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ON NEAR EQUITABLE TOTAL DOMINATION IN GRAPHS

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ABSTRACT

In this paper, we initiate a study on new domination parameter near equitable total domination number of a graph. We defined total domination and near equitable total domination in graphs. The minimal near equitable total dominating sets are established. Compute $\gamma_{net}(G)$ for some standard graphs. The relation between $\gamma_{net}(G)$ and $\gamma_t(G)$ are obtained. Also, characterization is given for near equitable total domination set is minimal.

Keywords: *Total Domination Number, Near Equitable Domination Number, Near Equitable Total Domination Number, Minimal Near Equitable Total Domination Number.*

1. INTRODUCTION

By a graph $G = (V, E)$ we mean a finite, undirected graph with neither loops nor multiple edges. The order and size of *G* are denoted by *n* and *m* respectively. For graph theoretic terminology we refer to Chartrand and Lesnaik. An excellent treatment of the fundamentals of domination is given in the book by Haynes. A survey of several advanced topics in domination is given in the book edited by Haynes. Various types of domination parameters have been defined and studied by several authors and more than 75 models of domination parameters are listed in the appendix of Haynes, Cockayne introduced the concept of total domination in graphs. Swaminathan introduced the concept of equitable domination in graphs, by considering the following real world problems; In a network vertices with nearly equal capacity may interact with each other in a better way. In this society persons with nearly equal status, tend to be friendly. In an industry, employees with nearly equal powers form association and move closely. Equitability among citizens in terms of wealth, health, status etc is the goal of a democratic nation. In order to study this practical concept a graph model is to be created. In this paper, we use near equitable dominating set and near equitable domination number of a graph idea to develop the concept of near equitable total dominating set and near equitable total domination number of a graph.

2. PRELIMINARIES

Definition 2.1: A subset *D* of *V* is called a dominating set of *G* if every vertex in *V* – *D* is adjacent to atleast one vertex in *D*. The minimum cardinality of a minimal dominating set is called the domination number of *G* and is denoted by $\gamma(G)$.

Definition 2.2: A subset *D* of *V* is called an equitable dominating set if for every vertex $v \in V - D$ there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$. The minimum cardinality of such an equitable dominating set of *G* is called the equitable domination number of *G* and is denoted by $\gamma_e(G)$.

Definition 2.3: A dominating set D of G is called a total dominating set if the induced subgraph <*D*> has no isolated vertices. The minimum cardinality of such a total dominating set of *G* is called the total domination number of *G* and is denoted by $\gamma_t(G)$.

Definition 2.4: A subset *D* of *V* is called an equitable total dominating set of *G*, if *D* is an equitable dominating set and the induced subgraph<*D*> has no isolated vertices. The minimum cardinality of such a equitable total dominating sets is the equitable total domination number and is denoted by $\gamma_{et}(G)$.

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Definition 2.5: Let $G = (V, E)$ be a graph, $D \subseteq V(G)$ and u be any vertex in D. The out degree of u with respect to D denoted by $od_D(u)$, is defined as $od_D(u) = |N(u) \cap (V - D)|$.

Definition 2.6: Let D be a dominating set of a graph G. Then D is called a near equitable dominating set of G if for evey $v \in V - D$, there exists a vertex $u \in D$ such that u is adjacent to v and $|od_D(u) - od_{V-D}(v)| \leq 1$. The minimum cardinality of such a near equitable dominating sets is called the near equitable domination number of G and is denoted by $\gamma_{ne}(G)$.

Definition 2.7: A double star is the tree obtained from two disjoint stars $K_{1,n}$ and $K_{1,m}$ by connecting their centers.

Definition 2.8: An equitable dominating set D is said to be a minimal equitable dominating set if no proper subset of D is an equitable dominating set.

3. NEAR EQUITABLE TOTAL DOMINATION NUMBER OF GRAPHS

Definition 3.1 Let D be a total dominating set of a graph G. Then D is called a near equitable total dominating set of G if for every $v \in V - D$, there exists a vertex $u \in D$ such that u is adjacent to v and $|od_D(u) - od_{V-D}(v)| \leq 1$. The minimum cardinality of such a near equitable total dominating set is called the near equitable total domination number of G and is denoted by $\gamma_{net}(G)$.

Example 3.2:

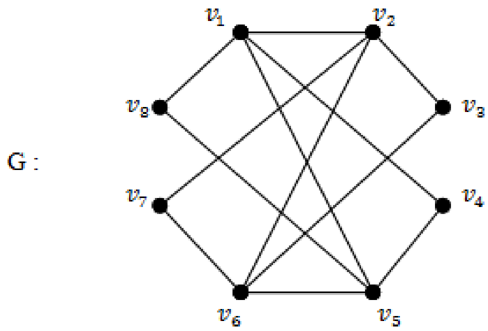


Figure-1

The near equitable total dominating sets of graph G are $D_1 = \{v_1, v_2, v_5, v_6\}$, $D_2 = \{v_1, v_2, v_4, v_7\}$, $D_3 = \{v_1, v_2, v_5, v_6, v_7\}$ and $D_4 = \{v_1, v_2, v_5, v_6, v_8\}$.

Therefore, $\gamma_{net}(G) = 4$.

Example 3.3 It is obvious that any near equitable total dominating set in a graph G is also a total dominating set, and thus we obtain $\gamma_t(G) \leq \gamma_{net}(G)$.

