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EQUITABLE VERTEX COLORING OF SUDHA GRAPHS $S(n, m)$

S. SUDHA & G. M. RAJA*

RIASM, University of Madras, India.

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ABSTRACT

Sudha et al. [7] found equitable vertex coloring of Sudha graph $S(n, 2)$ and found its chromatic number is either 3 or 4 according to $n \equiv 0(mod\ 3)$ or $n \not\equiv 0(mod\ 3)$. In this paper, we have extended our discussion of equitable vertex coloring to the Sudha graph $S(n, m)$ for $m = 3, 4$ & 5 and its chromatic number lies between 2 and 4.

AMS CLASIFICATION CODE: *05C15.*

Keywords: *Sudha graph, Equitable coloring, Color classes, Chromatic number of equitable coloring.*

1. INTRODUCTION

Mayer [1] introduced the definition of a equitable vertex chromatic number of a graph. An equitable vertex coloring is an assignment of colors to the vertices of a graph in such a way that no two adjacent vertices have the same color and the number of vertices in any two color classes differ atmost by one. Application of equitable vertex coloring is found in scheduling and timetabling problems. D. Grittner [2] discussed adout the equitable chromatic number of a complete bipartite graph. Dorothee [3] found the equitable vertex coloring of complete multipartite graphs and Lih [4] elaborately discussed about the equitable vertex coloring of graphs, in particular about bipartite graphs and trees in his handbook. Sudha *et al.* [5] introduced $S(n, m)$ graph and found total coloring of $S(n, m)$ graph. Sudha *et al.* [6] found equitable chromatic number of $S(n, 2)$ is either 3 or 4 according to $n \equiv 0(mod\ 3)$ or $n \not\equiv 0(mod\ 3)$.

Definition 1: Graph vertex coloring is the coloring of the vertices of a graph with the minimum number of colors without any two adjacent vertices having the same color.

Definition 2: In vertex coloring of a graph, the set of vertices of same color are said to be in that color class.

In k-coloring of a graph, there are k color classes. They are represented by $C[1], C[2], \ldots$, where $1,2, \ldots$ denote the colors.

Definition 3: A graph G is said to be equitable k-vertex colorable if its vertex set V can be partitioned into k subsets $V_1, V_2, \ldots, V_k$, such that each V_i is an independent set and the condition $\left||V_i| - |V_j|\right| \leq 1$ holds for all $1 \leq i \leq k$, $1 \leq j \leq k$.

The smallest integer k for which G is equitable k-vertex coloring is known as the equitable vertex chromatic number of G and is denoted by $\chi_{ev} G$.

Definition 4: The Sudha graph $S(n, m)$ is a graph with the vertex set $\{v_i \ / \ 1 \leq i \leq n\}$ and the edges are defined as for $1 \leq i \leq n$,

(i) v_i is adjacent to v_{i+1} if $i + 1 \leq n$ and v_n is adjacent to v_1,
(ii) v_i is adjacent to v_{i+m} if $i + m \leq n$ and
(iii) v_i is adjacent to v_{i+n-m} if $i + m > n$.

Corresponding Author: G. M. Raja, RIASM, University of Madras, India.*

2. EQUITABLE VERTEX COLORING OF $S(n,3)$ GRAPHS

Theorem 1: Sudha graph $S(n,3)$ admits equitable vertex coloring and its chromatic number is either 2 or 3 according to n is $even$ or n is odd.

Proof: Let $v_1, v_2, v_3, \dots, v_{n-1}, v_n$ be the vertices of the graph $S(n,3)$ and its edges are defined as follows:
for $1 \le i \le n$,

(i) v_i is adjacent to v_{i+1} if $i+1 \le n$ and v_n is adjacent to v_1,
(ii) v_i is adjacent to v_{i+3} if $i+3 \le n$ and
(iii) v_i is adjacent to v_{i+n-3} if $i+3 > n$ as shown in figure 1.

