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NOTE ON COMPLEMENT GRAPHS

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ABSTRACT

In this article we generalize the complement for all types of graphs and we proved some primary results on complement of graphs.

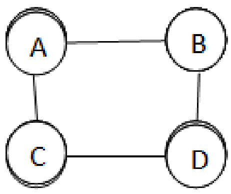
Key words: *Subgraph, Complement of graph.*

1. INTRODUCTION

There are various types of graphs depending upon the number of vertices, number of edges, interconnectivity, and their overall structure. We will discuss only a complement of graphs in this chapter. Note that the edges in graph-I are not present in graph-II and vice versa. Hence, the combination of both the graphs gives a final graph of 'n' vertices.

1.1. Definition: A graph 'G' is defined as G = (V, E) Where V is a set of all vertices and E is a set of all edges in the graph.

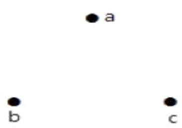
Example:



In the above example, AB, AC, CD, and BD are the edges of the graph. Similarly, A, B, C, and D are the vertices of the graph.

1.2. Definition: A graph having no edges is called a Null Graph.

Example:



1.3. Definition: A simple graph with 'n' mutual vertices is called a complete graph and it is denoted by 'K_n'. In the graph, a vertex should have edges with all other vertices, and then it is called a complete graph.

In other words, if a vertex is connected to all other vertices in a graph, then it is called a complete graph.

Example: In the following graphs I and II, each vertex in the graph is connected with all the remaining vertices in the graph except by itself

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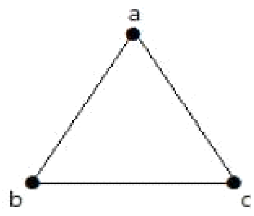
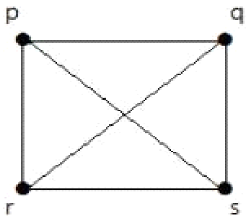
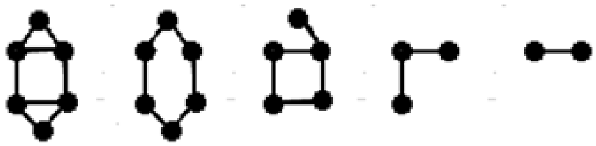




I   II

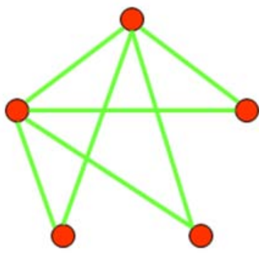
1.4. Definition: Let H be a graph with vertex set V(H) and edge set E(H), and similarly let G be a graph with vertex set V(G) and edge set E(G). Then, we say that H is a subgraph of G if V(H)$\subseteq$ (V(G) and E(H)$\subseteq$ (E(G). In such a case, we also say that G is a subgraph of H.

Example: Last four graphs are sub graphs for first graph in the below diagram.

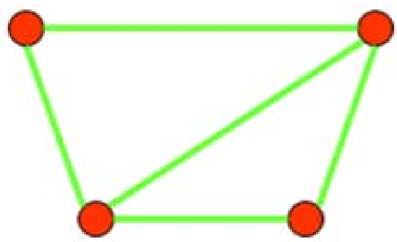


1.5. Definition: The union of 2 simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the simple graph $G = (V, E)$ with vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2$. The union is denoted by $G_1 \cup G_2$.

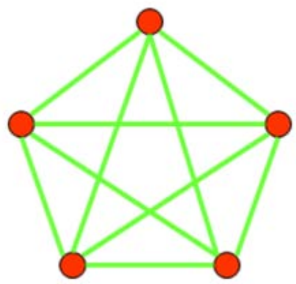
Example:



Graph G_1



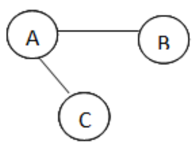
Graph G_2



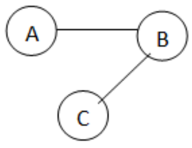
$G_1 \cup G_2$ Graph

1.6. Definition: The intersection of graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the graph $G = (V, E)$ with vertex set $V = V_1 \cap V_2$ and edge set $E = E_1 \cap E_2$. The intersection is denoted by $G_1 \cap G_2$.

