International Journal of Mathematical Archive-9(1), 2018, 170-173
Available online through www.ijma.info ISSN 2229 – 5046

N-DIMENSIONAL KALUZA-KLEIN TWO FLUID COSMOLOGICAL MODEL

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(Received On: 10-12-17; Revised & Accepted On: 30-12-17)

ABSTRACT

N-dimensional two fluid Kaluza-Klein space time is consider in general theory of relativity. In these models one fluid is the radiation distribution which represents the cosmic microwave background and the other fluid is the perfect fluid representing the matter content of the universe. For solving the Einstein's field equations the relation between metric coefficients a&b and $\gamma -$law for equation of state are used. Also, some physical and kinematical properties of the model are studied.

Key words: *n-dimensional Kaluza-Klein, radiation distribution.*

1. INTRODUCTION

The idea of higher-dimensional theory was originated in super string and super gravity theories with other fundamental forces in nature. The solution of Einstein field equation in higher-dimensional space-time believed to be of physical relevance possibly at the extremely early times before the universe underwent compactification transitions. It is well known that our universe was much small in its early stage than it is today. Indeed the present four dimensional stage of the universe could have been preceded by a higher-dimensional stage, which at later times becomes effectively four dimensional in the sense that the extra dimensions became unobservable small due to dynamical contraction.The study of higher dimensional cosmological models is motivated mainly by the possibility of geometrically unifying the fundamental interactions of the universe. Kaluza (1921) and Klein (1926) theory is most significant theory because it was one of the early possibilities of unification of gravity with electromagnetism and it has been elegantly presented in terms of geometry.In a certain sense, it is just ordinary gravity in free space described in five dimension instead of four. Overduin and Wesson (1997) have presented a review of Kluza-Klein theory. Ponce (1988), Chi (1990) Fukui (1993), Liu and Wesson (1994), Coley (1994) have studied Kaluza Klein cosmological models with different matters. Palatnik (2007) Constructed Schwarschild solution for 3 space and n time dimensions in Kaluza-Klein theory. Friedman Robertson Walker spatially homogeneous and isotropic models are widely considered as good approximation of the present and early stage of the universe. Two-fluid FRW models of the universe have been investigated where one fluid is the radiation field corresponding to the observed cosmic background radiation, while a perfect fluid is chosen to represent the matter content of the universe(Coley and Tupper,1986, Coley,1988). Chodos and Detweller (1980), Ibanez and Verdaguer (1986), Gleisur and Diaz (1988), Banerjee and Bhui (1990), Reddy and VenkateswaraRao (2001), Chatterjee (1993), Khadekar*et.al.*(2005) and Adhav *et.al*. (2008) have studied the multidimensional cosmological models in Einstein's general relativity theory.Also, Reddy D.R.K. and Naidu, R.L. (2007) have discuss string cosmological models in Kaluza-Klein five dimensional space time in the framework of Saez and Ballester (1985) scalar tensor theory of gravitation. Mete *et.al.* (2013) studied Higher dimensional plane symmetric cosmological model with Two fluid source in General Relativity

In this paper, we discussed n-dimensional Kaluza-Klein cosmological model in General theory of relativity in the presence of twofluid sources.Our paper is organized as follows. In section 2, we derive the Einstein's field equations and their solutions. Section3, is mainly concerned with the physical Kinematical properties of the model. The last section contains some conclusion.

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2. FIELD EQUATIONS AND SOLUTIONS OF THE MODEL

We consider the n-dimensional Kaluza Klein space time in the form

$$ds^2 = -dt^2 + a^2 \sum_{i=1}^{n-2} dx_i{}^2 + b^2 dx_{n-1}{}^2, \tag{1}$$

Where *a* and *b* are the functions of time *'t'* only.

The energy momentum tensor of the source is given by

$$T_{ij} = T_{ij}^{(m)} + T_{ij}^{(r)} \tag{2}$$

Where $T_{ij}^{(m)}$ is the energy momentum tensor for the matter field described by a perfect fluid density ρ_m, pressure p_m and four velocity $u_i^{(m)}$, where

$$g^{ij} u_i^{(m)} u_j^{(m)} = 1 \tag{3}$$

$T_{ij}^{(r)}$ denotes the energy momentum tensor for the radiation field with density ρ_r. Pressure $p_r = \frac{1}{3}\rho_r$ and four-velocity $u_i^{(r)}$, where

$$g^{ij} u_i^{(r)} u_j^{(r)} = 1 \tag{4}$$

Thus

$$T_{ij}^{(m)} = (\rho_m + p_m) u_i^{(m)} u_j^{(m)} + p_m g_{ij} \tag{5}$$

$$T_{ij}^{(r)} = \frac{4}{3}\rho_r u_i^{(m)} u_j^{(r)} + \frac{1}{3}\rho_r g_{ij} \tag{6}$$

Using energy conditions $\rho_m + p_m > 0$, $\rho_r > 0$, velocity

$$u_1^{(m)} = u_2^{(m)} = u_3^{(m)} = u_1^{(r)} = u_2^{(r)} = u_3^{(r)} = 0$$

$$u_4^{(m)} = u_4^{(r)} = 1$$

In commoving system of coordinates

$$T_0^o = \rho_m + 2p_m + \frac{5}{3}\rho_r$$

and $$T_1^1 = T_2^2 = T_3^3 = T_4^4 = T_{n-1}^{n-1} = p_m + \frac{1}{3}\rho_r \tag{7}$$

