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# STABILITY OF VISCOELASTIC FLUIDS IN ROTATING FLUIDS THROUGH POROUS MEDIUM HEATED FROM BELOW

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*ABSTRACT*

*In this paper, the linear stability of a saturated rotating porous layer of viscoelastic fluid which is heated from below has been studied. The modified Darcy Law is used to explain the fluid motion. The linear stability theory is employed to find the critical condition of the onset of convective motion. It has been observed that the critical Rayleigh-Darcy number increases with increasing Taylor-Darcy number. In limiting cases, some previous results for viscoelastic fluids in the absence of rotation are recovered from our results.*

***Keywords:*** *Viscoelastic fluids; thermal convection; porous media; rotation; linear stability*

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## INTRODUCTION:

The convective motion driven by buoyancy forces has attracted the researchers' interest. In this phenomenon, the porous media are used actively under investigation. It is well known that buoyancy force driven convection has a wide range of application in many engineering fields like geothermal utilization, nuclear waste disposal, agriculture product storage system, packed-bed catalyst reactors, pollutant transport in underground, heat removal of nuclear power plant and food process industry[1,2].

The stability problem for porous layer of viscoelastic fluid heated from below has been discussed by Sokolov and Tanner [3]. With Newtonian fluid system of slow heating Horton and Rogers [4], Lapwood [5] conducted theoretical analysis on the critical condition of the onset of buoyancy driven motion in fluid saturated horizontal porous layer. Katto and Masuoka [6] showed experimentally the effect of Darcy number on convection. They employed the Darcy's law to express the fluid characteristics in porous layters. In case of Newtonian fluid the stability analysis has been conducted under the principle of exchange of stability. However the viscoelastic fluid like polymeric fluid can exhibit markedly different stability properties. For Rayleigh Benard problem, Vest and Arpaci [7] and Koka and Ierley [8] analyzed over stability of Maxwell fluid. They confirmed that the buoyancy forces could induce the time periodic instability before the exchange of stability. The theoretical, experimental and numerical methods of [5, 6, 9, 10] investigation for thermal convection instability in porous media are involved. In modeling flows in a fluid saturated porous medium, the simplest but most widely used model in the engineering fields is well known Darcy law which states that the proportionality between flow rate and pressure gradient. In the literature, many researchers have used a modified Darcy law which extended the Darcy equation to include a time derivative term. However, these models can only apply to characterize Newtonian fluids in porous media. On the other hand, basic understanding of convection and heat transfer in a non-Newtonian fluid saturated porous medium is of considerable importance. Due to various applications in engineering like enhanced oil recovery, ceramic processing and geothermal receivers come in contact with non-Newtonian fluid in porous media. The increasing attention given to this topic, there exist no simple model have been used to study the non-Newtonian fluids in porous media. Rudraiah et al. [11] and Bertal and Cafaro [12], used a Maxwell Jeffrey model to investigate a viscoelastic fluid in a porous layer heated from below. Shenoy [13] gave

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a review of flow of power law in porous media more recently. Tan and Masuka [14] proposed a Darcy Maxwell model by introducing a flow resistance term to overcome the disadvantage encountered in the modified Darcy Maxwell Jeffery model. Hayat et al. [15] considered a second grade fluid saturated in a porous space past a stretching sheet. The main characteristic of the thermal convection instability of fluids is that there exist two different instability mechanism either exchange of stability of overstability [16].

The study on the effect of rotation on convection instability has its origin in theoretical and practical applications [17, 18, 19]. We examined the convective instability of a Newtonian fluid layer subject to Coriolis force using variously procedure Chandrasekhar [17]. He found that necessary condition for overstability to arrive is that the Prandtl number must be smaller than 1. The rotation effect on convective instability for a viscoelastic fluid layer was investigated by Bhatia and Steiner [20] which shows that the rotation has a destabilizing influence on the overstable mode convection. These results are in contrast with stabilizing influence of rotation in the case of an ordinary viscous fluid. The effect of rotation on convection instability in a Newtonian fluid saturated porous medium has also been investigated [18, 21-24]. Vadasz [22] and Govender [23] investigated the Coriolis effect on thermal convection derived by centrifugal buoyancy in which gravity buoyancy was neglected. In the paper the linear analysis for thermal convection in a viscoelastic fluid saturated porous layer subject to uniform rotation is conducted. The effect of relaxation, retardation and rotation on instabilities is analyzed. The solution corresponding to stationary is investigated and variations of Nusselt number in the neighborhood of the critical conditions are determined by using non-linear analysis results.

