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A STUDY ON (3, (c_1, c_2)) - REGULAR BIPOLAR FUZZY GRAPH

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ABSTRACT

In this paper, we defined d_3-degree of a vertex in bipolar fuzzy graphs, total d_3 -degree of a vertex in bipolar fuzzy graphs, (3, (c_1, c_2))-regular bipolar fuzzy graphs and totally (3, (c_1, c_2)) - regular bipolar fuzzy graphs. (3, (c_1, c_2))-regular bipolar fuzzy graphs and totally (3, (c_1, c_2))-regular bipolar fuzzy graphs are compared through various examples. A necessary and sufficient condition under which they are equivalent is provided. Also, (3, (c_1, c_2))-regularity on some bipolar fuzzy graphs whose underlying crisp graphs are a path on six vertices P_6, a Corona graph $C_n \circ K_1$ (n≥5), a Wagner graph and a cycle C_n (n≥5) is studied with some membership function. Some properties of (3, (c_1, c_2))-regular bipolar fuzzy graphs studied and they are examined for totally (3, (c_1, c_2))-regular bipolar fuzzy graphs.

Keywords: *d_3- degree of a vertex in bipolar fuzzy graphs, total d_3- degree of a vertex in bipolar fuzzy graphs, (3, (c_1, c_2))-regular bipolar fuzzy graphs, totally (3, (c_1, c_2))-regular bipolar fuzzy graphs.*

1. INTRODUCTION

In 1965, Lofti A. Zadeh introduced the concept of fuzzy subset of a set as method of representing the phenomena of uncertainty in real life situation. In 1975, Azriel Rosenfeld introduced the concept of fuzzy graphs. It has been growing fast and has numerous applications in various fields. Nagoor Gani and Radha introduced regular fuzzy graphs, total degree and totally regular fuzzy graphs, In 1994 W.R.Zhang initiated the concept of bipolar fuzzy sets as generalisation of fuzzy sets. S. Meena Devi and M. Andal introduced d_3 -degree of a vertex in fuzzy graphs and discussed some properties of d_3-degree of a vertex in fuzzy graphs. Bipolar fuzzy sets are extension of fuzzy sets with membership values in [−1, 1]. Throughout this paper, the vertices take the membership values (m_1^+, m_1^-) and edges take the membership values (m_2^+, m_2^-) where $m_1^+, m_2^+ \in [0, 1]$ and $m_1^-, m_2^- \in [-1, 0]$.

2. PRELIMINARIES

Definition 2.1: For a given graph G, the d_3-degree of a vertex v in G, denoted by $d_3(v)$ means number of vertices at a distance three away from v.

Definition 2.2: A graph G is said to be (3, k)-regular (d_3 –regular) if $d_3(v)$ = k, for all v in G. We observe that (3, k)-regular and d_3-regular graphs are same.

Definition 2.3: A fuzzy graph G is a pair of functions G: (σ, μ) where σ: V→ [0, 1] is a fuzzy subset of a non empty set V and μ:V x V → [0,1] is symmetric fuzzy relation on σ such that for all u, v in V the relation μ(uv) ≤ σ(u) Λ σ(v) is satisfied.

A fuzzy graph G is complete if μ(uv) = σ(u) Λ σ(u) for all u, v ϵ V where uv denotes the edge between u and v.

Definition 2.4: Let G: (σ, μ) be fuzzy graph. The degree of a vertex u is $d_G(u) = \sum_{u \neq v} \mu(uv)$ for uv ∈ E and μ(uv) = 0 for uv not in E, this equivalent to $d_G(u) = \sum_{uv \in E} \mu(uv)$

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The minimum degree of G is $\delta(G) = \Lambda \{d(v): v \in V\}$.

The maximum degree of G is $\Delta(G) = \vee \{d(v): v \in V\}$.

The order of a fuzzy graph is $O(G) = \sum \sigma(u)$.

Definition 2.5: The strength of connectedness between two vertices u and v is $\mu^{\infty}(uv)=\sup\{\mu^k(uv) / k = 1,2,\ldots\}$ where $(uv) = \sup\{\mu(uu_1)\Lambda\mu(u_1u_2)\Lambda\ldots\ldots\Lambda\mu(u_{k-1}v) | u_1, u_2, \ldots\ldots u_{k-1} \in V\}$.

Definition 2.6: The Wagner graph is a 3-regular with 8 vertices and 12 edges.

Definition 2.7: The Corona of two graphs G_1 and G_2 is the graph $G = G_1 \circ G_2$ formed from one copy of G_1 and $|v(G_1)|$ copies of G_2 where i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.
[Note that G_1 is a cycle of length ≥ 5 and G_2 is K_1].

Example 2.8:

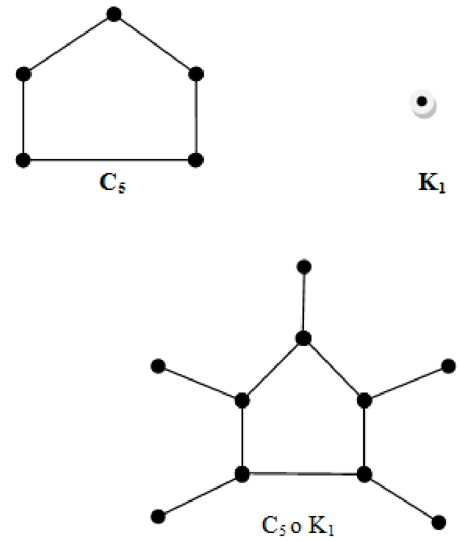

Figure-1

Definition 2.9: A bipolar fuzzy graph with an underlying set V is defined to be the pair G = (A, B) where $A = (m_1^+, m_1^-)$ is a bipolar fuzzy set on V and $B = (m_2^+, m_2^-)$ is a bipolar fuzzy set on E such that $m_2^+(x, y) \leq \min\{(m_1^+(x), m_1^+(y)\}$ and $m_2^-(x, y) \geq \max\{m_1^-(x), m_1^-(y)\}$ for all $(x, y) \in E$. Here A is called a bipolar fuzzy vertex set of V and B is called a bipolar fuzzy edge set of E.

Definition 2.10: The positive degree of a vertex $u \in G$ is $d^+(u) = m_2^+(u, v)$. The negative degree of a vertex $u \in G$ is $d^-(u) = m_2^-(u, v)$. The degree of the vertex u is defined as $d(u)= (d^+(u), d^-(u))$.

Definition 2.11: Let G = (A, B) be a bipolar fuzzy graph where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ are two bipolar fuzzy sets on a non-empty finite set V . Then G is said to be regular bipolar fuzzy graph if all the vertices have same positive and negative membership values.

Definition 2.12: Let G = (A, B) be a bipolar fuzzy graph where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ be two bipolar fuzzy sets on a non-empty finite set V . G is said to be a totally regular bipolar fuzzy graph if all the vertices of G has same total degree (k_1, k_2). It is denoted by (k_1, k_2) - totally regular bipolar fuzzy graph.

3. d_3-DEGREE AND TOTAL D_3-DEGREE OF A VERTEX IN BIPOLAR FUZZY GRAPHS

Definition 3.1: Let G = (A, B) be a bipolar fuzzy graph. The positive d_3 - degree of a vertex $u \in G$ is defined as $d_3^+(u) = m_2^{(3,+)}(u, v)$ where $m_2^{(3,+)}(u, v) = \sup\{m_2^+(u, u_1) \wedge m_2^+(u_1, u_2) \wedge m_2^+(u_2, v) : u, u_1, u_2, v\}$ is the shortest path connecting u and v of length 3. The negative d_3 - degree of a vertex $u \in G$ is defined as $d_3^-(u) = m_2^{(3,-)}(u, v)$ where $m_2^{(3,-)}(u, v) = \inf\{m_2^-(u, u_1) \vee m_2^-(u_1, u_2) \vee m_2^-(u_2, v) : u, u_1, u_2, v\}$ is the shortest path connecting u and v of length 3. Also $\mu(uv)=0$, for uv not in E.

The d_3 - degree of a vertex $u \in G$ is defined as $d_3(u) = (d_3^+(u), d_3^-(u))$

The minimum d_3 degree of G is $\delta_3(G) = \Lambda\, \{d_3(v): v\in V\}$.

The maximum d_3 degree of G is $\Delta_3(G) = V\, \{d_3(v): v\in V\}$.

