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FUZZY ASSIGNMENT PROBLEM USING OAM WITH HEXAGONAL FUZZY NUMBER

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ABSTRACT

In this research paper, we used ones assignment method to find the fuzzy optional total cost of fuzzy assignment problem. Hence the robust ranking technique with hexogonal fuzzy number is presented, a numerical example is examined for the proposed method.

Keywords: *fuzzy assignment problem, ones assignment algorithm, fuzzy number, robust's ranking technique.*

1. INTRODUCTION

The term Assignment Problem(AP) first appeared in Votaw and Orden(1952) [6]. APs are widely applied in manufacturing and service systems. Assignment Problem (AP) is used worldwide in solving real world problems. The goal of the assignment problem is to minimize the cost or time of completing a number of jobs by a number of persons. An assignment problem plays an important role in an assigning of persons to jobs, or classes to rooms, operators to machines, drivers to trucks, trucks to routes, or problems to research teams, etc. To find solution to assignment problems, various algorithm such as linear programming [2, 3, 5, 6, 8], Hungarian algorithm [7], neural network [4], genetic algorithm [1] have been developed.

This paper is organized as follows: Section 2 deals with some basic terminology of fuzzy theory. In section 3 , provides mathematical formulation of fuzzy assignment problem. In section 4 & 5, describes robust's ranking Technique andalgorithm of ones assignment method. In section 6, to illustrate the proposed method a numerical example with graphical representation is discussed and followed by the conclusion are given in Section 7.

2. PRELIMINARIES

Definition 2.1 (Fuzzy Set): Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function $\mu_A : X \to [0,1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element x in fuzzy set A for each $x \in X$.

Definition 2.2 (Normal fuzzy set): A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exist at least one $x \in X$ such that $\mu_A(x) = 1$.

Definition 2.3 (Convex): A fuzzy set $\widetilde{A}$ is convex $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda x_2)) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}(x_2)})$, $x_1, x_2 \in X$ and $\lambda \in [0,1]$. Alternatively, a fuzzy set is convex if all α - level sets are convex.

Definition 2.4 (Fuzzy Number): A fuzzy set $\widetilde{A}$ on R must possess at least the following three properties to qualify as a fuzzy number,

(i) $\widetilde{A}$ must be a normal fuzzy set,
(ii) ${}^{\alpha}A\sim$ must be closed interval for every $\alpha \in [0, 1]$
(iii) The support of $\widetilde{A}$, must be bounded.

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Definition 2.5 (Hexagonal Fuzzy Number): A fuzzy number $\widetilde{A_H}$ is a hexagonal fuzzy number denoted by $\widetilde{A_H}$ = $(a_1,a_2,a_3,a_4,a_5,a_6)$ where $(a_1,a_2,a_3,a_4,a_5,a_6)$ are real numbers and its membership function $\mu_{\widetilde{A_H}}$ is given below.

$$\mu_{\widetilde{A_H}}(x) = \begin{cases} 0 & for \quad x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & for \quad a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & for \quad a_2 \le x \le a_3 \\ 1 & for \quad a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right) & for \quad a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) & for \quad x > a_6 \end{cases}$$

3. MATHEMATICAL FORMULATION OF FUZZYASSIGNMENT PROBLEM

Suppose there are n jobs to be performed and n persons are available for doing these jobs. Assume that each person can do one job at a time and each job can be assigned to one person only. Let C_{ij} be the fuzzy cost (payment) if j^{th} job is assigned to i^{th} person. The problem is to find an assignment x_{ij} so that the total cost for performing all the jobs is minimum.

The chosen fuzzy assignment problem may be formulated into the following fuzzy linear programming problem (FLPP):

$$\text{minimize } z = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}\, X_{ij}$$

Subject to

$$\sum_{j=1}^{n} X_{ij} = 1 \quad j, \qquad j = 1,2,\dots,n.$$

$$\sum_{i=1}^{n} X_{ij} = 1 \quad i, \qquad i = 1,2,\dots,n.$$

$$X_{ij} = 0 \text{ or } 1 \quad i,j$$

where C_{ij} is a fuzzy number.

$$\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}\, X_{ij} :$$

Total fuzzy cost for performing all the jobs.

4. ROBUST'S RANKING TECHNIQUE

Robust ranking technique which satisfy compensation, linearity, and additively properties and provides results which are consist human intuition. If is a fuzzy number then the Robust Ranking is defined by

$$\mathrm{R}(\tilde{a}) = \int_0^1 0.5(a_1 \;, a_u)\, d\alpha$$

Where (a_i, a_u) is the α- level cut of the fuzzy number $\tilde{a}$. This method is for ranking the objective values. The Robust ranking index $\mathrm{R}(\tilde{a})$ gives the representative value of fuzzy number $\tilde{a}$.

5. ONES ASSIGNMENT METHOD

In this section we introduce Ones Assignment Problem for solving fuzzy assignment problem to obtained the fuzzy total optimal cost with fuzzy numbers.

