy,ABy}(t) \end{matrix}\right\}.M_{ABx,Qy}(2kt)$$

for all $x, y \in X$ and $t > 0$, *and*

$$N^2_{Px,Qy}(kt) \ \nabla \ [N_{ABx,Px}(kt).N_{STy,Qy}(kt)] \leq \min\left\{\begin{matrix} N_{ABx,STy}(t), N_{Px,ABy}(t), N_{Qy,STy}(t) \\ , N_{Px,STy}(t), N_{Qy,ABy}(t) \end{matrix}\right\}.N_{ABx,Qy}(2kt)$$

for all $x, y \in X$ and $t > 0$,

(1) If one of $P(X), ST(X), AB(X), Q(X)$ is a complete subspace of X then :
  (a) P and AB have a coincidence point and
  (b) Q and ST have a coincidence point.
(2) $AB = BA$, $ST = TS$, $PB = BP$, $QT = TQ$,
(3) The pair $\{P, AB\}$ and $\{Q, ST\}$ are weakly compatible, Then $A, B, S, T, P,$ and T have unique common fixed point in X.

Proof: Let x_0 be an arbitrary point in X, then by (1) there exists $x_1, x_2 \in X$ such that, $Px_0 = STx_1y_0$ and $Qx_1 = ABx_2 = y_1$. Inductively, we can construct sequence $\{x_n\}$ and $\{y_n\}$ in X such that,

$$Px_{2n} = STx_{2n+1} = y_{2n} \text{ and } Qx_{2n+1} = ABx_{2n+2} = y_{2n+1} \text{ for } n = 0, 1, 2, \ldots\ldots$$

On taking $x = x_{2n}$ and $y = x_{2n+1}$ in (2), we have

$$M^2_{Px_{2n},Qx_{2n+1}}(kt) \star [M_{ABx_{2n},Px_{2n}}(kt).M_{STx_{2n+1},Qx_{2n+1}}(kt)] \geq \min\begin{bmatrix} M_{ABx_{2n},STx_{2n+1}}(t), M_{Px_{2n},ABx_{2n+1}}(t), M_{Qx_{2n+1},STx_{2n+1}}(t) \\ , M_{Px_{2n},STx_{2n+1}}(t), M_{Qx_{2n+1},ABx_{2n+1}}(2t) \end{bmatrix} M_{ABx_{2n},Qx_{2n+1}}(2kt)$$

and

$$N^2_{Px_{2n},Qx_{2n+1}}(kt) \ \nabla \ [N_{ABx_{2n},Px_{2n}}(kt).N_{STx_{2n+1},Qx_{2n+1}}(kt)] \leq \min\begin{bmatrix} N_{ABx_{2n},STx_{2n+1}}(t), N_{Px_{2n},ABx_{2n+1}}(t), N_{Qx_{2n+1},STx_{2n+1}}(t) \\ , N_{Px_{2n},STx_{2n+1}}(t), N_{Qx_{2n+1},ABx_{2n+1}}(2t) \end{bmatrix} N_{ABx_{2n},Qx_{2n+1}}(2kt)$$

$$M^2_{y_{2n},y_{2n+1}}(kt) \star [M_{y_{2n-1},y_{2n}}(kt).M_{y_{2n},y_{2n+1}}(kt)] \geq \min\begin{bmatrix} M_{y_{2n-1},y_{2n}}(t), M_{y_{2n},y_{2n}}(t), M_{y_{2n+1},y_{2n}}(t) \\ , M_{y_{2n},y_{2n}}(t), M_{y_{2n+1},y_{2n}}(t) \end{bmatrix} M_{y_{2n-1},y_{2n+1}}(2kt)$$

and

$$N^2_{y_{2n},y_{2n+1}}(kt) \ \nabla \ [N_{y_{2n-1},y_{2n}}(kt).N_{y_{2n},y_{2n+1}}(kt)] \leq \min\begin{bmatrix} N_{y_{2n-1},y_{2n}}(t), N_{y_{2n},y_{2n}}(t), N_{y_{2n+1},y_{2n}}(t) \\ , N_{y_{2n},y_{2n}}(t), N_{y_{2n+1},y_{2n}}(t) \end{bmatrix} N_{y_{2n-1},y_{2n+1}}(2kt)$$