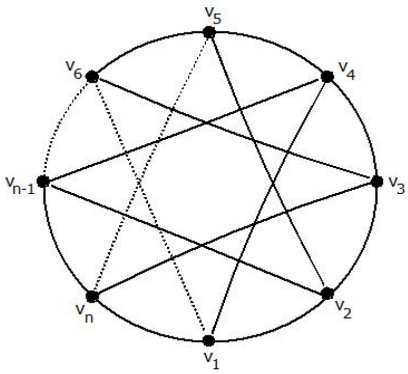

Figure-1: Sudha graph $S(n,3)$

There are two cases:

(i) n is odd
(ii) n is even
Here there are three types
(a) $n = 7 + 6j, j = 1,2,3,\dots$
(b) $n = 9 + 6j, j = 1,2,3,\dots$
(c) $n = 11 + 6j, j = 1,2,3,\dots$

The function f from the vertex set of $S(n,3)$ to the set of colors $\{1,2,3\}$ is defined as follows :

Case-(i): Let n be $even$.
The vertices of $S(n,3)$ are colored as
for $1 \le i \le n$,

$$f(v_i) = \begin{cases} 1 & i \equiv 1(mod\ 2) \\ 2 & i \equiv 0(mod\ 2) \end{cases}$$

The color classes $C[1]$ and $C[2]$ satisfy the condition $\big||C[i]| - |C[j]|\big| \le 1, 1 \le i \le 2,\ 1 \le j \le 2$. Since

$$|C[1]| = |C[2]| = \frac{n}{2}.$$

Sudha graph $S(n,3)$ has equitable vertex coloring with this type of coloring and hence $\chi_{ev}\big(S(n,3)\big) = 2$ if n is $even$.

Case-(ii): Let n be odd.

Type-(a): Let $n = 7 + 6j,\ j = 1,2,3,\dots$
The vertices of $S(n,3)$ are colored as
for $1 \le i \le \frac{n-1}{3}$

$$f(v_i) = \begin{cases} 1 & i\ is\ odd \\ 2 & i\ is\ even \end{cases}$$

for $\frac{n-1}{3} < i \le \frac{2(n-1)}{3}$

$$f(v_i) = \begin{cases} 3 & i\ is\ odd \\ 2 & i\ is\ even \end{cases}$$

for $\frac{2(n-1)}{3} < i \le n-1$

$$f(v_i) = \begin{cases} 3 & i\ is\ odd \\ 1 & i\ is\ even \end{cases}$$

and $f(v_n) = 2.$

The color classes $C[1], C[2]$ and $C[3]$ satisfy the condition $|(|C[i]| - |C[j]|)| \leq 1, 1 \leq i \leq 3, 1 \leq j \leq 3$. Since

$$|C[1]| = |C[3]| = \frac{n-1}{3}$$

$$\text{and} \quad |C[2]| = \frac{n+2}{3}.$$

Sudha graph $S(n,3)$ has equitable vertex coloring with this type of coloring and hence $\chi_{ev}(S(n,3)) = 3$ if $n = 7 + 6j$, $j = 1, 2, 3, \dots$.

Type-(b): Let $n = 9 + 6j,\ j = 0,1,2,\ \dots$.
The vertices of $S(n,3)$ are colored as
for $1 \leq i \leq \frac{n-3}{3}$

$$f(v_i) = \begin{cases} 1 & i \text{ is odd} \\ 2 & i \text{ is even} \end{cases}$$

for $\frac{n-3}{3} < i \leq \frac{2(n-3)}{3}$

$$f(v_i) = \begin{cases} 3 & i \text{ is odd} \\ 2 & i \text{ is even} \end{cases}$$

for $\frac{2(n-3)}{3} < i \leq n-3$

$$f(v_i) = \begin{cases} 3 & i \text{ is odd} \\ 1 & i \text{ is even} \end{cases}$$

$$f(v_{n-2}) = 3,$$

$$f(v_{n-1}) = 1,$$

$$\text{and} \quad f(v_n) = 2.$$

The color classes $C[1], C[2]$ and $C[3]$ satisfy the condition $|(|C[i]| - |C[j]|)| \leq 1, 1 \leq i \leq 3, 1 \leq j \leq 3$. Since

$$|C[1]| = |C[2]| = |C[3]| = \frac{n}{3}$$

Sudha graph $S(n,3)$ has equitable vertex coloring with this type of coloring and hence $\chi_{ev}(S(n,3)) = 3$ if $n = 9 + 6j$, $j = 0, 1, 2, \dots$.