Step-1: In a minimization (maximization) case, find the minimum (maximum) element of each row in the assignment matrix (say a_i) and write it on the right hand side

$$\begin{array}{c} \begin{bmatrix} 1 & 2 & 3 & \dots & n \\ a_{1,1} & a_{1,2} & a_{1,3} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & a_{2,3} & \dots & a_{2,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & a_{n,3} & \dots & a_{n,n} \end{bmatrix} \begin{array}{c} a_1 \\ a_2 \\ \cdot \\ \cdot \\ a_n \end{array} \end{array}$$

Then divide each element of ith row of the matrix by ai. These operations create at least one ones in each rows. In term of ones for each row and column do assignment, otherwise go to step 2.

Step-2: Find the minimum (maximum) element of each column in assignment matrix (b_j), and write it below j^{th} column. Then divide each element of j^{th} column of the matrix by (bj).These operations create at least one ones in each columns. Make assignment in terms of ones. If no feasible assignment can be achieved from step (1) and (2) then go to step3.

$$\begin{array}{cc} \begin{array}{c} 1 \\ 2 \\ \cdot \\ \cdot \\ n \end{array} & \begin{bmatrix} 1 & 2 & 3 & \dots & n \\ a_{1,1}/a_1 & a_{1,2}/a_1 & a_{1,3}/a_1 & \dots & a_{1,n}/a_1 \\ a_{2,1}/a_2 & a_{2,2}/a_2 & a_{2,3}/a_2 & \dots & a_{2,n}/a_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n,1}/a_n & a_{n,2}/a_n & a_{n,3}/a_n & \dots & a_{n,n}/a_n \end{bmatrix} \\ & \begin{array}{ccccc} b_1 & b_2 & . & . & b_n \end{array} \end{array}$$

Note: In a maximization case, the end of step 2 we have a matrix, which all elements are along to [0, 1], and the greatest element is one.

Step-3: Draw the minimum number of lines to cover all the ones of the matrix. If the number of drawn lines less than n, then the complete assignment is not possible, while if the number of lines is exactly equal to n, then the complete assignment is obtained.

Step-4: If a complete assignment program is not possible in step 3, then select the smallest (largest) element (saydij) out of those which do not lie on any of the lines in the above matrix. Then divide by dij each element of the uncovered rows or columns, which dij lies on it. This operation creates some new ones to this row or column.

If still a complete optimal assignment is not achieved in this new matrix, then use step 4 and 3 iteratively. By repeating the same procedure the optimal assignment will be obtained.

Priority plays an important role in this method, when we want to assign the ones. Priority rule, for minimization (maximization) assignment problem, assign the ones on the rows which have smallest (greatest) element on the right hand side, respectively.

6 NUMERICAL EXAMPLES

Let us consider a Fuzzy Assignment Problem with rows representing 3 persons A, B, C and columns representing 3 jobs, job1, job2, job3. The cost matrix [aij] is given whose elements are hexagonal fuzzy numbers. The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum

$$[a_{ij}] = \begin{bmatrix} (3,7,11,15,19,24) & (13,18,23,28,33,40) & (6,13,20,28,36,45) \\ (16,19,24,29,34,39) & (3,5,7,9,10,12) & (5,7,10,13,17,21) \\ (11,14,17,21,25,30) & (7,9,11,14,18,22) & (2,3,4,6,7,9) \end{bmatrix}$$

Solution: In Conformation to model the fuzzy assignment problem can be formulated in the following
Min {R (3,7,11,15,19,24) x_{11} + R (13,18,23,28,33,40) x_{12} + R (6,13,20,28,36,45) x_{13} + R (16,19,24,29,34,39) x_{21} + R (3,5,7,9,10,12) x_{22} + R (5,7,10,13,17,21) x_{23} + R (11,14,17,21,25,30) x_{31} + R (7,9,11,14,18,22) x_{32} + R (2,3,4,6,7,9) x_{33}}

Subject to
$x_{11} + x_{12} + x_{13} = 1 \quad x_{11} + x_{21} + x_{31} = 1$
$x_{21} + x_{22} + x_{23} = 1 \quad x_{12} + x_{22} + x_{32} = 1$
$x_{31} + x_{32} + x_{33} = 1 \quad x_{13} + x_{23} + x_{33} = 1$
$x_{ij} \in [0; 1]$

Now we calculate the membership function of hexagonal fuzzy number (3, 7, 11, 15, 19, 24) using Ranking of hexagonal fuzzy number

$$\widetilde{A_H} = \left(\frac{2a_1+3a_2+4a_3+4a_4+3a_5+2a_6+}{18}\right)\left(\frac{5}{18}\right)$$

and the value is obtained. (i.e.) $a_{11} = 4$.

proceeding similarly, we get $a_{12} = 7$, $a_{13} = 7$, $a_{21} = 7$, $a_{22} = 2$, $a_{23} = 3$, $a_{31} = 5$, $a_{32} = 4$, $a_{33} = 1$. Then apply the Robust's Ranking Technique,

$$\mathrm{R}(\tilde{a}) = \int_0^1 0.5(a_1\ , a_u)\, d\alpha$$

We get the ranking indices for the fuzzy cost a_{ij} are calculated as:

$$\mathrm{R}\,(\widetilde{a_{11}}) = \mathrm{R}\,(13, 18, 23, 28, 33, 40) = \int_0^1 0.5(4) d\alpha\ = 2$$

(i:e:) R $(\widetilde{a_{11}}) = 2$.