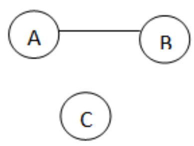
Example:



Graph G_1



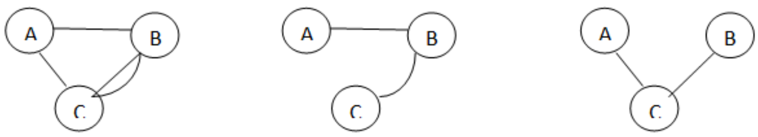
Graph G_2



Graph $G_1 \cap G_2$

1.7. Definition: The complement of a graph G is the graph having the same vertex set as G such that two vertices are adjacent if and only the same two vertices are non-adjacent in G. We denote the complement of a graph G by G^c or G^1 or $\overline{G}$

Example:



Graph G Subgraph H Complement of H^c

1.8. Notation: Since the complete graph on n vertices has nc_2 edges, it follows that if G is a graph on n vertices with m edges, then G^c is also a graph on n vertices but with $nc_2 - m$ edges.

2. OUR APPROACH ON COMPLEMENT OF GRAPHS

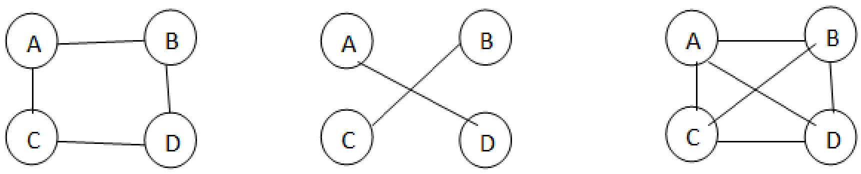
2.1. Lemma: Let G be a graph and H is a subgraph of G with the same vertices set of G then H^c is also subgraph of G.

The subgraphs with the same vertices set of graph G contains only its complement. So every subgraph of graph G has may not have its complement.

2.2. Theorem: Let G be a graph and H is a subgraph of G with the same vertices set of G then $G = H \cup H^c$

Proof: Let G be a graph with vertices set V and edges set E. H be a subgraph of G with vertices set V and edges set E_1 then H^c has with the same vertices set V and edges set E_2 which is different from E_1. Since all the edges of E contained in any one of the edge sets E_1 and E_2, $E = E_1 \cup E_2$. Therefore, $G = H \cup H^c$.

Example:

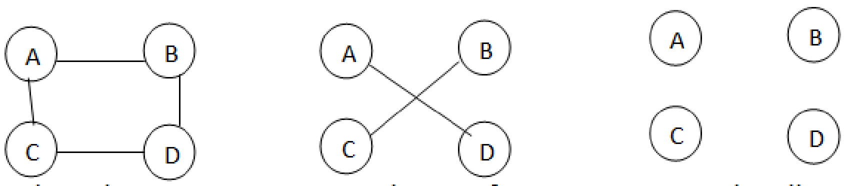


Subgraph H Complement of H or H^c $G = H \cup H^c$

2.3. Theorem: Let G be a graph and H is a subgraph of G with the same vertices set of G then $H \cap H^c$ is null graph

Proof: Let G be a graph with vertices set V and edges set E. H be a subgraph of G with vertices set V and edges set E_1 then H^c has with the same vertices set V and edges set E_2 which is different from E_1 implies $E_1 \cap E_2$ is empty set. So, $H \cap H^c$ has vertices set V and no edges. $H \cap H^c$ is null graph.

Example: Consider the above example



Subgraph H Complement of H or H^c $H \cap H^c$ is null graph

2.4. Notations:
1) The complement of graph G is null graph and null graph complement is graph G. G and null graphs are improper subgraphs of graph G.
2) Let G be a graph and H is a subgraph of G with the same vertices set of G then $(H^c)^c = H$.
3) Let G be graph with subgraphs $H_1, H_2, H_3, \ldots, H_n$ then $(H_1 \cup H_2 \cup H_3 \cup \ldots \cup H_n)^c = H_1^c \cap H_2^c \cap H_3^c \cap \ldots \cap H_n^c$
4) Let G be graph with subgraphs $H_1, H_2, H_3, \ldots, H_n$ then $(H_1 \cap H_2 \cap H_3 \cap \ldots \cap H_n)^c = H_1^c \cup H_2^c \cup H_3^c \cup \ldots \cup H_n^c$.
5) Observe that the trivial graph on 1 vertex and no edges is clearly self-complement.

Similarly, we can prove the following theorems

2.5. Theorem: Let G be graph with all possible n proper subgraphs $H_1, H_2, H_3, \ldots, H_n$ then $G = H_1^c \cup H_2^c \cup H_3^c \cup \ldots \cup H_n^c$.

2.6. Theorem: Let G be graph withal possible n proper subgraphs H_1, H_2, H_3,…., H_n then $H_1^c \cap H_2^c \cap H_3^c \cap \ldots\ldots \cap H_n^c$ is null graph.

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