Type-(c): Let $n = 11 + 6j,\ j = 0, 1, 2, \dots$.
The vertices of $S(n,3)$ are colored as
for $1 \leq i \leq \frac{n+1}{3}$

$$f(v_i) = \begin{cases} 1 & i \text{ is odd} \\ 2 & i \text{ is even} \end{cases}$$

for $\frac{n+1}{3} < i \leq \frac{2(n+1)}{3}$

$$f(v_i) = \begin{cases} 3 & i \text{ is odd} \\ 2 & i \text{ is even} \end{cases}$$

for $\frac{2(n+1)}{3} < i \leq n$

$$f(v_i) = \begin{cases} 3 & i \text{ is odd} \\ 1 & i \text{ is even} \end{cases}$$

The color classes $C[1], C[2]$ and $C[3]$ satisfy the condition $|(|C[i]| - |C[j]|)| \leq 1, 1 \leq i \leq 3, 1 \leq j \leq 3$. Since

$$|C[1]| = \frac{n-2}{3}$$

$$\text{and} \quad |C[2]| = |C[3]| = \frac{n+1}{3}$$

Sudha graph $S(n,3)$ has equitable vertex coloring with this type of coloring and hence $\chi_{ev}(S(n,3)) = 3$ if $n = 11 + 6j,\ j = 0, 1, 2, \dots$.

Therefore the equitable vertex chromatic number of $S(n,3)$ is either 2 or 3 according to n is $even$ or n is odd.

Illustration 6.1.10: Consider the graph $S(15,3)$. Using theorem 1 case (ii), type (b), we assign the color 1 to the vertices $v_1, v_3, v_{10}, v_{12}, v_{14}$, color 2 to the vertices $v_2, v_4, v_6, v_8, v_{15}$ and color 3 to the vertices $v_5, v_7, v_9, v_{11}, v_{13}$ shown in figure 2.

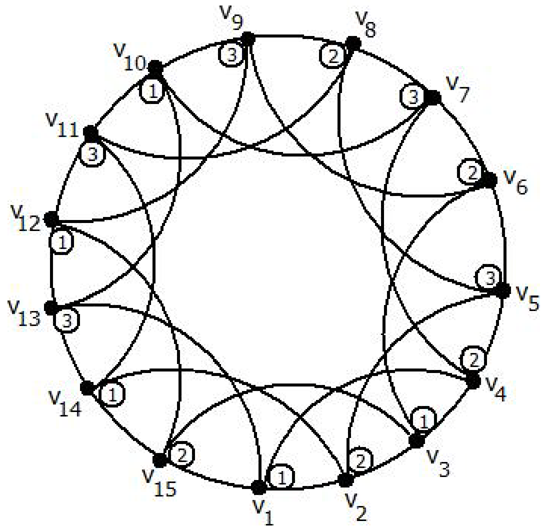

Figure-2: Sudha graph $S(15,3)$

Here $|C[1]| = 5, |C[2]| = 5$ and $|C[3]| = 5$. They satisfy the conditions $|(|C[i]| - |C[j]|)| < 1, 1 \leq i \leq 3, 1 \leq j \leq 3$. This type of coloring on Sudha graph $S(15,3)$ satisfy the conditions for equitable vertex coloring.

Hence $\chi_{ev}\big(S(15,3)\big) = 3$.

Illustration 6.1.11: Consider the graph $S(17,3)$. Using theorem 6.1.6 case (ii) type (c), we assign the color 1 to the vertices $v_1, v_3, v_5, v_{14}, v_{16}$, color 2 to the vertices $v_2, v_4, v_6, v_8, v_{10}, v_{12}$ and color 3 to the vertices $v_7, v_9, v_{11}, v_{13}, v_{15}, v_{17}$ as shown in figure 3.