Similarly,

R $(\widetilde{a_{12}}) = 3.5$, R $(\widetilde{a_{13}}) = 3.5$, R $(\widetilde{a_{21}}) = 3.5$, R $(\widetilde{a_{22}}) = 1$, R $(\widetilde{a_{23}}) = 1.5$, R $(\widetilde{a_{31}}) = 2.5$, R $(\widetilde{a_{32}}) = 2$, R $(\widetilde{a_{33}}) = 0.5$

We replace these values for their corresponding $\widetilde{a_{ij}}$ in which result in a convenient assignment problem in the linear programming problem

$$\begin{bmatrix} 2 & 3.5 & 3.5 \\ 3.5 & 1 & 1.5 \\ 2.5 & 2 & 0.5 \end{bmatrix}$$

We solve it by ones assignment method to get the following optimal solution.

Step-1: In a minimization case, find the minimum element of each row in the assignment matrix (say a_i) and write it on the right hand side of the matrix. Then divide each element of i^{th} row of the matrix by a_i. These operations create at least one ones in each rows. In term of ones for each row and column do assignment, otherwise go to step 2.

$$\approx \begin{array}{cc} & \text{min} \\ \begin{bmatrix} 2 & 3.5 & 3.5 \\ 3.5 & 1 & 1.5 \\ 2.5 & 2 & 0.5 \end{bmatrix} & \begin{array}{c} 2 \\ 1 \\ 0.5 \end{array} \end{array}$$

$$\approx \begin{bmatrix} 1 & 1.75 & 1.75 \\ 3.5 & 1 & 1.5 \\ 5 & 4 & 1 \end{bmatrix}$$

Step-2: Find the minimum element of each column in assignment matrix (b_j), and write it below j^{th} column. Then divide each element of j^{th} column of the matrix by b_j. These operations create at least one ones in each columns.

$$\approx \begin{array}{c} \begin{bmatrix} 1 & 1.75 & 1.75 \\ 3.5 & 1 & 1.5 \\ 5 & 4 & 1 \end{bmatrix} \\ \begin{array}{ccc} 1 & 1 & 1 \end{array} \end{array}$$

$$\approx \begin{bmatrix} 1 & 1.75 & 1.75 \\ 3.5 & 1 & 1.5 \\ 5 & 4 & 1 \end{bmatrix}$$

Step-3: Draw the minimum number of lines to cover all the ones of the matrix. If the number of drawn lines less than n, then the complete assignment is not possible, while if the number of lines is exactly equal to n, then the complete assignment is obtained.

Make assignment in terms of ones

$$\approx \begin{bmatrix} (1) & 1.75 & 1.75 \\ 3.5 & (1) & 1.5 \\ 5 & 4 & (1) \end{bmatrix}$$

We can assign the ones and the solution is (1, 1), (2, 2) and (3, 3).

The fuzzy optimal total cost

$$\widetilde{a_{11}} + \widetilde{a_{22}} + \widetilde{a_{33}} = \mathrm{R}\,(3,7,11,15,19,24)\, x_{11} + \mathrm{R}\,(3,5,7,9,10,12)\, x_{22} + \mathrm{R}\,(2,3,4,6,7,9)\, x_{33}$$

Table-2.7.1: Values of the fuzzy optimal total cost (8, 15, 22, 30, 36, 45) of the hexagonal fuzzy number

A	$P_1(u)$	$P_2(u)$
0.1	9.4	43.2
0.15	10.1	42.3

0.2	10.8	41.4
0.25	11.5	40.5
0.3	12.2	39.6
0.35	12.9	38.7
0.4	13.6	37.8
0.45	14.3	36.9
0.5	15	36
	$Q_1(v)$	$Q_2(v)$
0.5	15	36
0.55	15.7	35.4
0.6	16.4	34.8

Figure-1: Graphical representation of the fuzzy total optimal cost (8, 15, 22, 30, 36, 45) of Hexagonal Fuzzy Number.

7. CONCLUSION

In this research article, the robust ranking technique used for calculating the total cost of the fuzzy assignment problem using ones assignment method with hexagonal fuzzy number. The Robust ranking method used to rank the objective value of the objective function for transform the fuzzy assignment problem to a crisp one. The proposal method is simple to find a total of assignment problem.

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