Remark 3.4 For any connected graph G of order n, $\gamma_t(G) = \gamma_{et}(G) = \gamma_{net}(G) = 2$ if and only if $n = 3$.
For example,

1. Consider the connected graph of 3 vertices

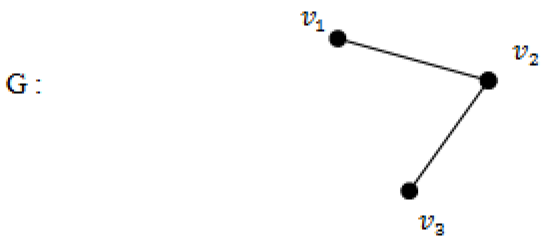


Figure-2

Here, $D = \{v_1, v_2\}$ is a minimum total dominating set, minimum equitable total dominating set and minimum near equitable total dominating set.

Therefore, $\gamma_t(G) = \gamma_{et}(G) = \gamma_{net}(G) = 2$.

2. Consider the connected graph of 4 vertices

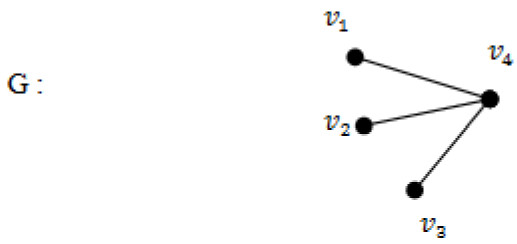


Figure-3

Here, $D = \{v_1, v_4\}$ is a minimum total dominating set and minimum near equitable total dominating set but not a equitable total dominating set.

Therefore, $\gamma_t(G) = \gamma_{net}(G) = 2$ but $\gamma_{et}(G) \neq 2$.

3.Consider the connected graph of 5 vertices

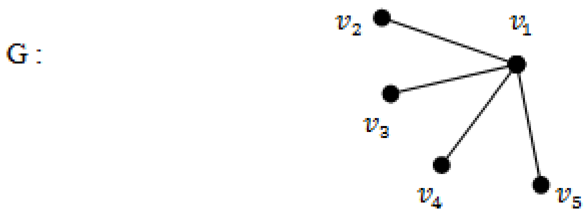


Figure-4

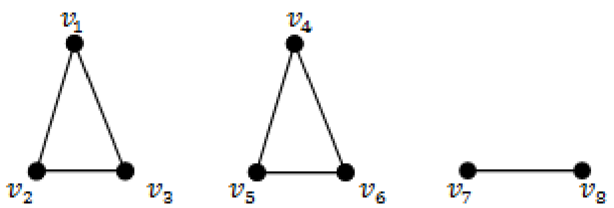
Here, $D = \{v_1, v_4, v_5\}$ is a minimum near equitable total dominating set but not a minimum total dominating set and equitable total dominating set.

Therefore, $\gamma_t(G) = 2$ but $\gamma_{et}(G) \neq \gamma_{net}(G) \neq 2$.

Remark 3.5: For $G \cong nK_3 \cup mK_2,\ m, n \geq 1, \gamma_t(G) = \gamma_{et}(G) = \gamma_{net}(G) = 2(n + m)$.

For example,

Let $m = 1, n = 2$.



$G = 2K_3 \cup K_2$

Figure-5

Here, $D = \{v_1, v_3, v_4, v_5, v_7, v_8\}$ is a minimum total dominating set, minimum equitable total dominating set and minimum near equitable total dominating set.

Therefore,$\gamma_t(G) = \gamma_{et}(G) = \gamma_{net}(G) = 2\ (2 + 1) = 6$.

Definition 3.6: Let G be a graph and let D be a near equitable total dominating set of G. The near equitable total neighbourhood of $u \in D$, denoted by ${N_D}^{net}(u)$ is defined as
${N_D}^{net}(u) = \{v \in V - D: v \in N(u), \mid od_D(u) - od_{V-D}(v) \mid \leq 1\}$.

Example 3.7:

G :

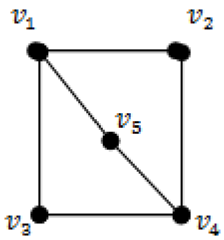


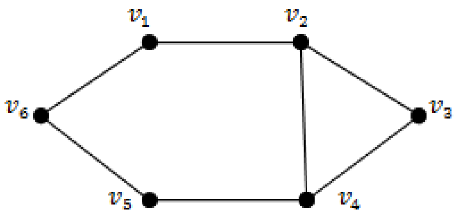
Figure-6

Let $D = \{v_1, v_{2,} v_5\}$ be a near equitable total dominating set. Then $V - D = \{v_3, v_4\}$.

Here, $N_D{}^{net}(v_1) = \{v_3\}$, $N_D{}^{net}(v_2) = \{v_4\}$, $N_D{}^{net}(v_5) = \{v_4\}$.

Definition 3.8: Let G be a graph and let D be a near equitable total dominating set of G. The maximum and minimum near equitable total degree of D are denoted by $\Delta_D{}^{net}$ and $\delta_D{}^{net}$, respectively.
That is $\Delta_D{}^{net} = \max_{u\in D} |N_D{}^{net}(u)|$ and $\delta_D{}^{net} = \min_{u\in D} |N_D{}^{net}(u)|$.