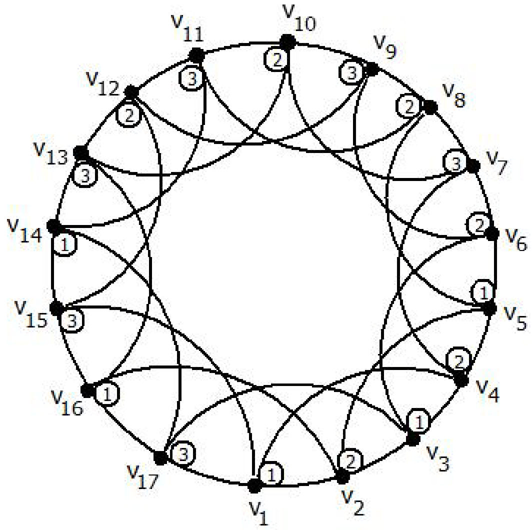

Figure-3: Sudha graph $S(17,3)$

Here $|C[1]| = 5, |C[2]| = 6$ and $|C[3]| = 6$. They satisfy the condition $|(|C[i]| - |C[j]|)| \leq 1, 1 \leq i \leq 3, 1 \leq j \leq 3$. This type of coloring on Sudha graph $S(17,3)$ satisfy the conditions for equitable vertex coloring.

Hence $\chi_{ev}\big(S(17,3)\big) = 3$.

3. EQUITABLE VERTEX COLORING OF $S(n,4)$ GRAPHS

Theorem 2: Sudha graph $S(n,4)$ admits equitable vertex coloring and its chromatic number is 3.

Proof: Let $v_1, v_2, v_3, \ldots, v_{n-1}, v_n$ be the vertices of the graph $S(n,4)$ and its edges are defined as follows:
for $1 \leq i \leq n$,

(i) v_i is adjacent to v_{i+1} if $i + 1 \leq n$ and v_n is adjacent to v_1,
(ii) v_i is adjacent to v_{i+4} if $i + 4 \leq n$ and
(iii) v_i is adjacent to v_{i+n-4} if $i + 4 > n$ as shown in figure 4.

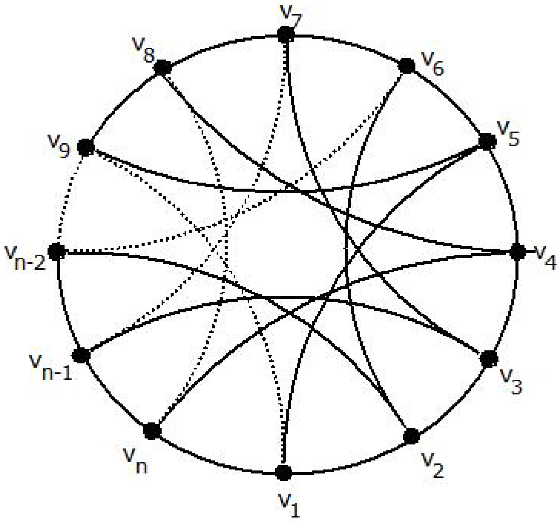

Figure-4: Sudha graph $S(n, 4)$

There are $three$ cases:

(i) $n = 10 + 6j,\ j = 0, 1, 2, \dots$

(ii) $n = 13 + 6j,\ j = 0, 1, 2, \dots$

(iii) $n \neq 10 + 6j$ and $n \neq 13 + 6j,\ j = 0, 1, 2, \dots$

The function f from the vertex set of $S(n, 4)$ to the set of colors $\{1, 2, 3\}$ is defined as follows :

Case-(i): Let $= 10 + 6j,\ j = 0, 1, 2, \dots$.

The vertices of $S(n, 4)$ are colored as

for $1 \leq i \leq \frac{n}{2}$

$$f(v_i) = \begin{cases} 1 & i \equiv 1(mod\ 3) \\ 2 & i \equiv 2(mod\ 3) \\ 3 & i \equiv 0(mod\ 3) \end{cases}$$

for $\frac{n}{2} < i < n$

$$f(v_i) = \begin{cases} 2 & i \equiv 1(mod\ 3) \\ 3 & i \equiv 2(mod\ 3) \\ 1 & i \equiv 0(mod\ 3) \end{cases}$$

and $f(v_n) = 3.$

The color classes $C[1], C[2]$ and $C[3]$ satisfy the condition $|(|C[i]| - |C[j]|)| \leq 1, \quad 1 \leq i \leq 3,\ 1 \leq j \leq 3$. Since

$$|C[1]| = \frac{n+2}{3}$$

and $|C[2]| = |C[3]| = \frac{n-1}{3}$.