Example 3.2: Let G = (A, B) be a bipolar fuzzy graph, where A = (m_1^+, m_1^-) and B = (m_2^+, m_2^-) are two bipolar fuzzy sets on a non-empty finite set V and E ⊆ V×V. Where V = $\{v_1, v_2, v_3, v_4, v_5\}$.

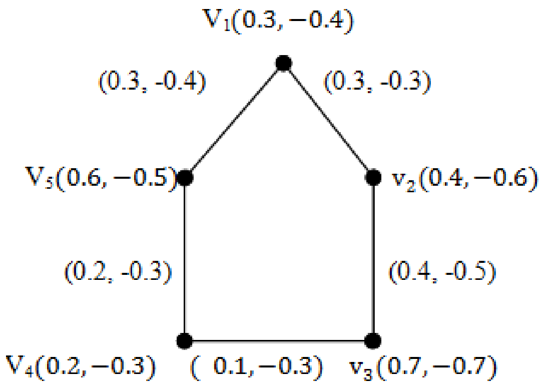

Figure-2

Here, $d_3(v_1) = (0.3 \wedge 0.2 \wedge 0.1, -0.4\vee-0.3\vee -0.3) + (0.3 \wedge 0.4 \wedge 0.1, -0.3\vee-0.5\vee-0.3) = (0.1, -0.3) + (0.1, -0.3)$
$= (0.2, -0.6)$.

$d_3(v_2) = (0.3 \wedge 0.3 \wedge 0.2, -0.3\vee-0.4\vee-0.3) + (0.4 \wedge 0.1 \wedge 0.2, -0.5\vee-0.3\vee-0.3) = (0.2, -0.3) + (0.1, -0.3)$
$= (0.3, -0.6)$

$d_3(v_3) = (0.4 \wedge 0.3 \wedge 0.3, -0.5\vee-0.3\vee-0.4) + (0.1 \wedge 0.2 \wedge 0.3, -0.3\vee-0.3\vee-0.4) = (0.3, -0.3) + (0.1, -0.3)$
$= (0.4, -0.6)$.

$d_3(v_4) = (0.1 \wedge 0.4 \wedge 0.3, -0.3\vee-0.5\vee-0.3 + (0.2 \wedge 0.3 \wedge 0.3, -0.3\vee-0.4\vee-0.3) = (0.1, -0.3) + (0.2, -0.3)$
$= (0.3, -0.6)$.

$d_3(v_5) = (0.2 \wedge 0.1 \wedge 0.4, -0.3\vee-0.3\vee-0.5) + (0.3 \wedge 0.3 \wedge 0.4 , -0.4\vee-0.3\vee-0.5) = (0.1, -0.3) + (0.3, -0.3)$
$= (0.4, -0.6)$.

Example 3.3: Let G = (A, B) be a bipolar fuzzy graph, where A = (m_1^+, m_1^-) and B = (m_2^+, m_2^-) are two bipolar fuzzy sets on a non-empty finite set V and E ⊆ V×V. Where V = $\{v_1, v_2, v_3, v_4, v_5, v_6\}$.

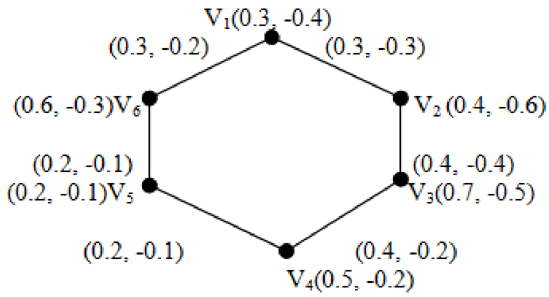

Figure-3

Here, $d_3(v_1) = (0.3 \wedge 0.2 \wedge 0.2, -0.2\vee-0.1\vee-0.1) + (0.3 \wedge 0.4 \wedge 0.4, -0.3\vee-0.4\vee-0.2) = (0.2, -0.1) + (0.3, -0.2)$
$= (0.5, -0.3)$.

$d_3(v_2) = (0.3 \wedge 0.3 \wedge 0.2, -0.3\vee-0.2\vee-0.1\} + (0.4 \wedge 0.4 \wedge 0.2, -0.4\vee-0.2\vee-0.1) = (0.2, -0.1) + (0.2, -0.1)$
$= (0.4, -0.2)$.

$d_3(v_3) = (0.4 \wedge 0.3 \wedge 0.3, -0.4\vee-0.3\vee-0.2) + (0.4 \wedge 0.2 \wedge 0.2, -0.2\vee-0.1\vee-0.1) = (0.3, -0.2) + (0.2, -0.1)$
$= (0.5, -0.3)$.

$d_3(v_4) = (0.4 \wedge 0.4 \wedge 0.3, -0.2\vee-0.4\vee-0.3) + (0.2 \wedge 0.2\wedge 0.3, -0.1\vee-0.1\vee-0.2) = (0.3, -0.2) + (0.2, -0.1)$
$= (0.5, -0.3)$.

$d_3(v_5) = (0.2 \wedge 0.4 \wedge 0.4, -0.1 \vee -0.2 \vee -0.4) + (0.2 \wedge 0.3 \wedge 0.3, -0.1 \vee -0.2 \vee -0.3) = (0.2, -0.1) + (0.2, -0.1)$
$= (0.4, -0.2)$.

$d_3(v_6) = (0.2 \wedge 0.2 \wedge 0.4, -0.1 \vee -0.1 \vee -0.2) + (0.3 \wedge 0.3 \wedge 0.4, -0.2 \vee -0.3 \vee -0.4) = (0.2, -0.1) + (0.3, -0.2)$
$= (0.5, -0.3)$.

Definition 3.4: The total degree of a vertex u ∈ V is denoted by td(u) and defined as td(u) = ($td^+(u)$, $td^-(u)$) where $td^+(u) = m_2^+(u, v) + m_1^+(u)$ and $td^-(u) = m_2^-(u, v) + m_1^-(u)$.

The minimum td_3-degree of G is $t\delta_3(G) = \wedge\{td_3(v) : v \in V\}$.

The maximum td_3-degree of G is $t\Delta_3(G) = \vee\{td_3(v) : v \in V\}$.

Example 3.5: Let G = (A, B) be a bipolar fuzzy graph, where A = (m_1^+, m_1^-) and B = (m_2^+, m_2^-) are two bipolar fuzzy sets on a non-empty finite set V and E ⊆ V×V. Where V = {v_1, v_2, v_3, v_4, v_5, v_6}.

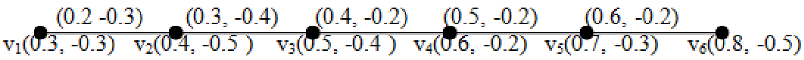

Figure-4

Here, $d_3(v_1) = (0.2 \wedge 0.3 \wedge 0.4, -0.3 \vee -0.4 \vee -0.2) = (0.2, -0.2)$.
$d_3(v_2) = (0.3 \wedge 0.4 \wedge 0.5, -0.4 \vee -0.2 \vee -0.2) = (0.3, -0.2)$.
$d_3(v_3) = (0.4 \wedge 0.5 \wedge 0.6, -0.2 \vee -0.2 \vee -0.2) = (0.4, -0.2)$.
$d_3(v_4) = (0.4 \wedge 0.3 \wedge 0.2, -0.2 \vee -0.4 \vee -0.3) = (0.2, -0.2)$.
$d_3(v_5) = (0.5 \wedge 0.4 \wedge 0.3, -0.2 \vee -0.2 \vee -0.4) = (0.3, -0.2)$.
$d_3(v_6) = (0.6 \wedge 0.5 \wedge 0.4, -0.2 \vee -0.2 \vee -0.2) = (0.4, -0.2)$.
$td_3(v_1) = (0.2, -0.2) + (0.3, -0.3) = (0.5, -0.5)$.
$td_3(v_2) = (0.3, -0.2) + (0.4, -0.5) = (0.7, -0.7)$.
$td_3(v_3) = (0.4, -0.2) + (0.5, -0.4) = (0.9, -0.6)$.
$td_3(v_4) = (0.2, -0.2) + (0.6, -0.2) = (0.8, -0.4)$.
$td_3(v_5) = (0.3, -0.2) + (0.7, -0.3) = (1.0, -0.5)$.
$td_3(v_6) = (0.4, -0.2) + (0.8, -0.5) = (1.2, -0.7)$.