Example 3.9:



G

Figure-7

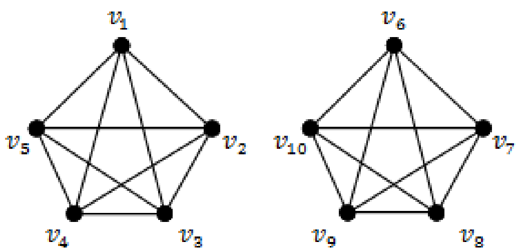
Let $D = \{v_2, v_4, v_5\}$ is a minimum near equitable total dominating set. Then $V - D = \{v_1, v_3, v_6\}$

Here, $N_D{}^{net}(v_2) = \{v_1, v_3\}$, $N_D{}^{net}(v_4) = \{v_3\}$, $N_D{}^{net}(v_5) = \{v_6\}$.

Therefore, $\Delta_D{}^{net} = 2$ and $\delta_D{}^{net} = 1$.

Remark 3.10: Let $G \cong mK_n, m \geq 1, n \geq 3$, if D is a near equitable total dominating set of G, then $\Delta(G) = \delta(G) = n - 1$ and $\Delta_D{}^{net} = \delta_D{}^{net} = n - 2$.

For example,



$G = 2K_5$

Figure-8

Let $D = \{v_1, v_4, v_6, v_8\}$ be a near equitable total dominating set.

Then $V - D = \{v_2, v_3, v_5, v_7, v_9, v_{10}\}$

Here, $N_D{}^{net}(v_1) = \{v_2, v_{3,} v_5\}$, $N_D{}^{net}(v_4) = \{v_2, v_{3,} v_5\}$, $N_D{}^{net}(v_6) = \{v_7, v_{9,} v_{10}\}$, $N_D{}^{net}(v_8) = \{v_7, v_{9,} v_{10}\}$.

Therefore, $\Delta_D{}^{net} = 3$ and $\delta_D{}^{net} = 3$.

Also, $\Delta(G) = 4$ and $\delta(G) = 4$.

Hence $\Delta(G) = \delta(G) = 4$ and ${\Delta_D}^{net} = {\delta_D}^{net} = 3$.

Remark 3.11: If *D* is a near equitable total dominating set of graph *G*, then ${\Delta_D}^{net} \leq \Delta(G)$.

Consider the above example, the remark is clear.

Remark 3.12: If *D* is a near equitable total dominating set of a tree *T*, then $\delta(T) \leq {\delta_D}^{net}$.

For example,

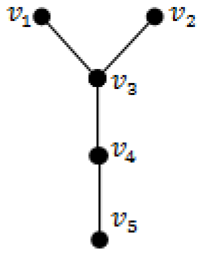


Figure-9

Here, $D = \{v_3, v_4, v_5\}$ is a minimum near equitable total dominating set.

Here, ${N_D}^{net}(v_3) = \{v_1, v_2\}$, ${N_D}^{net}(v_4) = \{\emptyset\}$, and ${N_D}^{net}(v_5) = \{\emptyset\}$.

Therefore, ${\delta_D}^{net} = 1$.

Also, $\delta(T) = 1$.

Hence $\delta(T) \leq {\delta_D}^{net}$.

Theorem 3.13:

1. For any path P_n,
$$\gamma_{net}(P_n) = \begin{cases} \frac{n}{2} & if \quad n \equiv 0 (mod\ 4) \\ \left\lfloor \frac{n}{2} \right\rfloor + 1 & otherwise \end{cases}$$
2. For any cycle C_n with $n \geq 3$ vertices,
$$\gamma_{net}(C_n) = \begin{cases} \frac{n}{2} & if \quad n \equiv 0 (mod\ 4) \\ \left\lfloor \frac{n}{2} \right\rfloor + 1 & otherwise \end{cases}$$

Proof:

1. As the degree of any vertex of P_n is either 1 or 2. Clearly, any total dominating set is near equitable. Therefore, $\gamma_t(P_n) = \gamma_{net}(P_n)$.

2. As C_n is a connected 2 – regular graph, any total dominating set is near equitable. Hence $\gamma_t(C_n) = \gamma_{net}(C_n)$.

Theorem 3.14:
For any complete graph K_n with $n \geq 6$ vertices,
$$\gamma_{net}(K_n) = \begin{cases} \frac{n}{2} & if\ n\ is\ even \\ \left\lfloor \frac{n}{2} \right\rfloor & if\ n\ is\ odd \end{cases}$$

Proof: Let $V = \{v_1, v_2, \dots, v_n\}$ be the vertex set of K_n. We consider the following cases.

Case-1: *n* is even.
Consider a total dominating set $D = \{v_1, v_2, \dots\dots, v_{\frac{n}{2}}\}$ such that $|D| = \frac{n}{2}$. Then $od_D(v_i) = \frac{n}{2}$ and $od_{V-D}(v_i) = \frac{n}{2}$. Then $|od_D(v_i) - od_{V-D}(v_i)| \leq 1$, for all $v_i \in V - D$. Therefore, *D* is a near equitable total dominating set of K_n.

Now, if $D_1 = \{v_1, v_2, \ldots, v_{\frac{n}{2}-1}\}$ is a near equitable total dominating set of K_n, then $|od_{D_1}(v_i) - od_{V-D_1}(v_i)| = 2$, a contradiction.

Therefore, D is a minimum near equitable total dominating set.