Sudha graph $S(n, 4)$ has equitable vertex coloring with this type of coloring and hence $\chi_{ev}(S(n, 4)) = 3$ if $n = 10 + 6j,\ j = 0, 1, 2, \dots$.

Case-(ii): Let $n = 13 + 6j,\ j = 0, 1, 2, \dots$

The vertices of $S(n, 4)$ are colored as

for $1 \leq i \leq \frac{n+3}{2}$

$$f(v_i) = \begin{cases} 1 & i \equiv 1(mod\ 3) \\ 2 & i \equiv 2(mod\ 3) \\ 3 & i \equiv 0(mod\ 3) \end{cases}$$

for $\frac{n+3}{2} < i < n$

$$f(v_i) = \begin{cases} 2 & i \equiv 1(mod\ 3) \\ 3 & i \equiv 2(mod\ 3) \\ 1 & i \equiv 0(mod\ 3) \end{cases}$$

and $f(v_n) = 3.$

The color classes $C[1], C[2]$ and $C[3]$ satisfy the condition $|(|C[i]| - |C[j]|)| \leq 1,\ 1 \leq i \leq 3,\ 1 \leq j \leq 3$. Since

$$|C[1]| = \frac{n+2}{3}$$

$$\text{and} \quad |C[2]| = |C[3]| = \frac{n-1}{3}.$$

Sudha graph $S(n,4)$ has equitable vertex coloring with this type of coloring and hence $\chi_{ev}(S(n,4)) = 3$ if $n = 13 + 6j,\ j = 0,1,2,\dots$.

Case-(iii): Let $n \not\equiv 10 + 6j$ and $n \not\equiv 13 + 6j, j = 0,1,2,\dots$

The vertices of $S(n,4)$ are colored as

for $1 \leq i \leq n$

$$f(v_i) = \begin{cases} 1 & i \equiv 1 (mod\ 3) \\ 2 & i \equiv 2 (mod\ 3) \\ 3 & i \equiv 0 (mod\ 3) \end{cases}$$

The color classes $C[1], C[2]$ and $C[3]$ satisfy the condition $|(|C[i]| - |C[j]|)| \leq 1, 1 \leq i \leq 3,\ 1 \leq j \leq 3$. Since

$$|C[1]| = \frac{n+2}{3}$$

$$\text{and} \quad |C[2]| = |C[3]| = \frac{n-1}{3}.$$

Sudha graph $S(n,4)$ has equitable vertex coloring with this type of coloring and hence $\chi_{ev}(S(n,4)) = 3$ if $n \not\equiv 10 + 6j$ and $\not\equiv 13 + 6j, j = 0,1,2,\dots$.

Therefore the equitable vertex chromatic number of $S(n,4)$ is 3.

Illustration 3: Consider the graph $S(15,4)$. Using theorem 2 case (iii), we assign the color 1 to the vertices $v_1, v_4, v_7, v_{10}, v_{13}$, color 2 to the vertices $v_2, v_5, v_8, v_{11}, v_{14}$ and color 3 to the vertices $v_3, v_6, v_9, v_{12}, v_{15}$ as shown in figure 5.

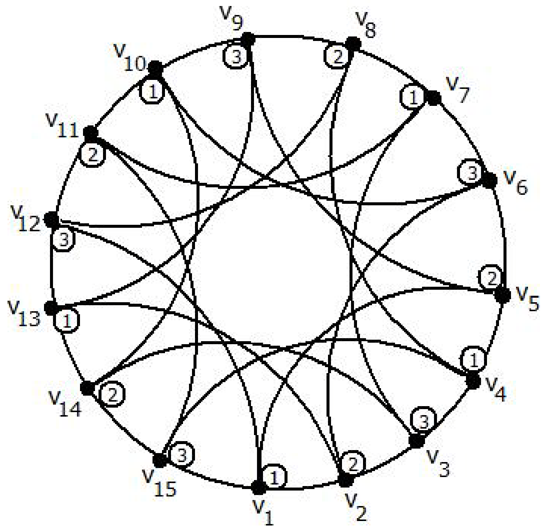

Figure-5: Sudha graph $S(15,4)$

Here $|C[1]| = 5, |C[2]| = 5$ and $|C[3]| = 5$. They satisfy the conditions $|(|C[i]| - |C[j]|)| < 1, 1 \leq i \leq 3, 1 \leq j \leq 3$. This type of coloring on Sudha graph $S(15,4)$ satisfy the conditions for equitable vertex coloring.