**FORMULATIONS:**

The system is considered of an infinite horizontal porous layer of vertical height $(d)$ which is confined between two rigid boundaries as shown in Fig. 1. The porous medium is homogeneous and isotropic and saturated with visco-elastic fluid. The porous layer heated slowly from below employing the Boussinesq approximation and modified Darcy model. The porous layer subject to rotation with uniform angular velocity $(\omega)$. The governing equation for the system is given as:

$$\nabla.\vec{U} = 0 \tag{1}$$

$$\frac{\mu}{K}\left(1+\bar{\varepsilon}\frac{\partial}{\partial t}\right)\vec{U} = \left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)\left(-\nabla P+\rho g-\frac{2\rho_0\vec{U}}{\omega}\hat{K}\right) \tag{2}$$

$$\frac{\partial T}{\partial t} + (\vec{U}.\nabla)T = \alpha\nabla^2 T \tag{3}$$

$$\rho = \rho_0\left[1-\beta(T-T_1)\right] \tag{4}$$

where $\vec{U}$ is the velocity vector $K$ is the permeability, $\mu$ is the viscosity, $\bar{\varepsilon}$ is strain retardation time or $\bar{\lambda}$ is the stress relaxation time and $P$ is the pressure, $T$ is the temperature, $\rho$ is the density, $g$ is the gravitational acceleration, $\alpha$ is the effective thermal diffusivity and $\beta$ is the thermal expansion coefficient, $\phi$ is the porosity of the porous medium, $\hat{k}$ is the unit vector in the $z$-direction.

At quiescent state temperature varies across the layer thickness. The basic state of the system is

$$\left.\begin{aligned} \vec{U}_b &= (0,0,0) \\ \rho &= \rho_b(z) \\ P &= P_b(z) \\ T &= T_b(z) \end{aligned}\right\} \tag{5}$$

Using above equation in (1), (2), (3) and (4), we have

$$-\frac{dP_b}{dz} + \rho_b\, g = 0 \tag{6}$$

$$\rho_b = \rho_0 \left[1-\beta(T_b - T_1)\right] \tag{7}$$

$$T_b = T_1 - \frac{\Delta T}{d} z \tag{8}$$

where $b$ denote the basic state of the system.

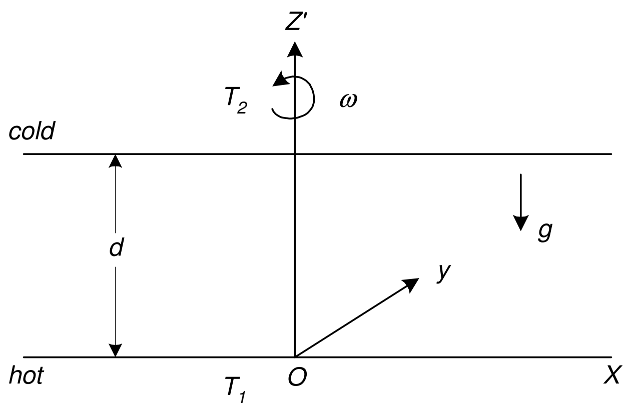


**Figure 1:** Schematic diagram of system considered here

To study the stability of the system the perturbation used on the basic state in the form

$$\vec{U} = U_b + U', \quad T = T_b + T', \quad P = P_b + P' \tag{9}$$

Using equation (9) in (1), (2), (3) and (4), we have

$$\nabla . U' = 0 \tag{10}$$

$$\frac{\mu}{k}\left(1+\bar{\varepsilon}\frac{\partial}{\partial t}\right)U' = \left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)\left(-\nabla P' + \rho_0 \beta T' g - \frac{2\omega\rho_0 \bar{U}}{\phi}\hat{k}\right) \tag{11}$$

$$\frac{\partial T'}{\partial t} + (\vec{U}'.\nabla)T' - \frac{\Delta T}{d}W' = \alpha \nabla^2 T' \tag{12}$$

where $W'$ is the $z$ component of velocity

For dimensionless,

$$\lambda = \frac{\bar{\lambda}\alpha}{d^2}, \quad \varepsilon = \frac{\bar{\varepsilon}\alpha}{d^2}$$

Now choosing $\frac{\alpha}{d}, \Delta T, \frac{\mu\alpha}{d^2}$ and $\frac{d^2}{\alpha}$ to solve the velocity, temperature, pressure and time, respectively.