Case-2: n is odd.
Consider a total dominating set $D = \{v_1, v_2, \ldots, v_{\lfloor \frac{n}{2} \rfloor}\}$ such that that $|D| = \lfloor \frac{n}{2} \rfloor$. Then $od_D(v_i) = \lfloor \frac{n}{2} \rfloor + 1$, $od_{V-D}(v_i) = \lfloor \frac{n}{2} \rfloor$. Then $|od_D(v_i) - od_{V-D}(v_i)| \leq 1$, for all $v_i \in V - D$. Therefore, D is a near equitable total dominating set of K_n. Now, if $D_1 = \{v_1, v_2 \ldots, v_{\lfloor \frac{n}{2} \rfloor - 1}\}$ is a near equitable total dominating set of K_n, then $|od_{D_1}(v_i) - od_{V-D_1}(v_i)| = 3$, a contradiction. Therefore, D is a minimum near equitable total dominating set.

Theorem 3.15:
For the complete bipartite graph $G \cong K_{n,m}$ with $1 < m \leq n$, we have

$$\gamma_{net}(K_{n,m}) = \begin{cases} m-1, & if\ n = m\ and\ m \geq 3; \\ m, & if\ \ n - m = 1\ and\ m, n > 2; \\ n-1, & if\ n - m \geq 2. \end{cases}$$

Proof: Let $V_1 = \{u_1, u_2, \ldots, u_n\}$ and $V_2 = \{v_1, v_2, \ldots, v_m\}$ be the bipartition of $K_{n,m}$.

We consider the following cases.

Case-1: $n = m \geq 3$.

We consider the following subcases.

Subcase-1.1: $n = m = 3$.

Let $D = \{u_1, v_1\}$ be a minimum total dominaing set of $K_{n,m}$. Then, $|od_D(u_1) - od_{V_2-D}(v_i)| \leq 1$, for all $v_i \in V_2 - D$ and $|od_D(v_1) - od_{V_1-D}(u_i)| \leq 1$, for all $u_i \in V_1 - D$. Hence, D is a near equitable total dominating set of $K_{n,m}$. Therefore, $\gamma_{net}(K_{n,m}) \leq \gamma_t(K_{n,m})$. But we have, $\gamma_t(K_{n,m}) \leq \gamma_{net}(K_{n,m})$. Hence $\gamma_t(K_{n,m}) = \gamma_{net}(K_{n,m})$. Thus, D is a minimum near equitable total dominating set.

Subcase-1.2: $n = m \geq 4$.

We have the following subsubcases.

Subsubcase-1.2.1: n and m are odd.

Consider a total dominating set $D = \{u_1, u_2, \ldots, u_{\lfloor \frac{n}{2} \rfloor}, v_1, v_2, \ldots, v_{\lfloor \frac{m}{2} \rfloor}\}$ such that $|D| = m - 1$. Then $od_D(u_i) = \lfloor \frac{m}{2} \rfloor$, $od_D(v_i) = \lfloor \frac{n}{2} \rfloor$, $od_{V_1-D}(u_j) = \lceil \frac{m}{2} \rceil$ and $od_{V_2-D}(v_j) = \lceil \frac{n}{2} \rceil$. Since $n = m$, we have, $|od_D(u_i) - od_{V_2-D}(v_j)| \leq 1$, for all $v_j \in V_2 - D$ and $|od_D(v_i) - od_{V_1-D}(u_j)| \leq 1$, for all $u_j \in V_1 - D$. Therefore, D is a near equitable total dominating set. Now, if $D_1 = \{u_1, u_2, \ldots, u_s, v_1, v_2, \ldots, v_{n-s-2}\}$, $s < \lfloor \frac{n}{2} \rfloor$ is a near equitable total dominating set of $K_{n,m}$, then $|od_{D_1}(u_i) - od_{V_2-D_1}(v_j)| = 2$, and $|od_{D_1}(v_i) - od_{V_1-D_1}(u_j)| =$, a contradiction. Therefore, D is a minimum near equitable total dominating set.

Subsubcase-1.2.2: n and m are even.

Consider a total dominating set $D = \{u_1, u_2, \ldots, u_{\frac{n}{2}}, v_1, v_2, \ldots, v_{\frac{m}{2}-1}\}$ such that $|D| = m - 1$. Then $od_D(u_i) = \frac{m}{2} + 1$, $od_D(v_i) = \frac{n}{2}$, $od_{V_1-D}(u_j) = \frac{m}{2} - 1$ and $od_{V_2-D}(v_j) = \frac{n}{2}$. Since $n = m$, we have, $|od_D(u_i) - od_{V_2-D}(v_j)| \leq 1$, for all $v_j \in V_2 - D$ and $|od_D(v_i) - od_{V_1-D}(u_j)| \leq 1$, for all $u_j \in V_1 - D$. Therefore, D is a near equitable total dominating set. Now, if $D_1 = \{u_1, u_2, \ldots, u_s, v_1, v_2, \ldots, v_{n-s-2}\}$, $s < \lfloor \frac{n}{2} \rfloor$ is a near equitable total dominating set of $K_{n,m}$, then $|od_{D_1}(u_i) - od_{V_2-D_1}(v_j)| = 2$, and $|od_{D_1}(v_i) - od_{V_1-D_1}(u_j)| = 2$, a contradiction. Therefore, D is a minimum near equitable total dominating set.

Case-2: $n \neq m$. We consider the following subcases.

Subcase-2.1: $n-m=1$ and $n,m>2$.