4. (3, (c_1, c_2))-REGULAR AND TOTALLY (3, (c_1, c_2))-REGULAR BIPOLAR FUZZY GRAPHS

Definition 4.1: Let G = (A, B) be a bipolar fuzzy graph. If $d_3(u) = (c_1, c_2)$ for all u ∈ V, then G is said to be a (3, (c_1,c_2)) - regular bipolar fuzzy graph.

Example 4.2: Let G = (A, B) be a bipolar fuzzy graph where A = (m_1^+, m_1^-) and B = (m_2^+, m_2^-) be two bipolar fuzzy sets on a non-empty finite set V and E ⊆ V×V, where V = {v_1, v_2 , v_3 , v_4, v_5, v_6.

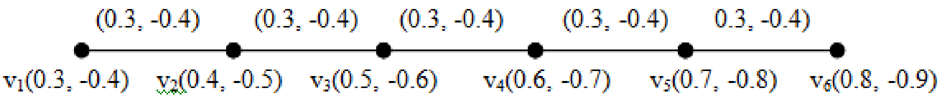

Figure-5

Here $d_3(v_1) = (0.3, -0.4)$, $d_3(v_2) = (0.3, -0.4)$, $d_3(v_3) = (0.3, -0.4)$, $d_3(v_4) = (0.3, -0.4)$, $d_3(v_5) = (0.3, -0.4)$, $d_3(v_6) = (0.3, -0.4)$. So, G is (3, (0.3, -0.4))- regular bipolar fuzzy graph.

Example 4.3: Let G = (A, B) be a bipolar fuzzy graph where A = (m_1^+, m_1^-) and B = (m_2^+, m_2^-) be two bipolar fuzzy sets on a non-empty finite set V and E ⊆ V×V, where V = {v_1, v_2 , v_3 , v_4, v_5}.

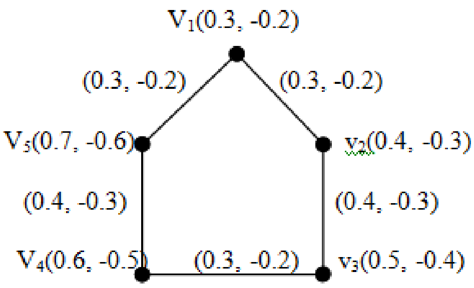

Figure-6

Here, $d_3(v_1)$ = (0.6, -0.4), $d_3(v_2)$ = (0.6, -0.4), $d_3(v_3)$ = (0.6, -0.4), $d_3(v_4)$ = (0.6, -0.4), $d_3(v_5)$ = (0.6, -0.4). So G is (3, (0.6, -0.4))- regular bipolar fuzzy graph.

Example 4.4: Let G = (A, B) be a bipolar fuzzy graph where A = (m_1^+, m_1^-) and B = (m_2^+, m_2^-) be two bipolar fuzzy sets on a non-empty finite set V and E ⊆ V×V, where V = {v_1, v_2 , v_3, v_4, v_5, v_6, v_7, v_8}.

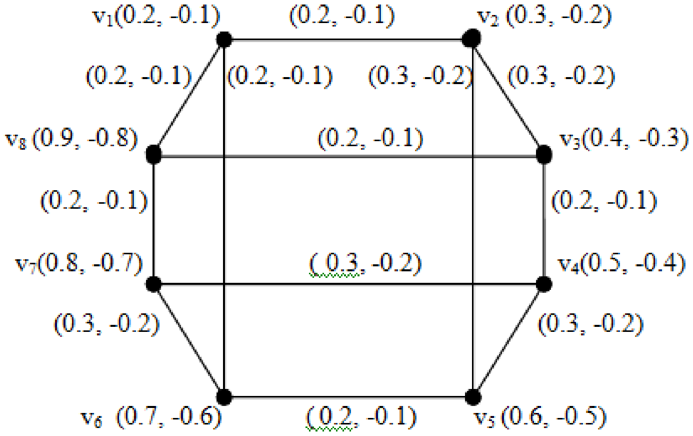

Figure-7

$$\begin{aligned} \text{Here, } d_3(v_1) &= (0.2\wedge 0.2\wedge 0.3, -0.1\text{-}0.1\vee\text{-}0.2) + (0.2\wedge 0.2\wedge 0.2, -0.1\vee\text{-}0.1\vee\text{-}0.1) + (0.2\wedge 0.3\wedge 0.2, -0.1\vee\text{-}0.2\vee\text{-}0.1) \\ &\quad + (0.2 \wedge 0.3 \wedge 0.3, -0.1\vee\text{-}0.2\vee\text{-}0.2) + (0.2 \wedge 0.2 \wedge 0.3, -0.1\vee\text{-}0.1\vee\text{-}0.2) \\ &= (0.2, -0.1) + (0.2, -0.1) + (0.2, -0.1) + (0.2, -0.1) + (0.2, -0.1) = (1.0, -0.5) \end{aligned}$$

$$\begin{aligned} d_3(v_2) &= (0.2 \wedge 0.2 \wedge 0.2, -0.1\vee\text{-}0.1\vee\text{-}0.1) + (0.2 \wedge 0.2 \wedge 0.3, -0.1\vee\text{-}0.1\vee\text{-}0.2) + (0.3\wedge 0.2\wedge 0.3, -0.2\vee\text{-}0.1\vee\text{-}0.2\} \\ &\quad + (0.3 \wedge 0.2 \wedge 0.2, -0.2\vee\text{-}0.1\vee\text{-}0.1) + (0.3 \wedge 0.2 \wedge 0.3, -0.2\vee\text{-}0.1\vee\text{-}0.2) \\ &= (1.0, -0.5) \end{aligned}$$

$$\begin{aligned} d_3(v_3) &= (0.3 \wedge 0.2 \wedge 0.2, -0.2\vee\text{-}0.1\vee\text{-}0.1) + (0.3 \wedge 0.3 \wedge 0.2, -0.2\vee\text{-}0.2\vee\text{-}0.1) + (0.2 \wedge 0.3 \wedge 0.2, -0.1\vee\text{-}0.2\vee\text{-}0.1) \\ &\quad + (0.2 \wedge 0.3 \wedge 0.3, -0.1\vee\text{-}0.2\vee\text{-}0.2) + (0.2 \wedge 0.2 \wedge 0.3, -0.1\vee\text{-}0.1\vee\text{-}0.2) \\ &= (1.0, -0.5) \end{aligned}$$

$$\begin{aligned} d_3(v_4) &= (0.2 \wedge 0.3 \wedge 0.2, -0.1\vee\text{-}0.2\vee\text{-}0.1) + (0.2 \wedge 0.2 \wedge 0.2, -0.1\vee\text{-}0.1\vee\text{-}0.1) + (0.3 \wedge 0.2 \wedge 0.2, -0.2\vee\text{-}0.1\vee\text{-}0.1) \\ &\quad + (0.3 \wedge 0.3 \wedge 0.2, -0.2\vee\text{-}0.2\vee\text{-}0.1) + (0.3 \wedge 0.2 \wedge 0.2, -0.2\vee\text{-}0.1\vee\text{-}0.1) \\ &= (1.0, -0.5) \end{aligned}$$

$$\begin{aligned} d_3(v_5) &= (0.3 \wedge 0.2 \wedge 0.2, -0.2\vee\text{-}0.1\vee\text{-}0.1) + (0.3 \wedge 0.3 \wedge 0.2, -0.2\vee\text{-}0.2\vee\text{-}0.1) + (0.2 \wedge 0.3\wedge 0.2, -0.1\vee\text{-}0.2\vee\text{-}0.1) \\ &\quad + (0.2 \wedge 0.2 \wedge 0.2, -0.1\vee\text{-}0.1\vee\text{-}0.1) + (0.3 \wedge 0.2 \wedge 0.2, -0.2\vee\text{-}0.1\vee\text{-}0.1) \\ &= (1.0, -0.5) \end{aligned}$$

$$\begin{aligned} d_3(v_6) &= (0.2 \wedge 0.3 \wedge 0.2, -0.1\vee\text{-}0.2\vee\text{-}0.1) + (0.2 \wedge 0.3 \wedge 0.3, -0.1\vee\text{-}0.2\vee\text{-}0.2) + (0.3 \wedge 0.2 \wedge 0.2, -0.2\vee\text{-}0.1\vee\text{-}0.1) \\ &\quad + (0.3 \wedge 0.3 \wedge 0.2, -0.2\vee\text{-}0.2\vee\text{-}0.1) + (0.2 \wedge 0.2 \wedge 0.3, -0.1\vee\text{-}0.1\vee\text{-}0.2) \\ &= (1.0, -0.5) \end{aligned}$$