$$M^2_{y_{2n},y_{2n+1}}(kt) \star [M_{y_{2n-1},y_{2n}}(kt).M_{y_{2n},y_{2n+1}}(kt)] \geq M_{y_{2n-1},y_{2n}}(t).M_{y_{2n-1},y_{2n+1}}(2kt)$$

and

$$N^2_{y_{2n},y_{2n+1}}(kt) \ \nabla [N_{y_{2n-1},y_{2n}}(kt).N_{y_{2n},y_{2n+1}}(kt)] \leq N_{y_{2n-1},y_{2n}}(t).N_{y_{2n-1},y_{2n+1}}(2kt)$$

$$M_{y_{2n},y_{2n+1}}(kt).[M_{y_{2n-1},y_{2n}}(kt) \star M_{y_{2n},y_{2n+1}}(kt)] \geq M_{y_{2n-1},y_{2n}}(t).M_{y_{2n-1},y_{2n+1}}(2kt)$$

and

$$N_{y_{2n},y_{2n+1}}(kt).[N_{y_{2n-1},y_{2n}}(kt) \nabla N_{y_{2n},y_{2n+1}}(kt)] \leq N_{y_{2n-1},y_{2n}}(t).N_{y_{2n-1},y_{2n+1}}(2kt)$$

$$M_{y_{2n},y_{2n+1}}(kt).\, M_{y_{2n-1},y_{2n+1}}(2kt) \geq M_{y_{2n-1},y_{2n}}(t).\, M_{y_{2n-1},y_{2n+1}}(2kt)$$

and

$$N_{y_{2n},y_{2n+1}}(kt)\, .\, N_{y_{2n-1},y_{2n+1}}(2kt) \leq N_{y_{2n-1},y_{2n}}(t).\, N_{y_{2n-1},y_{2n+1}}(2kt)$$

$M_{y_{2n},y_{2n+1}}(kt) \geq M_{y_{2n-1},y_{2n}}(t)$ and $N_{y_{2n},y_{2n+1}}(kt) \leq N_{y_{2n-1},y_{2n}}(t)$

Similarly we can prove that,

$M_{y_{2n+1},y_{2n+2}}(kt) \geq M_{y_{2n},y_{2n+1}}(t)$ and $N_{y_{2n+1},y_{2n+2}}(kt) \leq N_{y_{2n},y_{2n+1}}(t)$

For $k \in (0,1)$ and all $t > 0$. Thus by lemma 2.8, $\{y_n\}$ is a Cauchy sequence in X.

Now suppose AB(X) is a complete. Note that the subsequence $\{y_{2n+1}\}$ is contained in AB(X) and has a limit in $AB(X)$ call it 'z'. let $w \in AB^{-1}(z)$, then $ABw = z$. we shall use the fact that subsequence $\{y_{2n}\}$ also converges to 'z'.

By putting $x = w$ and $y = x_{2n+1}$ in (2) and taking limit as $n \to \infty$, we have

$$M^2_{Pw,z}\,(kt) \star \left[M_{z,Pw}(kt).\, M_{z,z}(kt)\right] \geq \min\begin{bmatrix} M_{z,z}(t), M_{Pw,z}(t), M_{z,z}(t) \\ , M_{Pw,z}(t), M_{z,z}(t)\end{bmatrix} M_{z,z}(2kt)$$

and

$$N^2_{Pw,z}\,(kt) \nabla \left[N_{z,Pw}(kt).\, N_{z,z}(kt)\right] \leq \min\begin{bmatrix} N_{z,z}(t), N_{Pw,z}(t), N_{z,z}(t) \\ , N_{Pw,z}(t), N_{z,z}(t)\end{bmatrix} N_{z,z}(2kt)$$

Thus if follows that,

$M_{z,Pw}(kt) \geq 1$ and $N_{z,Pw}(kt) \leq 0$

Hence $z = Pw$. since $ABw = z$, thus we have $Pw = z = ABw$ that is w is coincidence point of P and ABw.

Since $P(X) \subset ST(X)$, $Pw = z$ implies that, $z \in ST(X)$. Let $v \in ST^{-1}z$ then $STv = z$.