Consider a total dominating set $D=\{u_1, u_2, \dots, u_{\lfloor \frac{n}{2} \rfloor}, v_1, v_2, \dots, v_{\lfloor \frac{m}{2} \rfloor}\}$ such that $|D|=m$. Since $n=m+1$, we have, $|od_D(u_i)-od_{V_2-D}(v_j)| \le 1$, for all $v_j \in V_2-D$ and $|od_D(v_i)-od_{V_1-D}(u_j)| \le 1$, for all $u_j \in V_1-D$. Therefore, D is a near equitable total dominating set. Now, if n is odd and m is even, we have, $|D|=m$. Consider a near equitable total dominating set $D_1=\{u_1, u_2, \dots, u_s, v_1, v_2, \dots, v_{n-s-2}\}$, $s<\left\lceil \frac{n}{2} \right\rceil$. Then, $|od_{D_1}(v_i)-od_{V_1-D_1}(u_j)|=2$. Similarly, if m is odd and n is even, $|od_{D_1}(v_i)-od_{V_1-D_1}(u_j)|=2$, a contradiction. Therefore, D is a minimum near equitable total dominating set.

Subcase-2.2: $n-m \ge 2$.

Consider a total dominating set $D=\{u_1, u_2, \dots, u_{n-m-1}, v_1, v_2, \dots, v_m\}$, $|D|=n-1$. Then, $|od_D(u_i)-od_{V_2-D}(v_j)|=0$, for all $v_j \in V_2-D$ and $|od_D(v_i)-od_{V_1-D}(u_j)|=1$, for all $u_j \in V_1-D$. Therefore, D is a near equitable total dominating set. Now, if $D_1=D-\{u_{n-m-1}\}$ or $D-\{v_m\}$ is a near equitable total dominating set, then $D_1=\{u_1, u_2, \dots, u_{n-m-2}, v_1, v_2, \dots, v_m\}$ or $D_1=\{u_1, u_2, \dots, u_{n-m-1}, v_1, v_2, \dots, v_{m-1}\}$. Therefore, $|od_{D_1}(v_i)-od_{V_1-D_1}(u_j)|=2$, a contradiction. Thus, D is a minimum near equitable total dominating set.

Remark 3.16 For the Wheel $W_{1,n}$, $\gamma_{net}(W_{1,n})=\begin{cases} 2, & if \ n=3,4; \\ \left\lceil \frac{n}{3} \right\rceil+1, & otherwise. \end{cases}$

For example,
Let $n=3$.

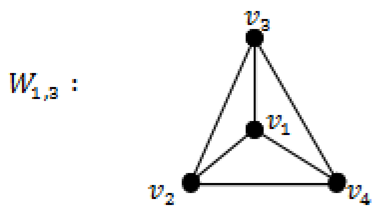


Figure-10

Here, $D=\{v_1, v_2\}$ is a minimum near equitable total dominating set.

Therefore, $\gamma_{net}(W_{1,3})=2$.

Let $n=5$.

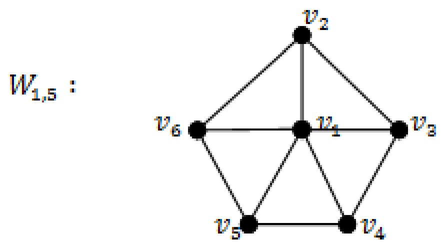


Figure-11

Here, $D=\{v_1, v_3, v_5\}$ is a minimum near equitable total dominating set.

Therefore, $\gamma_{net}(W_{1,5})=\left\lceil \frac{5}{3} \right\rceil+1=3$.

Remark 3.17: For any double star $S_{n,m}$, $\gamma_{net}(S_{n,m})=\begin{cases} 2, & if \ n,m \le 2; \\ n+m-2, & if \ n,m \ge 2 \text{ and } n(or)m \ge 3. \end{cases}$

For example,
(i) Let $n=1, m=2$

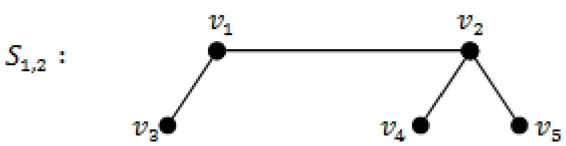


Figure-12

Here, $D = \{v_1, v_2\}$ is a minimum near equitable total dominating set.

Therefore, $\gamma_{net}(S_{1,2}) = 2$.

(ii) Let $n = m = 2$.

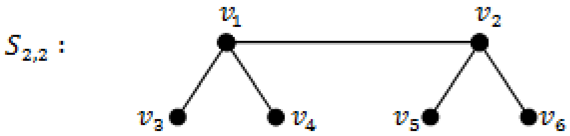


Figure-13

Here, $D = \{v_1, v_2\}$ is a minimum near equitable total dominating set.

Therefore, $\gamma_{net}(S_{2,2}) = 2$.

(iii) Let $n = 2, m = 3$.

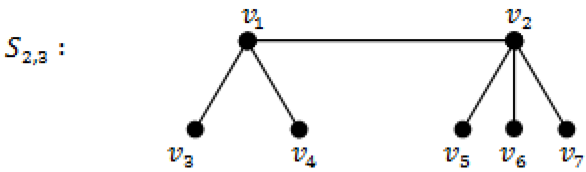


Figure-14

Here, $D = \{v_1, v_2, v_5\}$ is a minimum near equitable total dominating set.