$$\begin{aligned} d_3(v_7) &= (0.3 \wedge 0.2 \wedge 0.3, -0.2\vee\text{-}0.1\vee\text{-}0.2) + (0.3 \wedge 0.2 \wedge 0.2, -0.2\vee\text{-}0.1\vee\text{-}0.1) + (0.2 \wedge 0.2 \wedge 0.2, -0.1\vee\text{-}0.1\vee\text{-}0.1) \\ &\quad + (0.2 \wedge 0.2 \wedge 0.3, -0.1\vee\text{-}0.1\vee\text{-}0.2) + (0.3 \wedge 0.2 \wedge 0.3, -0.2\vee\text{-}0.1\vee\text{-}0.2) \\ &= 0.2 + 0.2 + 0.2 + 0.2 + 0.2 = 1.0 \end{aligned}$$

$$\begin{aligned} d_3(v_8) &= (0.2 \wedge 0.3 \wedge 0.2, -0.1\vee\text{-}0.2\vee\text{-}0.1) + (0.2 \wedge 0.3 \wedge 0.3, -0.1\vee\text{-}0.2\vee\text{-}0.2) + (0.2 \wedge 0.2 \wedge 0.3, -0.2\vee\text{-}0.2\vee\text{-}0.2) \\ &\quad + (0.2 \wedge 0.2 \wedge 0.2, -0.1\vee\text{-}0.1\vee\text{-}0.1) + (0.2 \wedge 0.2 \wedge 0.3, -0.1\vee\text{-}0.1\vee\text{-}0.2) \\ &= (1.0, -0.5) \end{aligned}$$

So G is (3, (1.0, -0.5))- regular bipolar fuzzy graph.

Example 4.5: Let G = (A, B) be a bipolar fuzzy graph where A = (m_1^+, m_1^-) and B = (m_2^+, m_2^-) be two bipolar fuzzy sets on a non-empty finite set V and E ⊆ V×V, where V = {v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}}.

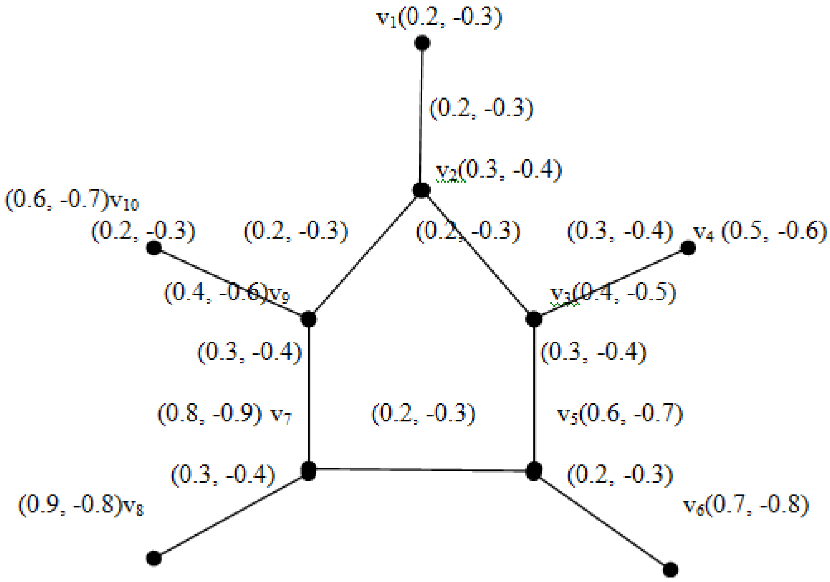

Figure 8

Here, $d_3(v_1)$ = (0.2∧0.2∧0.3, -0.3∨-0.3∨-0.4) + (0.2∧0.2∧0.3, -0.3∨-0.3∨-0.4) + (0.2∧0.2∧0.3, -0.3∨-0.3∨-0.4) + (0.2∧0.2∧0.2, -0.3∨-0.3∨-0.3) = (0.8, -1.2) .

$d_3(v_2)$ = (0.2∧0.3∧0.2, -0.3∨-0.4∨-0.3) + (0.2∧0.3∧0.2, -0.3∨-0.4∨-0.3) + (0.2∧0.3∧ 0.2, -0.3∨-0.4∨-0.3) + (0.2 ∧0.3∧0.3, -0.3∨-0.4∨-0.4) = (0.8, -1.2).

$d_3(v_3)$ = (0.3∧0.2∧0.3, -0.4∨-0.3∨-0.4) + (0.3∧0.2∧0.3, -0.4∨-0.3∨-0.4) + (0.2∧0.2∧0.2, -0.3∨-0.3∨-0.3) + (0.2∧0.2 ∧0.3, -0.3∨-0.3∨-0.4) = (0.8, -1.2).

$d_3(v_4)$ = (0.3 ∧0.2∧0.2, -0.4∨-0.3∨-0.3) + (0.3∧0.2∧0.2, -0.4∨-0.3∨-0.3) + (0.3∧0.3 ∧ 0.2, -0.4∨-0.4∨-0.3) + (0.3 ∧ 0.3 ∧ 0.2, -0.4∨-0.4∨-0.3) = (0.8, -1.2).

$d_3(v_5)$ = (0.3 ∧0.2∧0.2, -0.4∨-0.3∨-0.3) + (0.3∧0.2∧0.2, -0.4∨-0.3∨-0.3) + (0.2∧0.3∧ 0.2, -0.3∨-0.4∨-0.3) + (0.2 ∧ 0.3 ∧ 0.2, -0.3∨-0.4∨-0.3) = (0.8, -1.2).

$d_3(v_6)$ = (0.2∧0.3∧0.2, -0.3∨-0.4∨-0.3) + (0.2∧0.3∧0.3, -0.3∨-0.4∨-0.4) + (0.2∧0.2 ∧ 0.3, -0.3∨-0.3∨-0.4) + (0.2 ∧ 0.2 ∧ 0.3, -0.3∨-0.3∨-0.4) = (0.8, -1.2).

$d_3(v_7)$ = (0.3∧0.2∧0.2, -0.4∨-0.3∨-0.3) + (0.3∧0.2∧0.2, -0.4∨-0.3∨-0.3) + (0.2∧0.3∧ 0.2, -0.3∨-0.4∨-0.3) + (0.2 ∧ 0.3 ∧ 0.3, -0.3∨-0.4∨-0.4) = (0.8, -1.2).

$d_3(v_8)$ = (0.3∧0.3∧0.2, -0.4∨-0.4∨-0.3) + (0.3∧0.2∧0.3, -0.4∨-0.3∨-0.4) + (0.3∧0.2∧ 0.2, -0.4∨-0.3∨-0.3) + (0.3∧0.3∧0.2, -0.4∨-0.4∨-0.3) = (0.8, -1.2).

$d_3(v_9)$ = (0.2∧0.2∧0.3, -0.3∨-0.3∨-0.4) + (0.3∧0.2∧0.3, -0.4∨-0.3∨-0.4) + (0.2∧ 0.2∧0.3, -0.3∨-0.3∨-0.4) + (0.3 ∧ 0.2 ∧ 0.2, -0.4∨-0.3∨-0.3) = (0.8, -1.2).

$d_3(v_{10})$ = (0.2∧0.2∧0.2, -0.3∨-0.3∨-0.3) + (0.2∧0.3∧0.2, -0.3∨-0.4∨-0.3) + (0.2∧0.3∧ 0.3, -0.3∨-0.4∨-0.4) + (0.2 ∧ 0.2 ∧ 0.2, -0.3∨-0.3∨-0.3) = (0.8, -1.2).

So G is (3, (0.8, -1.2))- regular bipolar fuzzy graph.

Definition 4.6: Let G = (A, B) be a bipolar fuzzy graph. If each vertex of G has same total d_3 - degree, then G is said to be totally (3, (c_1, c_2)) - regular bipolar fuzzy graph.

Example 4.7: Let G = (**A**, B) be a bipolar fuzzy graph where A = (m_1^+, m_1^-) and B = (m_2^+, m_2^-) be two bipolar fuzzy sets on a non-empty finite set V and E ⊆ V×V, where V = {v_1, v_2 , v_3 , v_4, v_5}.