Hence $\chi_{ev}(S(15,4)) = 3$.

4. EQUITABLE VERTEX COLORING OF $S(n,5)$ GRAPHS

Theorem 3: Sudha graph $S(n,5)$ admits equitable vertex coloring and its chromatic number is lies between 2 to 4.

Proof: Let $v_1, v_2, v_3, \dots, v_{n-1}, v_n$ be the vertices of the graph $S(n,5)$ and its edges are defined as follows:

for $1 \leq i \leq n$

(i) v_i is adjacent to v_{i+1} if $i + 1 \leq n$ and v_n is adjacent to v_1,

(ii) v_i is adjacent to v_{i+5} if $i + 5 \leq n$ and

(iii) v_i is adjacent to v_{i+n-5} if $i + 5 > n$ as shown in figure 6.

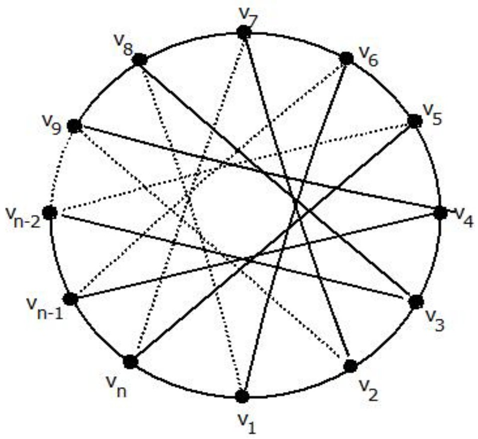

Figure-6: Sudha graph $S(n, 5)$

There are two cases :

(i) n is even

(ii) n is odd

Here there are three types

(a) $n \equiv 0(mod\ 3)$

(b) $n \not\equiv 0(mod\ 3)$ and $n = 7 + 4j,\ j = 1, 2, 3, ...$

(c) $n \not\equiv 0(mod\ 3)$ and $n = 9 + 4j,\ j = 1, 2, 3, ...$

The function f from the vertex set of $S(n, 5)$ to the set of colors $\{1, 2, 3, 4\}$ is defined as follows:

Case-(i): Let n be $even$.
The vertices of $S(n, 5)$ are colored as
for $1 \leq i \leq n$

$$f(v_i) = \begin{cases} 1 & i \equiv 1(mod\ 2) \\ 2 & i \equiv 0(mod\ 2) \end{cases}$$

The color classes $C[1]$ and $C[2]$ satisfy the condition $\big||C[i]| - |C[j]|\big| \leq 1, 1 \leq i \leq 2,\ 1 \leq j \leq 2$. Since

$$|C[1]| = |C[2]| = \frac{n}{2}.$$

Sudha graph $S(n, 5)$ has equitable vertex coloring with this type of coloring and hence $\chi_{ev}\big(S(n, 5)\big) = 2$ if n is $even$.

Case-(ii): Let n be odd.

Type-(a): Let $n \equiv 0(mod\ 3)$.
The vertices of $S(n, 5)$ are colored as
for $1 \leq i \leq n$

$$f(v_i) = \begin{cases} 1 & i \equiv 1(mod\ 3) \\ 2 & i \equiv 2(mod\ 3) \\ 3 & i \equiv 0(mod\ 3) \end{cases}$$

The color classes $C[1], C[2]$ and $C[3]$ satisfy the condition $|(|C[i]| - |C[j]|)| \leq 1,\ 1 \leq i \leq 3,\ 1 \leq j \leq 3$. Since

$$|C[1]| = |C[2]| = |C[3]| = \frac{n}{3}.$$

Sudha graph $S(n, 5)$ has equitable vertex coloring with this type of coloring and hence $\chi_{ev}\big(S(n, 5)\big) = 3$ if $n \equiv 0(mod\ 3)$.