Therefore, $\gamma_{net}(S_{2,3}) = 2 + 3 - 2 = 3$.

(iv) Let $n = 3, m = 3$.

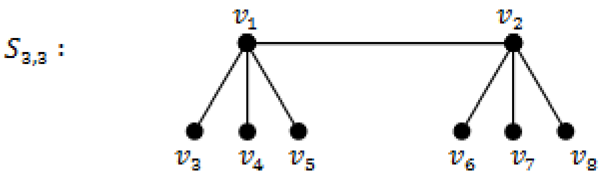


Figure-15

Here, $D = \{v_1, v_2, v_4, v_7\}$ is a minimum near equitable total dominating set.

Therefore, $\gamma_{net}(S_{3,3}) = 3 + 3 - 2 = 4$.

Theorem 3.18: Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be graphs such that $|V_1| = n$ and $|V_2| = m, n \leq m, m - n \leq 1$. Then $\gamma_{net}\ (G_1 + G_2) = n$.

Proof: Let $G = G_1 + G_2$. For any $u \in V_1$and $v \in V_2, u$ and v adjacent. Since $m - n \leq 1$, it follows that $|od_{V_1}(u) - od_{V_2}(v)| \leq 1$ in G. Since $n \leq m$, V_1 is a minimum near equitable total dominating set of G. Thus, $\gamma_{net}(G) = n$.

Theorem 3.19: Let T be a wounded spider obtained from the star $K_{1,n-1}, n \geq 5$ by subdividing m edges exactly once. Then

$$\gamma_{net}(T) = \gamma_t(T) = \begin{cases} n, & if\ m = n - 1; \\ n - 1, & if\ m = n - 2; \\ n - 2, & if\ m = n - 3. \end{cases}$$

Proof: Let $K_{1,n-1}$ be a star with central vertex u. Then $V(K_{1,n-1}) = \{u, u_1, u_2, \ldots, u_{n-1}\}$ with $\deg(u) = n - 1$. Let v_i be the vertex subdividing the edge uu_i. Then we consider the following cases.

Case-1: $m = n - 1$. Since $n \geq 5$ then $D = \{u, v_1, v_2, \ldots, v_{m-1}, v_{n-1}\}$ is a near equitable total dominating set and hence $\gamma_{net}(T) = |D| = n$. Also, $\gamma_t(T) = n$. Hence $\gamma_{net}(T) = \gamma_t(T) = n$.

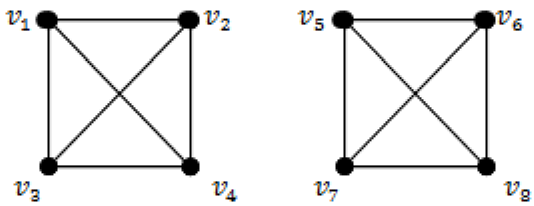
Case-2: $m = n - 2$. Since $n \geq 5$ then $D = \{u, v_1, v_2, \ldots\ldots, v_{m-1}, v_{n-2}\}$ is a near equitable total dominating set and hence $\gamma_{net}(T) = |D| = n - 1$. Also, $\gamma_t(T) = n - 1$. Hence $\gamma_{net}(T) = \gamma_t(T) = n - 1$.

Case-3: $m = n - 3$. Since $n \geq 5$ then $D = \{u, v_1, v_2, \ldots\ldots, v_{m-1}, v_{n-3}\}$ is a near equitable total dominating set and hence $\gamma_{net}(T) = |D| = n - 2$. Also, $\gamma_t(T) = n - 2$. Hence $\gamma_{net}(T) = \gamma_t(T) = n - 2$.

Corollary 3.20: Let T be a wounded spider obtained from the star $K_{1,n-1}, n \geq 5$ by subdividing m edges exactly once. Then $\gamma_{net}(T) = n - 2$ *and* $\gamma_t(T) \geq 2$ *if* $m < n - 3$.

Remark 3.21: Let $G \cong mK_n$, $m \geq 1$ and $n = 3, 4, 5$. Then $\gamma_{net}(G) = \gamma_t(G) = 2m$.
For example,

Let $m = 2, n = 4$.



$G = 2K_4$

Figure-16

Here, $D = \{v_1, v_3, v_5, v_8\}$ is a minimum near equitable total dominating set and minimum total dominating set.

Therefore, $\gamma_{net}(G) = \gamma_t(G) = 2 . 2 = 4$.

Remark 3.22: For any graph G with no isolated vertices, $2 \leq \gamma_t(G) \leq \gamma_{net}(G) \leq n$.

For Example,

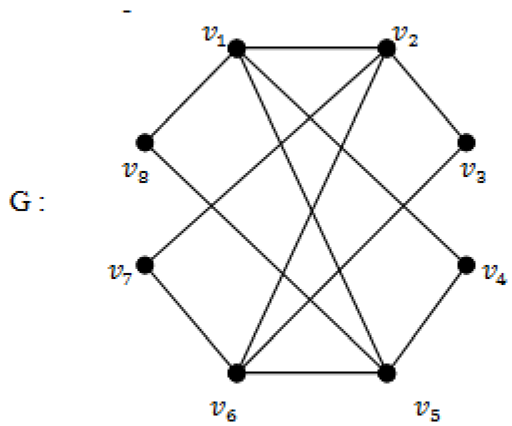


Figure-17

Here, $D = \{v_1, v_2, v_5, v_6\}$ is a minimum near equitable total dominating set and $D = \{v_1, v_2\}$ is a minimum total dominating set.