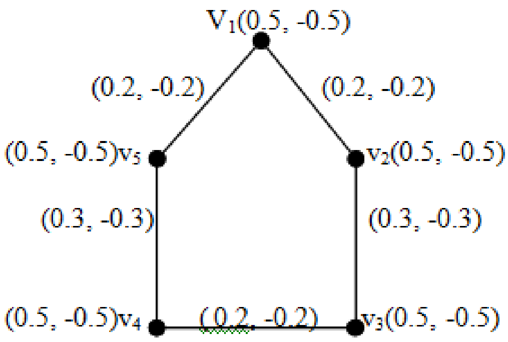

Figure-9

Here, $d_3(v_1)$ = (0.4, -0.4), $d_3(v_2)$ = (0.4, -0.4), $d_3(v_3)$ = (0.4, -0.4), $d_3(v_4)$ = (0.4, -0.4), $d_3(v_5)$ = (0.4, -0.4) and $td_3(v_1)$ = (0.9, -0.9), $td_3(v_2)$ = (0.9,-0.9), $td_3(v_3)$ = (0.9, -0.9), $td_3(v_4)$ = (0.9, -0.9), $td_3(v_5)$ = (0.9, -0.9). Each vertex has same total d_3-degree (0.9, -0.9). Hence G is totally (3, (0.9, -0.9))-regular bipolar fuzzy graph.

Example 4.8: A totally (3, (c_1, c_2))-regular bipolar fuzzy graph need not be a (3, (c_1, c_2))-regular bipolar fuzzy graph.

Let G = (A, B) be a bipolar fuzzy graph where A = (m_1^+, m_1^-) and B = (m_2^+, m_2^-) be two bipolar fuzzy sets on a non-empty finite set V and E ⊆ V×V, where V = {v_1, v_2, v_3, v_4, v_5}.

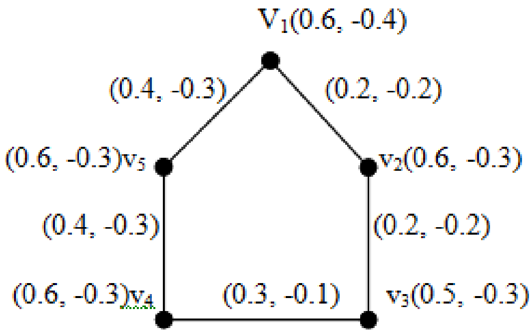

Figure-10

Here, $d_3(v_1)$ = (0.5, -0.2), $d_3(v_2)$ = (0.4, -0.3), $d_3(v_3)$ = (0.5, -0.3), $d_3(v_4)$ = (0.4, -0.3), $d_3(v_5)$ = (0.4, -0.3) and $td_3(v_1)$ = (1.0, -0.6), $td_3(v_2)$ = (1.0, -0.6), $td_3(v_3)$ = (1.0, -0.6), $td_3(v_4)$ = (1.0, -0.6), $td_3(v_5)$ = (1.0, -0.6). Each vertex has same total d_3-degree (1.0, -0.6). Hence G is totally (3, (1.0, -0.6))-regular bipolar fuzzy graph. But G is not (3, (c_1, c_2))-regular bipolar fuzzy graph.

Example 4.9: A (3, (c_1, c_2))-regular bipolar fuzzy graph need not be a totally (3, (c_1, c_2))-regular bipolar fuzzy graph.

Let G = (A, B) be a bipolar fuzzy graph where A = (m_1^+, m_1^-) and B = (m_2^+, m_2^-) be two bipolar fuzzy sets on a non-empty finite set V and E ⊆ V×V, where V = {v_1, v_2, v_3, v_4, v_5, v_6}.

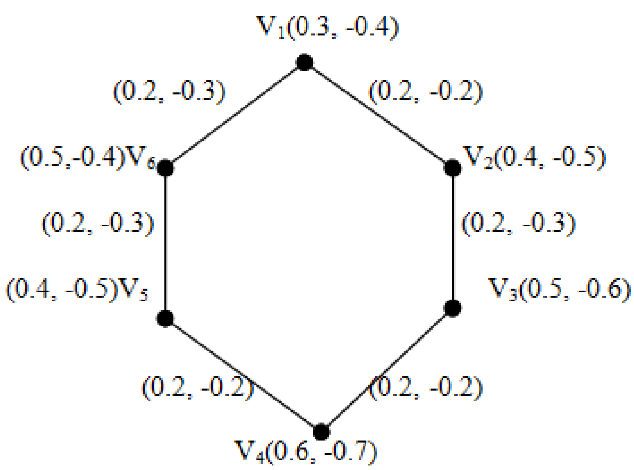

Figure-11

Here, $d_3(v_1) = (0.4, -0.4)$, $d_3(v_2) = (0.4, -0.4)$, $d_3(v_3) = (0.4, -0.4)$, $d_3(v_4) = (0.4, -0.4)$, $d_3(v_5) = (0.4, -0.4)$, $d_3(v_6) = (0.4, -0.4)$ and $td_3(v_1) = (0.7, -0.8)$, $td_3(v_2) = (0.8, -0.9)$, $td_3(v_3) = (0.9, -1.0)$, $td_3(v_4) = (1.0, -1.1)$, $td_3(v_5) = (0.8, -0.9)$, $td_3(v_6) = (0.9, -0.8)$. Each vertex has same d_3-degree (0.4, -0.4). So, G is (3, (0.4, -0.4))-regular bipolar fuzzy graph. But G is not a totally (3, (c_1, c_2))-regular bipolar fuzzy graph.

Example 4.10: A (3, (c_1, c_2))-regular fuzzy graph which is totally (3, (c_1, c_2))-regular fuzzy graph.

Let G = (A, B) be a bipolar fuzzy graph where A = (m_1^+, m_1^-) and B = (m_2^+, m_2^-) be two bipolar fuzzy sets on a non-empty finite set V and E ⊆ V×V, where V = {v_1, v_2, v_3, v_4, v_5, v_6}.

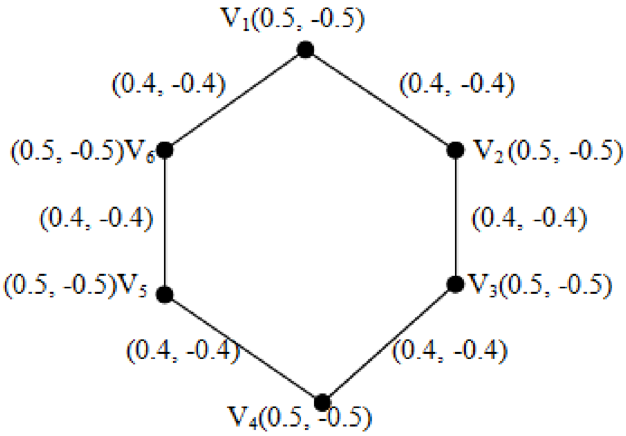

Figure-12

Here, $d_3(v_1) = (0.8, -0.8)$, $d_3(v_2) = (0.8, -0.8)$, $d_3(v_3) = (0.8, -0.8)$, $d_3(v_4) = (0.8, -0.8)$, $d_3(v_5) = (0.8, -0.8)$, $d_3(v_6) = (0.8, -0.8)$ and $td_3(v_1) = (1.3, -1.3)$, $td_3(v_2) = (1.3, -1.3)$, $td_3(v_3) = 1.3$, $td_3(v_4) = (1.3, -1.3)$, $td_3(v_5) = (1.3, -1.3)$ $td_3(v_6) = (1.3, -1.3)$. Each vertex has the same d_3-degree (0.8, -0.8). So, G is a (3, (0.8, -0.8))-regular bipolar fuzzy graph. Each vertex has the same total d_3-degree (1.3, -1.3). Hence G is a totally (3, (1.3, -1.3))-regular bipolar fuzzy graph.

Theorem 4.11: Let G = (A, B) be a bipolar fuzzy graph on $G^*(V, E)$. Then $A(u) = (k_1, k_2)$ for all u ∈ V if and only if the following conditions are equivalent.

i). G = (A, B) is a (3, (c_1, c_2)) - regular bipolar fuzzy graph.

ii). G = (A, B) is a totally (3, ($c_1 + k_1$, $c_2 + k_2$)) - regular bipolar fuzzy graph.

Proof: Suppose $A(u) = (k_1, k_2)$ for all u ∈ V .

Assume that G is a (3, (c_1, c_2)) - regular bipolar fuzzy graph, then $d_3(u) = (c_1, c_2)$ for all u ∈ V .

So $td_3(u) = d_3(u) + A(u) = (c_1, c_2) + (k_1, k_2) = (c_1 + k_1, c_2 + k_2)$.