Type-(b): Let $n \not\equiv 0(mod\ 3)$ and $= 7 + 4j,\ j = 1, 2, 3, ...$.
The vertices of $S(n, 5)$ are colored as
for $1 \leq i \leq n$

$$f(v_i) = \begin{cases} 1 & i \equiv 1(mod\ 4) \\ 2 & i \equiv 2(mod\ 4) \\ 3 & i \equiv 3(mod\ 4) \\ 4 & i \equiv 0(mod\ 4) \end{cases}$$

The color classes $C[1], C[2], C[3]$ and $C[4]$ satisfy the condition$|(|C[i]| - |C[j]|)| \leq 1$, $1 \leq i \leq 4$, $1 \leq j \leq 4$. Since

$$|C[1]| = |C[2]| = |C[3]| = \frac{n+1}{4}$$

and $|C[4]| = \frac{n-3}{4}$.

Sudha graph $S(n, 5)$ has equitable vertex coloring with this type of coloring and hence $\chi_{ev}\big(S(n, 5)\big) = 4$ if $n \not\equiv 0(mod\ 3)$ and $= 7 + 4j$, $j = 1, 2, 3, \ldots$.

Type-(c): Let $n \not\equiv 0(mod\ 3)$ and $= 9 + 4j$, $j = 1, 2, 3, \ldots$.
The vertices of $S(n, 5)$ are colored as
for $1 \leq i \leq n - 5$

$$f(v_i) = \begin{cases} 1 & i \equiv 1(mod\ 4) \\ 2 & i \equiv 2(mod\ 4) \\ 3 & i \equiv 3(mod\ 4) \\ 4 & i \equiv 0(mod\ 4) \end{cases}$$

$$f(v_{n-4}) = 2,$$

$$f(v_{n-3}) = 3,$$

$$f(v_{n-2}) = 4,$$

$$f(v_{n-1}) = 1,$$

and $f(v_n) = 2.$

The color classes $C[1], C[2], C[3]$ and $C[4]$ satisfy the condition $|(|C[i]| - |C[j]|)| \leq 1$, $1 \leq i \leq 4$, $1 \leq j \leq 4$. Since

$$|C[1]| = |C[3]| = |C[4]| = \frac{n-1}{4}$$

and $|C[4]| = \frac{n+3}{4}$.

Sudha graph $S(n, 5)$ has equitable vertex coloring with this type of coloring and hence $\chi_{ev}\big(S(n, 5)\big) = 4$ if $n \not\equiv 0(mod\ 3)$ and $= 9 + 4j$, $j = 1, 2, 3, \ldots$.

Therefore the equitable vertex chromatic number of $S(n, 5)$ is lies between 2 to 4.

Illustration 4: Consider the graph $S(17, 5)$. Using theorem 3 case (ii) type (c), we assign the color 1 to the vertices v_1, v_5, v_9, v_{16}, color 2 to the vertices $v_2, v_6, v_{10}, v_{13}, v_{17}$, color 3 to the vertices v_3, v_7, v_{11}, v_{14} and color 4 to the vertices v_4, v_8, v_{12}, v_{15} as shown in figure 7.

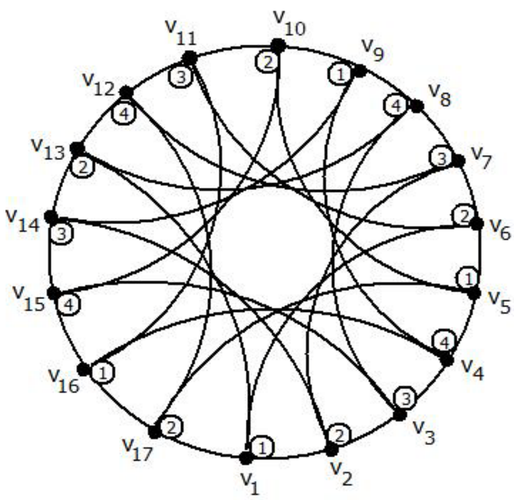

Figure-7: Sudha graph $S(17, 5)$

Here $|C[1]| = 4, |C[2]| = 5, |C[3]| = 4$ and $|C[4]| = 4$. They satisfy the conditions $|(|C[i]| - |C[j]|)| < 1, 1 \leq i \leq 4, 1 \leq j \leq 4$. This type of coloring on Sudha graph $S(17, 5)$ satisfy the conditions for equitable vertex coloring.
Hence $\chi_{ev}\big(S(17, 5)\big) = 4$.