Now from equation (10), (11) and (12), we have

$$\nabla . \vec{U} = 0 \tag{13}$$

$$\frac{1}{Da}\left(1+\varepsilon\frac{\partial}{\partial t}\right)\vec{U} = \left(1+\lambda\frac{\partial}{\partial t}\right)\left(-\nabla\rho - Ta^{1/2}\,\vec{U}\hat{k} + RaT'\hat{k}\right) \tag{14}$$

$$\frac{\partial T'}{\partial t} + (\vec{U}.\nabla)T' = \nabla^2 T' \tag{15}$$

 

The new dimensionless parameter $Da$ is the Darcy number, $Ta$ is the Taylor number and $Ra$ is the Rayleigh number. All the three parameter are defined as

$$Da = \frac{k}{d^2}, \quad Ta = \left(\frac{2\rho_0 \omega d^2}{\phi\,\mu}\right)^2 \text{ and } Ra = \left(\frac{\rho_0\ g\ \beta\ \Delta T\ d^3}{\mu\,\alpha}\right)$$

**LINEAR STABILITY ANALYSIS**

Taking curl of equation (14) to eliminate the pressure term and using the following

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \qquad \text{(Horizontal Laplacian Operator)}$$

$$R_D = R_a \,.\, D_a \qquad \text{(Rayleigh-Darcy number)}$$

$$T_D = T_a \,.\, D_a^2 \qquad \text{(Taylor-Darcy number)}$$

$$\left(1+\varepsilon\frac{\partial}{\partial t}\right)\nabla^2 W - R_D\left(1+\lambda\frac{\partial}{\partial t}\right)\nabla^2 T' + T_D\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial^2 W}{\partial z^2} = 0 \tag{16}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right) T = W \tag{17}$$

According to normal mode analysis

$$W(x, y, z, t)\, T\,(\eta, y, z, t) = W(z)\ \ T(z)\, e^{[\,i(a_x x + a_y y)+\sigma t\,]}$$

$$a = \left[a_x^2 + a_y^2\right]^{1/2}$$

$$(1+\varepsilon\sigma)\,(D^2 - a^2)W + R_D(1+\lambda\sigma)\ a^2 T' + T_D(1+\sigma x)\ D^2 W = 0 \tag{18}$$

$$(D^2 - a^2 - \sigma)\,T' = -W \tag{19}$$

Using boundary condition

$$W_1 = T' = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1$$

$$T' = D^2 T' = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1$$

$$(1+\varepsilon\sigma)\,(D^2 - a^2)\,(D^2 - a^2 - \sigma)T' - R_D(1+\sigma\lambda)a^2 T' + T_D(1+\sigma\lambda)D^2(D^2 - a^2 - \sigma)T' = 0 \tag{20}$$

Solution

$$T' = C_n \sin n\pi z$$

$$T' = C_n \sin \pi z\,, \quad n = 1$$

$$\sigma^2\,[\varepsilon\pi^2 + \varepsilon a^2 + \lambda\pi^2 T_D] + \sigma\,[\varepsilon\pi^4 + \varepsilon a^4 + 2\varepsilon\pi^2 a^2 - R_D\lambda a^2 + T_D\lambda\pi^4 + T_D\lambda\pi^2 a^2 + T_D\pi^2 + \pi^2 + a^2]$$
$$+ [\pi^4 + a^4 + 2\pi^2 a^2 - R_D a^2 + T_D\pi^4 + T_D\pi^2 a^2] = 0$$

or $$\sigma^2 A + \sigma B + C = 0 \tag{21}$$

It has two solutions, depending on whether the instability is steady or oscillatory. In case of steady, i.e., exchange of stabilities, we have $\sigma = 0$, at critical conditions

$$R_D = \frac{\pi^4 + a^4 + 2\pi^2 a^2 + \pi^4 T_D + \pi^2 a^2 T_D}{a^2}$$

$$R_D = \frac{\pi^4 + a^4 + 2\pi^2 a^2 + \pi^2 T_D\,(\pi^2 + a^2)}{a^2} \tag{22}$$

Now, the critical wave number obtained by minimizing $R_D$ with respect to $a$, i.e., $\dfrac{\partial R_D}{\partial a} = 0$.