Hence G is a totally (3, ($c_1 + k_1$, $c_2 + k_2$)) - regular bipolar fuzzy graph.

Thus (i) ⇒ (ii) is proved.

Suppose G is a totally (3, ($c_1 + k_1$, $c_2 + k_2$)) - regular bipolar fuzzy graph.
⇒ $td_3(u) = (c_1 + k_1, c_2 + k_2)$ for all u ∈ V
⇒ $d_3(u) + A(u) = (c_1 + k_1, c_2 + k_2)$ for all u ∈ V
⇒ $d_3(u) + (k_1, k_2) = (c_1, c_2) + (k_1, k_2)$ for all u ∈ V
⇒ $d_3(u) = (c_1, c_2)$ for all u ∈ V

Hence G is a (3, (c_1,c_2)) - regular bipolar fuzzy graph.
Thus (i) and (ii) are equivalent.

Conversely assume that (i) and (ii) are equivalent.

Let G be a (3, (c_1, c_2)) - regular bipolar fuzzy graph and totally (3, ($c_1 + k_1$, $c_2 + k_2$)) - regular bipolar fuzzy graph.
⇒ $td_3(u) = (c_1 + k_1, c_2 + k_2)$ and $d_3(u) = (c_1, c_2)$ for all u ∈ V
⇒ $d_3(u) + A(u) = (c_1 + k_1, c_2 + k_2)$ and $d_3(u) = (c_1, c_2)$ for all u ∈ V
⇒ $d_3(u) + A(u) = (c_1, c_2) + (k_1, k_2)$ and $d_3(u) = (c_1, c_2)$ for all u ∈ V
⇒ $A(u) = (k_1, k_2)$ for all u ∈ V .

Hence $A(u) = (k_1, k_2)$ for all $u \in V$.

Theorem 4.12: If a bipolar fuzzy graph G is both (3, (c_1, c_2))-regular and totally (3, (c_1, c_2))-regular then A(u) is constant function.

Proof: Let G be (3, (c_1, c_2)) -regular and totally (3, (c_1, c_2)) –regular bipolar fuzzy graph.
Then $d_3(u) = (k_1, k_2)$ and $td_3(u) = (k_3, k_4)$ for all $u \in V$.

Now $td_3(u) = (k_3, k_4)$ for all $u \in V$.
$\Rightarrow d_3(u) + A(u) = (k_3, k_4)$ for all $u \in V$.
$\Rightarrow (k_1, k_2) + A(u) = (k_3, k_4)$ for all $u \in V$.
$\Rightarrow A(u) = (k_3, k_4) - (k_1, k_2)$ for all $u \in V$.

Hence A(u) is a constant function.

Remark 4.13: The converse of theorem 4.12 is not true.

Let G = (A, B) be a bipolar fuzzy graph where A = (m_1^+, m_1^-) and B = (m_2^+, m_2^-) be two bipolar fuzzy sets on a non-empty finite set V and $E \subseteq V \times V$, where $V = \{v_1, v_2, v_3, v_4, v_5\}$.

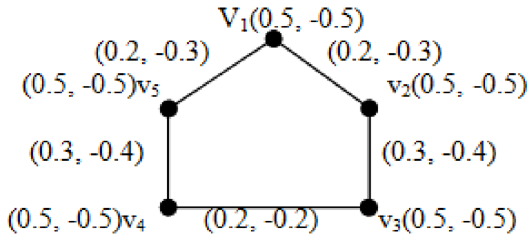

Figure-13

Here, $d_3(v_1) = (0.4, -0.4)$, $d_3(v_2) = (0.4, -0.5)$, $d_3(v_3) = (0.4, -0.5)$, $d_3(v_4) = (0.4, -0.5)$, $d_3(v_5) = (0.4, -0.5)$ and $td_3(v_1) = (0.9, -0.9)$, $td_3(v_2) = (0.9,-1.0)$, $td_3(v_3) = (0.9, -1.0)$, $td_3(v_4) = (0.9, -1.0)$, $td_3(v_5) = (0.9, -1.0)$. Here, A(u) is a constant function. But G is neither (3, k)-regular bipolar fuzzy graph nor totally (3, k)-regular bipolar fuzzy graph.

5. (3, (c_1, c_2))–REGULAR BIPOLAR FUZZY GRAPHS ON A PATH ON 6 VERTICES WITH SOME SPECIFIC MEMBERSHIP FUNCTIONS

In this section (3, (c_1, c_2))-regularity and totally (3, (c_1, c_2))-regularity on bipolar fuzzy graph whose underlying crisp graph is a path on 6 vertices is studied with some specific membership functions.

Theorem 5.1: Let G = (A, B) be a bipolar fuzzy graph such that $G^*(V, E)$ is a path on six vertices. If B is a constant function then G is a (3, (k_1, k_2)) - regular bipolar fuzzy graph.

Proof: Suppose that B is a constant function, say $B(uv) = (k_1, k_2)$, for all $uv \in E$. Then $d_3(u) = (k_1, k_2)$. Hence G is (3, (k_1, k_2)) - regular bipolar fuzzy graph.

Remark 5.2: The converse of Theorem 5.1 need not be true. For example consider G = (A, B) be bipolar fuzzy graph such that $G^*(V, E)$ is a path on six vertices.

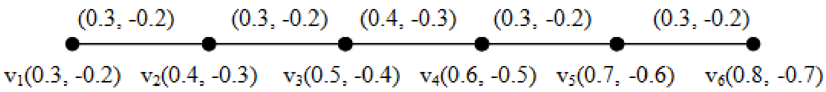

Figure-14

Here, $d_3(v_1) = (0.3, -0.2)$, $d_3(v_2) = (0.3, -0.2)$, $d_3(v_3) = (0.3, -0.2)$, $d_3(v_4) = (0.3, -0.2)$, $d_3(v_5) = (0.3, -0.2)$, $d_3(v_6) = (0.3, -0.2)$. Hence G is (3, (0.3, -0.2)) - regular bipolar fuzzy graph. But B is not a constant function.

Theorem 5.3: Let G = (A, B) be a bipolar fuzzy graph such that $G^*(V, E)$ is a path on six vertices. If alternate edges have the same membership values then G is a (3, (k_1, k_2)) - regular bipolar fuzzy graph where $(k_1, k_2) = \min\{(c_1, c_2), (c_3, c_4)\}$.

Proof: If alternate edges have same membership values, then $\mu(e_i) = \begin{cases} (c_1, c_2) \text{ if } i \text{ is odd} \\ (c_3, c_4) \text{ if } i \text{ is even} \end{cases}$.

If $(c_1, c_2) = (c_3, c_4)$, then B is constant function. So G is (3, (c_1, c_2)) -regular bipolar fuzzy graph.

If $(c_1, c_2) < (c_3, c_4)$, then $d_3(v) = (c_1, c_2)$, for all v∈ V. So G is (3, (c_1, c_2))-regular bipolar fuzzy graph.

If $(c_1, c_2) > (c_3, c_4)$, then $d_3(v) = (c_3, c_4)$, for all v∈ V. So G is (3, (c_3, c_4))-regular bipolar fuzzy graph.

Theorem 5.4: Let G = (A, B) be a bipolar fuzzy graph such that G*(V, E) is a path on six vertices. If the middle edge has positive membership value less than positive membership values of the remaining edges and negative membership value greater than the negative membership value of remaining edges, then G is a (3, (c_1, c_2)) - regular bipolar fuzzy graph where c_1 and c_2 are membership values of the middle edge.

For example, consider G = (A, B) be bipolar fuzzy graph such that G*(V, E) is a path on six vertices.

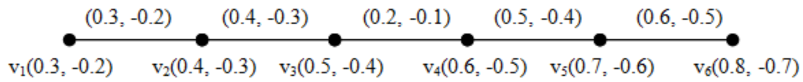

Figure-15

Here, $d_3(v_1)$ = (0.2, -0.1), $d_3(v_2)$ = (0.2, -0.1), $d_3(v_3)$ = (0.2, -0.1), $d_3(v_4)$ = (0.2, -0.1), $d_3(v_5)$ = (0.2, -0.1), $d_3(v_6)$ = (0.2, -0.1). Hence G is (3, (0.2, -0.1)) – regular bipolar fuzzy graph.

Remark 5.5: If A is a constant function, then Theorems 5.1, 5.3 and 5.4 hold good for totally (3, (c_1,c_2)) - regular bipolar fuzzy graphs.