Einstein's field equations in general relativity are

$$R_i^j - \frac{1}{2}R = -8\pi G T_i^j \tag{8}$$

Here, we consider geometrized units so that $8\pi G = C = 1$

The field equation (8) for the metric (1) with the help of (2) and (7) can be written as

$$(n-2)\frac{\dot{a}\dot{b}}{ab} + \left(\frac{n^2-5n+6}{2}\right)\left(\frac{\dot{a}}{a}\right)^2 = -\rho_m - 2p_m - \frac{5}{3}\rho_r \tag{9}$$

$$(n-3)\frac{\ddot{a}}{a} + \left(\frac{n^2-7n+12}{2}\right)\left(\frac{\dot{a}}{a}\right)^2 + (n-3)\frac{\dot{a}\dot{b}}{ab} + \frac{\ddot{b}}{b} = -p_m - \frac{1}{3}\rho_r \tag{10}$$

$$(n-2)\frac{\ddot{a}}{a} + \left(\frac{n^2-5n+6}{2}\right)\left(\frac{\dot{a}}{a}\right)^2 = -p_m - \frac{1}{3}\rho_r \tag{11}$$

Here over head dot denotes differentiation with to *t*

Equations (9) to (11) are three independent equations in five unknown a, b, p_m, ρ_m and ρ_r. For complete determinacy of the system two extra conditions are needed.

For this purpose, first we assume the relation between metric potential *a* and *b*as $a = b^n$
And secondly, we assume a $\gamma -$law for equation of state

$$p_m = (\gamma - 1)\rho_m, \quad 1 \le \gamma \le 2 \tag{12}$$

For $\gamma = 1$.

With the help of equation (12) the above equations (9)-(11), becomes

$$(n-2)\frac{\dot{a}\dot{b}}{ab} + \left(\frac{n^2 - 5n + 6}{2}\right)\left(\frac{\dot{a}}{a}\right)^2 = -\frac{5}{3}\rho_r \tag{13}$$

$$(n-3)\frac{\ddot{a}}{a} + \left(\frac{n^2 - 7n + 12}{2}\right)\left(\frac{\dot{a}}{a}\right)^2 + (n-3)\frac{\dot{a}\dot{b}}{ab} + \frac{\ddot{b}}{b} = -\frac{1}{3}\rho_r \tag{14}$$

$$(n-2)\frac{\ddot{a}}{a} + \left(\frac{n^2 - 5n + 6}{2}\right)\left(\frac{\dot{a}}{a}\right)^2 = -\frac{1}{3}\rho_r \tag{15}$$

Then equations (13)-(15) admit the exact solutions

$$a = N^n (k_1 t + k_2)^{\frac{n}{M+1}} \text{ and } b = N(k_1 t + k_2)^{\frac{1}{M+1}}, \text{where } N = (M+1)^{\frac{1}{M+1}} \tag{16}$$

$$p_m = \rho_m = 0 \tag{17}$$

$$\rho_r = \frac{k_3}{(k_1 t + k_2)^2} \tag{18}$$

$$\text{Where, } k_3 = \left[\frac{-3(n-2)n(n-M-1)k_1^{\,2} n}{(M+1)} - \frac{3}{2}\frac{(n^2 - 5n + 6)}{(M+1)^2}(k_1 n)^2\right] \tag{19}$$

With the help of equation (16), the metric (1) can be expressed as

$$ds^2 = -dt^2 + N^{2n}(k_1 t + k_2)^{\frac{2n}{M+1}} \sum_{i=1}^{n-2} dx_i^{\,2} + N^2 (k_1 t + k_2)^{\frac{2}{M+1}} dx_{n-1} \tag{20}$$

Through a proper choice of coordinates and constants of integration, the above equation (20) reduces to

$$ds^2 = -\frac{dT^2}{k_1^{\,2}} + N^{2n}(T)^{\frac{2n}{M_1+1}} \sum_{i=1}^{n-2} dx_i^{\,2} + N^2 (T)^{\frac{2}{M+1}} dx_{n-1}^2, \tag{21}$$

3. PHYSICAL AND KINEMATICAL PROPERTIES

Some physical and kinematical parameters for the model (21) are

$$\text{Proper Volume} \quad V^3 = \sqrt{-g} = k_4 (T)^{\frac{n^2 - 2n + 1}{M+1}}, \text{ where } k_4 = (N)^{\left(n^2 - 2n + \frac{1}{M+1}\right)} \tag{22}$$

$$\text{Expansions} \quad \text{Scalar}(\theta) = k_5 \frac{1}{T}, \text{where } k_5 = \frac{1}{3}\frac{k_1}{M+1}(n^2 - 2n + 1) \tag{23}$$

$$\text{Shear Scalar} \quad (\sigma^2) = k_6 \frac{1}{T^2}, \text{where } k_6 = \frac{(n-1)k_5^{\,2}}{6} \tag{24}$$

The model (21) has no initial singularity, while the density given by (18) possess initial singularities. However, as *T* increases these singularities vanish. The proper volume of the model given by (22) shows the anisotropic expansion of the universe with time. For the model (21), the expansion scalar θ and shear scalar σ^2 tends to zero as $T \to \infty$.

Also, since $\lim_{T\to\infty}\left(\frac{\sigma}{\theta}\right) \neq 0$

The model does not approach isotropy for large values of *T*.

For $\gamma = \frac{4}{3}$, and $\gamma = 2$ same solution of field equations (9)–(11) are obtain as given in equations (16) With different values of density and pressure.

4. CONCLUSION

In this paper, we have obtained higher dimensional Kaluza-Klein cosmological model in presence of the matter field described by a perfect fluid with density ρ_m, pressure p_m and radiation field with density ρ_r in general relativity. It is observed that the models is similar for different values γ and behave alike. Also it is non singular and does not approach isotropy for large values of *T*. For $\gamma = \frac{4}{3}$ and $\gamma = 2$ the radiation density, matter density and pressure diverges at the initial moment.

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Source of support: Nil, Conflict of interest: None Declared.