$$a_c^s = \pi\,(1+T_D)^{1/4} \tag{23}$$

Critical wave number for stationary convection

$R_{D,C}^{S} = \pi^2\,[1+(1+T_D)^{1/2}]^2$, for stationary convection

When $T_D \to 0$, then we have

$R_{D,C}^{S} = 4\pi^2$ and $a_C^S = \pi$

These results for viscoelastic fluids in a non-rotating porous medium and identical to those obtained by Kim et al. [25]

If $\sigma_i \neq 0,\ \sigma_r \to 0$

For disturbances, oscillatory instability which is sometimes called overstability sets in.

For the case of oscillatory mode, i.e., overstability.

If $B = 0$ and $A\,C > 0$

$$R_D^S = \frac{\varepsilon\pi^4 + \varepsilon a^4 + 2\varepsilon\pi^2 a^2 + T_D\lambda\pi^4 + T_D\lambda\pi^2 a^2 + \pi^2 T_D + \pi^2 + a^2}{\lambda a^2} \tag{24}$$

For minimum value of $R_D$, $\dfrac{\partial R_D^S}{\partial a} = 0$

$$a^2 = \left(\pi^2 + \frac{\lambda T_D}{\varepsilon}\pi^4 + \frac{T_D\,\pi^2}{\varepsilon}\right)^{1/2}$$

$$R_D^O = \frac{\varepsilon\pi^4 + \varepsilon a^4 + 2\varepsilon\pi^2 a^2 + T_D\lambda\pi^4 + T_D\lambda\pi^2 a^2 + T_D\pi^2 + \pi^2 + a^2}{\lambda a^2}$$

**RESULTS;**

The linear stability theory gives us the critical Darcy–Rayleigh number, but does not predict the amplitude of convective motion. The temporal growth rate is a complex number, in which real part of growth rate $(\sigma_r)$ is less than zero, then the system is always stable and when $\sigma_r$ it is real part of growth rate is greater than zero, the system becomes unstable. If $\sigma_i = 0$ and $\sigma_r$ changed from negative to positive as Rayleigh number increases the principle of exchange of stabilities is satisfied, or if $\sigma_i \neq 0$ and $\sigma_r$ changes from negative to positive as the Rayleigh number increases the overstability may be a preferred and oscillatory motion occurs.

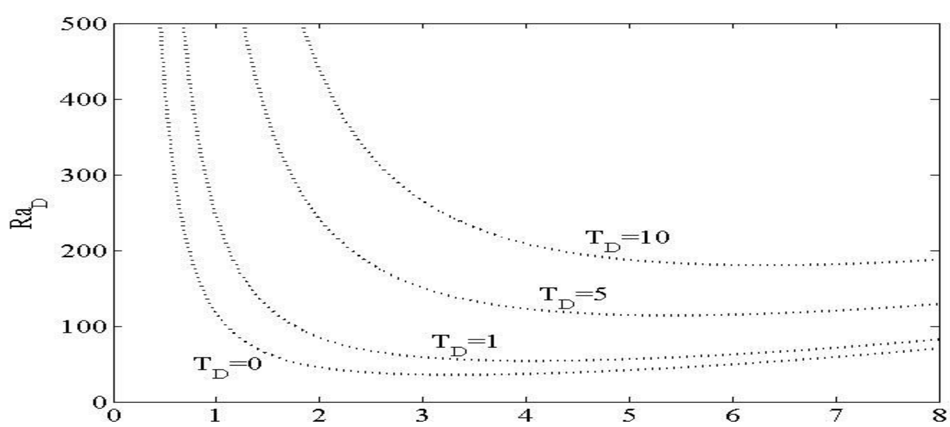


**Figure 2:** Curves for Neutral stability when $\varepsilon = 0.4, \lambda = 0.5$ for different values of Taylor -Darcy number.

The effect of rotation on the onset of instability has been studied in the figure 2. In this figure we have plotted the Rayleigh-Darcy number for the different values of Taylor-Darcy number $T_D$. For $T_D = 0$ curve is the same curve as obtained by Kim et al. [25]. On each curve, the minimum for $Ra_D$ will be the critical Rayleigh-Darcy number to mark the onset of convection. As Taylor-Darcy number increases, the critical value of Rayleigh-Darcy number increases.