6. (3, (c_1, c_2))–REGULAR BIPOLAR FUZZY GRAPHS ON WAGNER GRAPH WITH SOME SPECIFIC MEMBERSHIP FUNCTION

In this section (3, (c_1, c_2))-regularity and totally (3, (c_1, c_2))-regularity on bipolar fuzzy graph whose underlying crisp graph is a Wagner graph is studied with some specific membership function.

Theorem 6.1: Let G: (A, B) be a bipolar fuzzy graph such that G^*: (V, E) is Wagner graph. If B is constant function, then G is (3, (5c_1, 5c_2)) - regular bipolar fuzzy graph.

Proof: Suppose that, B is constant function say B(uv) = c for u v ∈ E. Then $d_3(v)$ = (5c_1, 5c_2) for all v ∈V.

Hence G is a (3, (5c_1, 5c_2)) – regular bipolar fuzzy graph.

Remark 6.2: Converse of the theorem 6.1 need not be true.

For example, Consider G: (A, B) be a bipolar fuzzy graph such that G^*:(V, E) is Wagner graph.

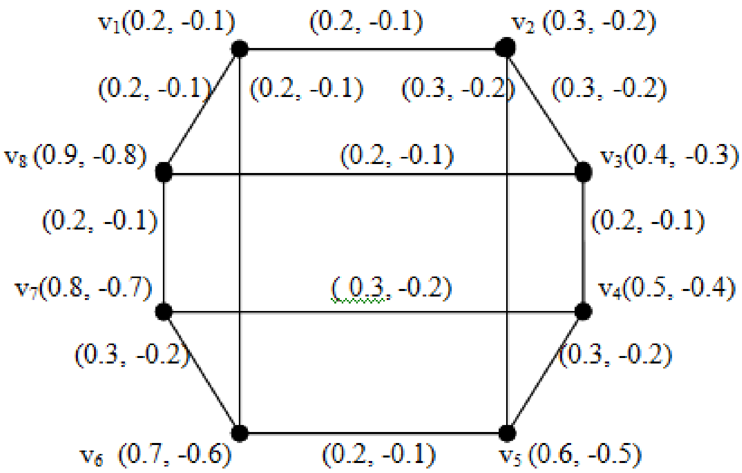

Figure-16

Here, $d_3(v_1)$ = (1.0, -0.5), $d_3(v_2)$ = (1.0, -0.5), $d_3(v_3)$ = (1.0, -0.5), $d_3(v_4)$ = (1.0, -0.5), $d_3(v_5)$ = (1.0, -0.5), $d_3(v_6)$ = (1.0, -0.5), $d_3(v_7)$ = (1.0, -0.5), $d_3(v_8)$ = (1.0, -0.5). So, G is (3, (1.0, -0.5))-regular bipolar fuzzy graph. But B is not a constant function.

Remark 6.3: The theorem 6.1 does not hold for totally (3, (c_1, c_2)) – regular bipolar fuzzy graphs.

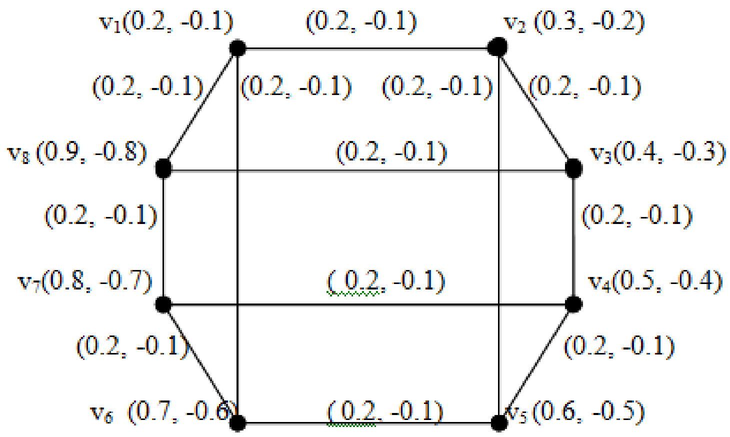

Figure-17

Here, $d_3(v_1)$ = (1.0, -0.5), $d_3(v_2)$ = (1.0, -0.5), $d_3(v_3)$ = (1.0, -0.5), $d_3(v_4)$ = (1.0, -0.5), $d_3(v_5)$ = (1.0, -0.5), $d_3(v_6)$ = (1.0, -0.5), $d_3(v_7)$ = (1.0, -0.5), $d_3(v_8)$ = (1.0, -0.5) and $td_3(v_1)$ = (1.2, -0.6), $td_3(v_2)$ = (1.3, -0.7), $td_3(v_3)$ = (1.4, -0.8), $td_3(v_4)$ = (1.5, -0.9), $td_3(v_5)$ = (1.6, -1.0), $td_3(v_6)$ = (1.7, -1.1), $td_3(v_7)$ = (1.8, -1.2), $td_3(v_8)$ = (1.9, -1.3). Hence G is (3, (1.0, -0.5)) - regular bipolar fuzzy graph but not totally (3, (c_1, c_2))-regular bipolar fuzzy graph. But B is a constant function.

7. (3, (c_1, c_2))–REGULAR BIPOLAR FUZZY GRAPHS ON CORONA GRAPH WITH SOME SPECIFIC MEMBERSHIP FUNCTION

In this section (3, (c_1, c_2))-regularity and totally (3, c_1, c_2))-regularity on bipolar fuzzy graph whose underlying crisp graph is a Corona graph is studied with some specific membership function.

Theorem 7.1: Let G: (A, B) be a bipolar fuzzy graph such that G^*: (V, E) is Corona graph $C_n \circ K_1$ where $n \geq 5$. If B is a constant function, then G is (3, ($4c_1$, $4c_2$)) - regular bipolar fuzzy graph.

Proof: Suppose that, B is a constant function say B(uv) = (c_1, c_2) for all u v ∈ E , then $d_3(v)$ = ($4c_1$, $4c_2$) for all v ∈V.

Hence G is (3, ($4c_1$, $4c_2$)) – regular bipolar fuzzy graph.

Remark 7.2: Converse of the theorem 7.1 need not be true.

For example, Consider G: (A, B) be a bipolar fuzzy graph such that G^*: (V, E) is Corona graph.

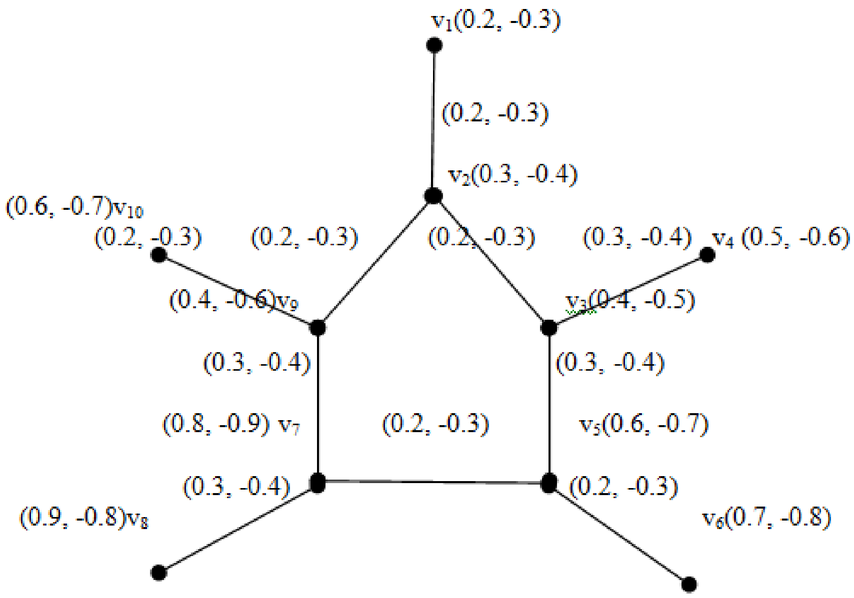

Figure-18

Here, $d_3(v_1)$ = (0.8, -1.2), $d_3(v_2)$ = (0.8, -1.2), $d_3(v_3)$ = (0.8, -1.2), $d_3(v_4)$ = (0.8, -1.2), $d_3(v_5)$ = (0.8, -1.2), $d_3(v_6)$ = (0.8, -1.2), $d_3(v_7)$ = (0.8, -1.2), $d_3(v_9)$ = (0.8, -1.2), $d_3(v_{10})$ = (0.8, -1.2), $d_3(v_{11})$ = (0.8, -1.2), $d_3(v_{12})$ = (0.8, -1.2). So G is (3, (0.8, -1.2)) - regular bipolar fuzzy graph. But B is not a constant function.