Therefore, $\gamma_{net}(G) = 4$ and $\gamma_t(G) = 2$.

Hence $2 \leq \gamma_t(G) \leq \gamma_{net}(G) \leq 8$.

4. MINIMAL NEAR EQUITABLE TOTAL DOMINATING SETS

Definition 4.1: A near equitable total dominating set D is said to be a minimal near equitable total dominating set if no proper subset of D is near equitable total dominating set.

Example 4.2:

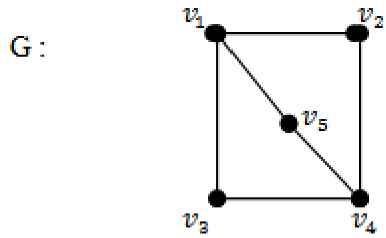


Figure-18

Here, $D = \{v_1, v_{2,} v_5\}$ is a minimal near equitable total dominating set.

Theorem 4.3: For any graph *G* without isolated vertices, a near equitable total dominating set *D* is minimal if and only if for every $u \in D$, one of the following two properties holds:
(i) There exists a vertex $v \in V - D$such that $N(v) \cap D = \{u\}$, $|\, od_D(u) - od_{V-D}(v)\,| \leq 1$.
(ii) $< D - \{u\} >$ contains no isolated vertices.

Proof: Assume that *D* is a minimal near equitable total dominating set and (i) and (ii) do not hold. Then for some $u \in D$, there exists $v \in V - D$ such that $|\ od_D(u) - od_{V-D}(v)\,| \leq 1$and for every $v \in V - D$, either $N(v) \cap D \neq \{u\}$, or $|\ od_D(u) - od_{V-D}(v)\ | \geq 2$ or both. Therefore $< D - \{u\} >$ contains an isolated vertex, contradiction to the minimality of *D*. Therefore (i) and (ii) holds.

Conversely, if for every vertex $u \in D$, the statement (i) or (ii) holds and *D* is not minimal. Then there exists $u \in D$ such that $D - \{u\}$ is a near equitable total dominating set. Therefore there exists $v \in D - \{u\}$ such that v near equitably dominates u. That is, $v \in N(u)$ and $|\, od_D(u) - od_{V-D}(v)\,| \leq 1$. Hence u does not satisfy (i). Then u must satisfy (ii) and there exists $v \in V - D$ such that $N(v) \cap D = \{u\}$ and $|\, od_D(u) - od_{V-D}(v)\,| \leq 1$. And also there exists $w \in D - \{u\}$ such that w is adjacent to v. Therefore $w \in N(v) \cap D$, $|\ od_D(u) - od_{V-D}(v)\,| \leq 1$ and $w \neq u$, a contradiction to $N(v) \cap D = \{u\}$. Hence *D* is a minimal near equitable total dominating set.

Remark 4.4: For any nontrivial connected graph *G*, $\left\lceil \frac{n}{1+\Delta(G)} \right\rceil \leq \gamma_{net}(G)$.

For Example,

Let $n = 6$.

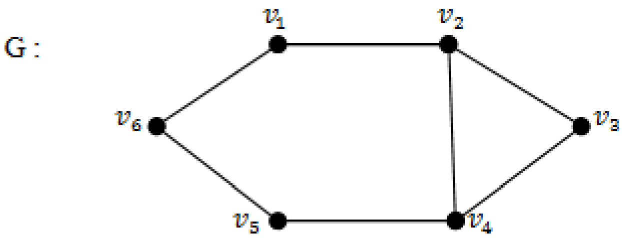


Figure-19

Here, $D = \{v_2, v_4, v_5\}$ is a minimum near equitable total dominating set.

Therefore, $\gamma_{net}(G) = 3$.

Also, $\Delta(G) = 3$.

Hence $\left\lceil \frac{6}{1+3} \right\rceil = 2 < 3$.

Remark 4.5: If $G(\neq K_{n,m};\ 1 < m \leq n;\ n - m \geq 2)$ has *n* vertices, no isolated vertices and $\Delta(G) < n - 1$, then
(i) $\gamma_{net}(G) + \gamma_{net}(\overline{G}) \leq 2 \left\lceil \frac{n}{2} \right\rceil$
(ii) $\gamma_{net}(G) \, . \, \gamma_{net}(\overline{G}) \leq \left(\left\lceil \frac{n}{2} \right\rceil \right)^2$

For example,

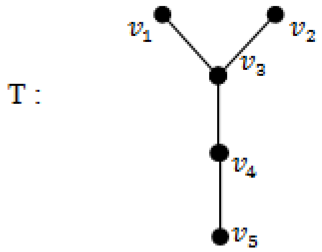


Figure-20

Here, $D = \{v_3, v_4, v_5\}$ is a minimum near equitable total dominating set.

Therefore, $\gamma_{net}(T) = 3$.

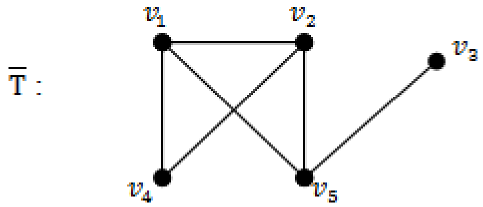


Figure-21

Here, $D = \{v_1, v_5\}$ is a minimum near equitable total dominating set.