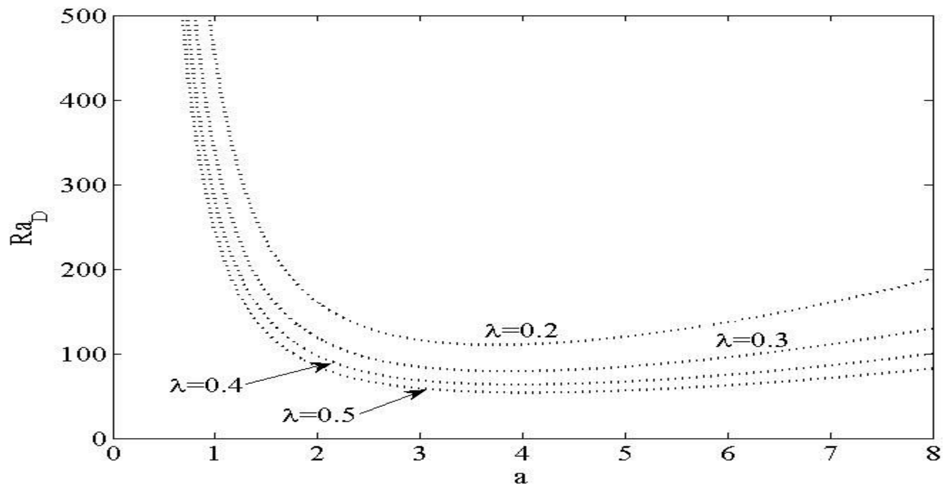


**Figure 3:** Curves for Neutral stability when $\varepsilon = 0.4, T_D = 1$ for different values of λ.

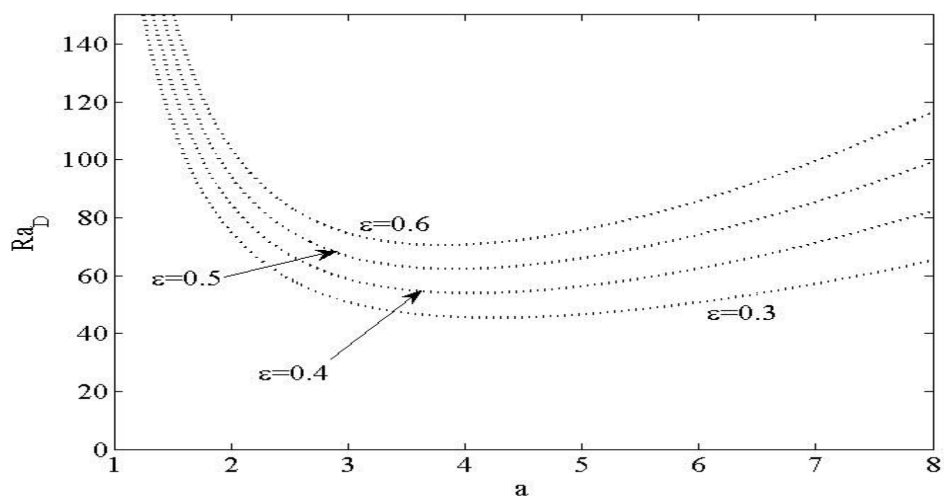


**Figure 4:** Curves for Neutral stability when $\lambda = 0.5, T_D = 1$ for different values of ε.

For $\varepsilon = 0.4, T_D = 1$, the neutral stability curves are obtained as a function of $\lambda$ as shown in Fig. 3. The critical value of Rayleigh-Darcy number increases the increasing $\lambda$ where $\lambda$ is the nondimensional relaxation time parameter. The effect of nondimensional retardation time parameter $\varepsilon$ has been studied in the figure 4. It has been observed that the Rayleigh-Darcy number increases as $\varepsilon$ increases.

## CONCLUSION:

In this paper the linear stability analysis for stationary and overstability are conducted for thermal convection in a viscoelastic fluid saturated porous layer subject to uniform rotation. The effect of relaxation, retardation and rotation on convective instabilities has been evaluated. Normal mode analysis is used, the convection motion is assumed to exhibit horizontal periodicity. It has been observed that the critical Rayleigh-Darcy number increases with increasing Taylor-Darcy number. The linear analysis results indicate that the critical Rayleigh-Darcy number for overstability increases, with increase in retardation time and Taylor-Darcy number, which decreases with increase in relaxation.

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