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MODIFIED GENERALIZED NEWTON'S METHOD WITHOUT MULTIPLICITY

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ABSTRACT

In this paper, we present Modified Generalized Newton's (MGN) Method for solving non-linear equation of the type f(x) =0 and is studied in a special case when the multiplicity of the root of the equation is not known in advance and Some numerical examples are given to illustrate the efficiency and the performance of the new method.

Key words: *N – R Method, Iterative method.*

AMS Classification: *65 H 10.*

SECTION: 1 INTRODUCTION

Solving non-linear equations is one of the most important problems in Numerical Analysis. In this paper, we consider iterative methods to find multiple root of a non-linear equation

$$f(x) = 0 \tag{1.1}$$

If η be a root of (1.1) with multiplicity m, then the Generalized Newton's method (GN) is defined as

$$x_{n+1} = x_n - m\frac{f(x_n)}{f^{I}(x_n)} \quad \text{(n=0, 1, 2,...)} \tag{1.2}$$

which has a second order convergence.

As given in Jain et. al [1], the formula (1.2) is rewritten by eliminating m, as

$$x_{n+1} = x_n - \frac{f(x_n)f^{I}(x_n)}{[f^{I}(x_n)]^2 - f(x_n)f^{II}(x_n)} \quad \text{(n=0, 1, 2...)} \tag{1.3}$$

and the secant method for the multiple root of (1.1) is given as

$$x_{n+1} = \frac{x_{n-1}f(x_n)f^{I}(x_{n-1}) - x_n\ f(x_{n-1})f^{I}(x_n)}{f(x_n)f^{I}(x_{n-1}) - f(x_{n-1})f^{I}(x_n)} \quad \text{(n=0, 1, 2...)} \tag{1.4}$$

The Generalized Extrapolated Newton's (GEN) method considered by Dr, V.B.Kumar, Vatti et.al [3] for finding a root of the equation (1.1) with multiplicity 'm' is given by

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$$x_{n+1} = x_n - m\,\alpha_n \frac{f(x_n)}{f^{|}(x_n)} \tag{1.5}$$

which has a third order convergence, where $\alpha_n = \dfrac{2}{m+1-m\omega_n}$

As it is well-known that any iterative method of the form $x_{n+1} = \phi(x_n)$ converges if $|\phi^{|}(x_n)| < 1$ for all x in I: $a \le x \le b$ such that f(a) f(b) < 0 and f(x) and $f^{||}(x)$ have the same sign for all successive approximations of x starting with x_0.

Hence the method (1.5) converges under the condition

$$\mu = |1 - m\alpha_n + m\alpha_n\,\omega_n| < 1 \text{ for all x in I} \tag{1.6}$$

Where $$\omega_n = \frac{f(x_n) f^{||}(x_n)}{[f^{|}(x_n)]^2} \tag{1.7}$$

In this paper by taking $m\alpha_n = \overline{\alpha}_n$, the MGN method is developed without 'm' i.e., without knowing the multiplicity of the root of equation f(x) =0 in section 2 and this method is compared with the other methods considered in this paper in section 3 through some examples.

SECTION: 2 MODIFIED GENERALIZED NEWTON'S (MGN) METHOD WITHOUT m

Taking $m\alpha_n = \overline{\alpha}_n$ in the method (1.5), we obtain

$$x_{n+1} = x_n - \overline{\alpha}_n \frac{f(x_n)}{f^{|}(x_n)} \tag{2.1}$$

The method (2.1) converges if

$$\overline{\mu} = |1 - \overline{\alpha}_n + \overline{\alpha}_n\,\omega_n| < 1 \tag{2.2}$$

where $$\omega_n = \frac{f(x_n) f^{||}(x_n)}{[f^{|}(x_n)]^2} \tag{2.3}$$

If $a_n \le \omega_n \le b_n$, initially a_0 and b_0 can be chosen as $a_0 = \dfrac{f(\alpha) f^{||}(\alpha)}{[f^{|}(\alpha)]^2}$ and $b_0 = \dfrac{f(\beta) f^{||}(\beta)}{[f^{|}(\beta)]^2}$

provided the required root of equation f(x) =0 lies in (α,β) and then the successive a's and b's are to be taken basing on the shrinkanized interval, then the process of minimization of $\overline{\mu}$ as done in [4] and [5] gives the optimal choice for the parameter $\overline{\alpha}_n$ as

$$\overline{\alpha}_n = \frac{2}{2-(a_n+b_n)} \tag{2.4}$$

with this choice of $\overline{\alpha}_n$ the $\overline{\mu}$ of (2.2) takes the form

$$\overline{\mu} = \left| \frac{2\omega_n - (a_n+b_n)}{2-(a_n+b_n)} \right| \tag{2.5}$$

If ω_n is either of a_n or b_n , (2.5) reduces to

$$\overline{\mu} = \left| \frac{(b_n - a_n)}{2-(a_n+b_n)} \right| \tag{2.6}$$

which will be always less than unity as long as both $\bar{a}$ and $\bar{b}$ in the interval (0,1). Hence the method (2.1) converges.

SECTION: 3 NUMERICAL EXAMPLES

We consider some examples given in [1] and [2] for finding the multiple roots of an equation using the methods discussed in this paper and the successive approximations of the roots are tabulated below until the functional value becomes negligible.

Table: 3.1

Finding the triple root of $f(x) = 27x^5 + 27x^4 + 36x^3 + 28x^2 + 9x+1$ lies in (-1, 0) with $x_1 = -1$.

	Method (1.5)	Method (1.3)	Method (1.4)	Method (2.1)
n	x_{n+1}	x_{n+1}	x_{n+1}	x_{n+1}
1	–0.454545454	–0.185185186	–0.379310344	–0.316193401
2	–0.336914193	–0.332661524	–0.333740364	–0.336715695
3	–0.33333594	–0.333328286	–0.333329916	–0.333344167

Table: 3.2

Finding the triple root of $f(x) = x^4 - x^3 - 3x^2 + 5x - 2$ lies in (0,2) with $x_1 = 0$.

	Method (1.5)	Method (1.3)	Method (1.4)	Method (2.1)
N	x_{n+1}	x_{n+1}	x_{n+1}	x_{n+1}
1	1.2	0.769230769	1.130434783	0.987726475
2	1.004081633	0.993071573	0.989619376	1.000853192
3	1.000001932	0.999994773	1.066880473	0.999986188
4	--	--	1.000075778	--

CONCLUSIONS

From the above tables, we see that the MGN method is efficient. It converges not only faster than the methods (1.3), (1.4) but also GEN method. In view of this fact, the MGN method can be viewed as a significant improvement compared with the previously known methods.

REFERENCES

[1] JAIN. M. K, IYENGAR. S. R. Kand JAIN. R. K., (2004), 'Numerical Methods of Scientific and Engineering Computation', New age International publishers, New Delhi, India.

[2] SASTRY. S. S, (2005), 'Introductory Methods of Numerical Analysis', Prentice – Hall of India, New Delhi, India.

[3] VATTI. V. B. K and MYLAPALLI. M. S. K., 'Generalized Extrapolated Newton Raphson Method' South East Asian Journal of Mathematics and mathematical sciences, 2010, vol.8 no.3, pp. 43-48.

[4] VATTI. V. B. K. and KONERU. S. R., Extrapolated accelerated Gauss-Seidel Methods, Math. Comp., 1987, Vol. 21.

[5] YOUNG. D. M., (1971), 'Iterative Solution of large linear systems', Academic Press, New York and London.

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