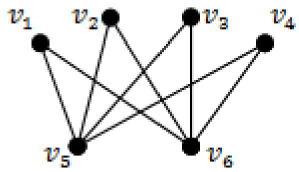
Therefore, $\gamma_{net}(\overline{T}) = 2$.
$\gamma_{net}(T) + \gamma_{net}(\overline{T}) = 3 + 2 = 5$ and $2\left\lceil\frac{n}{2}\right\rceil = 2\left\lceil\frac{5}{2}\right\rceil = 6$.

Hence $\gamma_{net}(T) + \gamma_{net}(\overline{T}) \le 2\left\lceil\frac{n}{2}\right\rceil$.
$\gamma_{net}(T) \,.\, \gamma_{net}(\overline{T}) = 3\,.\,2 = 6$ and $\left(\left\lceil\frac{n}{2}\right\rceil\right)^2 = \left(\left\lceil\frac{5}{2}\right\rceil\right)^2 = (3)^2 = 9$.

Hence $\gamma_{net}(T) \,.\, \gamma_{net}(\overline{T}) \le \left(\left\lceil\frac{n}{2}\right\rceil\right)^2$.

Remark 4.6: If $G(= K_{n,m};\ 1 < m \le n;\ n - m \ge 2)$ has n vertices, no isolated vertices and $\Delta(G) < n - 1$, then
(i) $\gamma_{net}(G) + \gamma_{net}(\overline{G}) \ge 2\left\lceil\frac{n}{2}\right\rceil$
(ii) $\gamma_{net}(G) \,.\, \gamma_{net}(\overline{G}) \ge \left(\left\lceil\frac{n}{2}\right\rceil\right)^2$

For example,



$K_{4,2}$

Figure-22

Here, $D = \{v_1, v_3, v_5\}$ is a minimum near equitable total dominating set.

Therefore, $\gamma_{net}(K_{4,2}) = 3$.

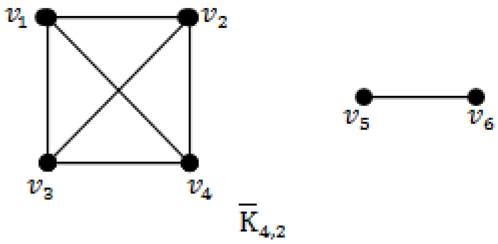


Figure-23

Here, $D = \{v_1, v_3, v_5, v_6\}$ is a minimum near equitable total dominating set.

Therefore, $\gamma_{net}(\overline{K}_{4,2}) = 4$.
$\gamma_{net}(K_{4,2}) + \gamma_{net}(\overline{K}_{4,2}) = 3 + 4 = 7$ and $2\left\lceil\frac{n}{2}\right\rceil = 2\left\lceil\frac{6}{2}\right\rceil = 6$.

Hence $\gamma_{net}(K_{4,2}) + \gamma_{net}(\overline{K}_{4,2}) \geq 2\left\lceil\frac{n}{2}\right\rceil$.
$\gamma_{net}(K_{4,2}) \,.\, \gamma_{net}(\overline{K}_{4,2}) = 3\,.\,4 = 12$ and $\left(\left\lceil\frac{n}{2}\right\rceil\right)^2 = \left(\left\lceil\frac{6}{2}\right\rceil\right)^2 = (3)^2 = 9$.

Hence $\gamma_{net}(K_{4,2}) \,.\, \gamma_{net}(\overline{K}_{4,2}) \geq \left(\left\lceil\frac{n}{2}\right\rceil\right)^2$.

5. CONCLUSION AND FUTURE STUDIES

In this paper total domination in graphs and near equitable total domination in graphs are compared through various examples. A necessary and sufficient condition under which they are equivalent is provided. The minimal near equitable total dominating sets are established. Compute $\gamma_{net}(G)$ and $\gamma_t(G)$ for some standard graphs. The results discussed may be used to study about various graphs invariants. For further investigation, the following open problem is suggested. "Give necessary and sufficient condition for a given graph *G* is entire equitable total dominating graph of some graph may be investigated".

REFERENCES

1. Ali Mohammed Sahal and Veena Mathad, *On Near Equitable Domination in Graphs*, Asian Journal of Current Engineering and Maths 3:2 March – April (2014) 39-46.
2. A. Anitha, S. Arumugam and Mustapha Chellali, *Equitable domination in graphs*, Discrete Mathematics, Algorithms and Applications, Vol.3, No.3(2011), 311-321.
3. E. J. Cockayne and S. T. Hedetniemi, *Towards a Theory of Domination in Graphs*, Networks, 7, 247-261 (1977).
4. E. J.Cockayne, R. M. Dawes and S. T. Hedetniemi, *Total Domination in Graphs*, Networks, 10, 211-219 (1980).
5. T. W. Haynes, S. T. Hedetniemi and O. J. Slater, *Domination in Graphs*, Advanced Topics, Marcel Dekker, New York, (1998).
6. V. R. Kulli, *Theory of Domination in Graphs*, Vishwa International Publications, Gulbarga, India (2010).
7. V. R. Kulli, B. Basavanagoud and Vijay V. Teli, *Equitable Total Domination in Graphs*, Advances in Domination Theory I, ed. V. R. Kulli, Vishwa International Publications, 156 (2012).
8. P. C. B. Lam and B. Wei, *On the Total Domination Number of a Graph*, Utilitas Maths; 72, 223-240 (2007).

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