5. EQUITABLE VERTEX COLORING OF $S(n,m)$ GRAPHS

Theorem 4: Sudha graph $S(n,m)$, n is even and m is odd admits equitable vertex coloring and its chromatic number is 2.

Proof: Let $v_1, v_2, v_3, \dots, v_{n-1}, v_n$ be the vertices of the graph $S(n,m)$ and its edges are defined as follows:
for $1 \le i \le n$

(i) v_i is adjacent to v_{i+1} if $i+1 \le n$ and v_n is adjacent to v_1,
(ii) v_i is adjacent to v_{i+m} if $i+m \le n$ and
(iii) v_i is adjacent to v_{i+n-m} if $i+m > n$.

The function f from the vertex set of $S(n,m)$, n is even and m is odd to the set of colors $\{1,2\}$ is defined as follows :

The vertices of $S(n,m)$, n is even and m is odd are colored as for $1 \le i \le n$

$$f(v_i) = \begin{cases} 1 & i \equiv 1 (mod\ 2) \\ 2 & i \equiv 0 (mod\ 2) \end{cases}$$

The color classes $C[1]$ and $C[2]$ satisfy the condition $\big||C[i]| - |C[j]|\big| \le 1$, $1 \le i \le 2$, $1 \le j \le 2$. Since

$$|C[1]| = |C[2]| = \frac{n}{2}.$$

Sudha graph $S(n,m)$, n is even and m is odd has equitable vertex coloring with this type of coloring and hence $\chi_{ev}(S(n,m)) = 2$ if n is even and m is odd.

Theorem 5: Sudha graph $S(n,m)$, n is odd and divisible by 3, m is odd and not divisible by 3 admits equitable vertex coloring and its chromatic number is 3.

Proof: Let $v_1, v_2, v_3, \dots, v_{n-1}, v_n$ be the vertices of the graph $S(n,m)$ and its edges are defined as follows:
for $1 \le i \le n$

(i) v_i is adjacent to v_{i+1} if $i+1 \le n$ and v_n is adjacent to v_1,
(ii) v_i is adjacent to v_{i+m} if $i+m \le n$ and
(iii) v_i is adjacent to v_{i+n-m} if $i+m > n$.

The function f from the vertex set of $S(n,m)$, n is odd and divisible by 3, m is odd and not divisible by 3 to the set of colors $\{1,2,3\}$ is defined as follows :

The vertices of $S(n,m)$, n is odd and divisible by 3, m is odd and not divisible by 3 are colored as for $1 \le i \le n$

$$f(v_i) = \begin{cases} 1 & i \equiv 1 (mod\ 3) \\ 2 & i \equiv 2 (mod\ 3) \\ 3 & i \equiv 0 (mod\ 3) \end{cases}$$

The color classes $C[1], C[2]$ and $C[3]$ satisfy the condition $\big||C[i]| - |C[j]|\big| \le 1$, $1 \le i \le 3$, $1 \le j \le 3$. Since

$$|C[1]| = |C[2]| = |C[3]| = \frac{n}{3}.$$

Sudha graph $S(n,m)$, n is odd and divisible by 3, m is odd and not divisible by 3 has equitable vertex coloring with this type of coloring and hence $\chi_{ev}(S(n,m)) = 3$ if n is odd and divisible by 3, m is odd and not divisible by 3.

CONCLUSION

We have established the equitable vertex coloring of Sudha graph $S(n,m)$ for any n, $m = 3, 4, 5$ and their equitable vertex chromatic number lies between 2 and 4 depending on the values of m and n.

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