By putting $x = x_{2n}$ and $y = v$ in (2) and taking $n \to \infty$ we have,

$$M^2_{z,Qv}\,(kt) \star \left[M_{z,z}(kt).\, M_{z,Qv}(kt)\right] \geq \min\begin{bmatrix} M_{ABx_{2n},STv}(t), M_{Px_{2n},ABv}(t), M_{Qv,STv}(t) \\ , M_{Px_{2n},STv}(t), M_{Qv,ABv}(t)\end{bmatrix} M_{z,Qv}(2kt)$$

and

$$N^2_{z,Qv}\,(kt) \star \left[N_{z,z}(kt).\, N_{z,Qv}(kt)\right] \leq \min\begin{bmatrix} N_{ABx_{2n},STv}(t), N_{Px_{2n},ABv}(t), N_{Qv,STv}(t) \\ , N_{Px_{2n},STv}(t), N_{Qv,ABv}(t)\end{bmatrix} N_{z,Qv}(2kt)$$

Thus we have $M_{z,Qv}(kt) \geq 1$ and $N_{z,Qv}(kt) \leq 0$. Thus $z = Qv$, since $STv = z$, we have $Qv = z = STv$ that is v is coincidence point of Q and ST. this proves (b), the remaining two cases pertain essentially to the previous cases. Indeed if P(x) or Q(x) is complete, then by(1), $z \in P(X) \subset ST(X)$ or $z \in Q(X) \subset AB(X)$, thus (a) and (b) are completely established.

Since the pair (P, AB) is weakly compatible therefore P and AB commute at there coincidence point that is $P(ABw) = AB(P)w$, that is $Pz = ABz$.

Since the pair(Q, ST) is weakly compatible therefore Q and ST commute at there coincidence point that is

$Q(STv) = ST(Q)v$, that is $Qz = STz$.

By putting $x = z$, and $y = x_{2n+1}$in (2) and taking limit at $n \to \infty$, we have

$$M^2_{Pz,z}\,(kt) \star \left[M_{ABz,Pz}(kt).\, M_{z,z}(kt)\right] \geq \min\begin{bmatrix} M_{ABz,z}(t), M_{Pz,z}(t), M_{Qz,z}(t) \\ , M_{Pz,z}(t), M_{z,z}(t)\end{bmatrix} M_{ABz,z}(2kt)$$

and

$$N^2_{Pz,z}\,(kt) \star \left[N_{ABz,Pz}(kt).\, N_{z,z}(kt)\right] \geq \min\begin{bmatrix} N_{ABz,z}(t), N_{Pz,z}(t), N_{z,z}(t) \\ , N_{Pz,z}(t), N_{z,z}(t)\end{bmatrix} N_{ABz,z}(2kt)$$

Thus we have $M_{z,Pz}(kt) \geq 1$ and $N_{z,Pz}(kt) \leq 0$. Thus $z = Pz$. So $Pz = ABz = z$.

By putting $x = x_{2n}$, $y = z$ in (2) and taking limit at $n \to \infty$ we have $M_{z,Qz}(kt) \geq 1$ and $N_{z,Qz}(kt) \leq 0$ thus, $z = Qz$, so $Qz = STz = z$.
By putting $x = z$, $y = Tz$ in (2) and using (4) we have $M_{z,Tz}(kt) \geq 1$ and $N_{z,Tz}(kt) \leq 0$, thus $z = Tz$. since $STz = z$ therefore $Sz = z$. to prove $Bz = z$ we put $x = Bz$, $y = z$ in (2) and using (4) we have

$M_{Bz,z}(kt) \geq 1$ and $N_{Bz,z}(kt) \leq 0$. Thus $z = Bz$.

since $ABz = z$, there fore $Az = z$. by combining the above results we have $Az = Bz = Sz = Tz = Pz = Qz = z$.

That is z is a common fixed point of A, B, S, T, P, Q.

UNIQUENESS

Let 'w' is another fixed point of A, B, S, T, P, Q different from 'z' then On taking $x = z$ and $y = w$ in (2), we have $M_{z,w}(kt) \geq 1$ and $N_{z,w}(kt) \leq 0$.

Hence, $z = w$ for all $x, y, \in X$ and $t > 0$. Therefore 'z' is the unique common fixed point of A, B, S, T, P, Q.

This completes the proof.

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