Remark 7.3: The theorem 7.1 does not hold for totally (3, (c_1, c_2))-regular bipolar fuzzy graph.

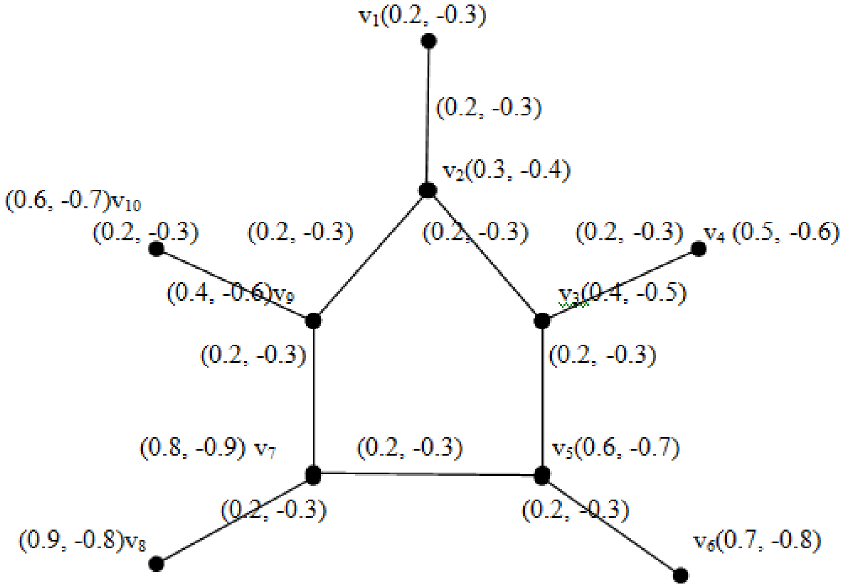

Figure-19

Here, $d_3(v_1)$ = (0.8, -1.2), $d_3(v_2)$ = (0.8, -1.2), $d_3(v_3)$ = (0.8, -1.2), $d_3(v_4)$ = (0.8, -1.2), $d_3(v_5)$ = (0.8, -1.2), $d_3(v_6)$ = (0.8, -1.2), $d_3(v_7)$ = (0.8, -1.2), $d_3(v_9)$ = (0.8, -1.2), $d_3(v_{10})$ = (0.8, -1.2), $d_3(v_{11})$ = (0.8, -1.2), $d_3(v_{12})$ = (0.8, -1.2). and $td_3(v_1)$ = (1.0, -1.5), $td_3(v_2)$ = (1.1, -1.6), $td_3(v_3)$ = (1.2, -1.7), $td_3(v_4)$ = (1.3, -1.8), $td_3(v_5)$ = (1.4, -1.9), $td_3(v_6)$ = (1.5, -2.0), $td_3(v_7)$ = (1.6, -2.1), $td_3(v_8)$ = (1.7, -2.0), $td_3(v_9)$ = (1.2, -1.8), $td_3(v_{10})$ = (1.4, -1.9). Hence G is (3, (0.8, -1.2)) - regular bipolar fuzzy graph but not totally (3, (c_1, c_2))-regular bipolar fuzzy graph. But B is a constant function.

8. (3, (C_1, C_2))-REGULAR BIPOLAR FUZZY GRAPHS ON A CYCLE OF LENGTH ≥ 5 WITH SOME SPECIFIC MEMBERSHIP FUNCTION

In this section (3, (c_1, c_2))-regularity and totally (3, (c_1, c_2))-regularity on bipolar fuzzy graph whose underlying crisp graph is a Cycle of length ≥ 5 is studied with some specific membership function.

Theorem 8.1: Let G=(A, B) be bipolar fuzzy graph such that G^*= (V, E) is cycle of length ≥ 5. If B is constant function, then G is (3, ($2c_1$, $2c_2$)) - regular bipolar fuzzy graph.

Proof: Suppose that, B is constant function say B(uv) = (c_1, c_2) for all uv∈E , then $d_3(v)$ = ($2c_1$, $2c_2$), for all v∈V. Hence G is (3, ($2c_1$, $2c_2$)) – regular bipolar fuzzy graph.

Remark 8.2: Converse of the theorem 8.1 need not be true.

For example, Consider G = (A, B) be fuzzy graph such that G^*:(V, E) is odd cycle of length seven.

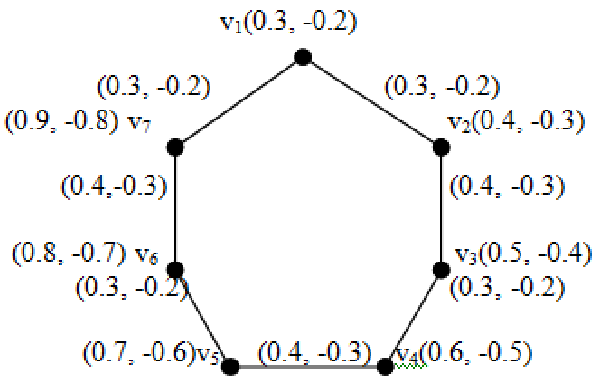

Figure-20

Here, $d_3(v_1)$ = (0.6,- 0.6), $d_3(v_2)$ = (0.6,- 0.6), $d_3(v_3)$ =(0.6,- 0.6), $d_3(v_4)$ = (0.6,- 0.6), $d_3(v_5)$ =(0.6,- 0.6), $d_3(v_6)$ = (0.6,- 0.6), $d_3(v_7)$ =(0.6,- 0.6). So G is (3, (0.6, - 0.6)) – regular bipolar fuzzy graph. But μ is not a constant function.

Remark 8.3: The theorem 8.1 does not hold for totally (3, (c_1, c_2))-regular bipolar fuzzy graphs**.**

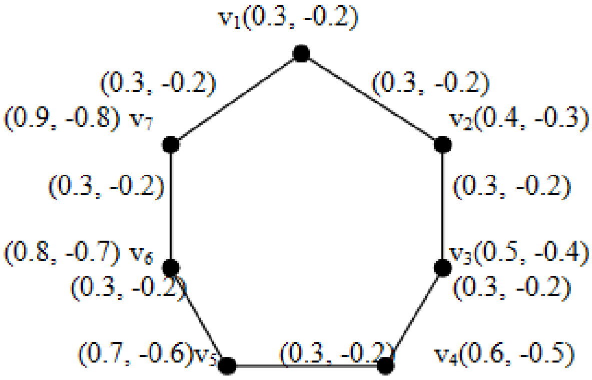

Figure-21

Here, $d_3(v_1)$ = (0.6,- 0.6), $d_3(v_2)$ = (0.6,- 0.6), $d_3(v_3)$ =(0.6,- 0.6), $d_3(v_4)$ = (0.6,- 0.6), $d_3(v_5)$ = (0.6,- 0.6), $d_3(v_6)$ = (0.6,- 0.6), $d_3(v_7)$ =(0.6,- 0.6). and $td_3(v_1)$ = (0.9, -0.8), $td_3(v_2)$ = (1.0, -0.9), $td_3(v_3)$ = (1.1, -1.0), $td_3(v_4)$ = (1.2, -1.1), $td_3(v_5)$ = (1.3, -1.2), $td_3(v_6)$ = (1.4, -1.3), $td_3(v_7)$ =(1.5, -1.4).So G is (3, (0.6,- 0.6))-regular fuzzy graph, but not totally(3, (c_1, c_2))-regular bipolar fuzzy graph. But B is a constant function.

Theorem 8.4: Let G= (A, B) be bipolar fuzzy graph such that G*: (V, E) is even cycle of length ≥ 5. If alternate edges have same membership values, then G is (3, (c_1, c_2))-regular bipolar fuzzy graph.

Proof: If alternate edges have same membership values, then B(e_i) =$\begin{Bmatrix} (c_{1,}, c_2\ ) if\ i\ is\ odd \\ (c_{3,}, c_4\ )\ if i\ is\ even \end{Bmatrix}$.

If $(c_{1,}, c_2\ )$ =$(c_{3,}, c_4)$, then B is constant function. So G is (3, (2c_1, 2c_2))-regular bipolar fuzzy graph.

If $(c_{1,}, c_2\ ) < (c_{3,}, c_4)$, then $d_3(v)$ = (2c_1, 2c_2), for all v∈ V. So G is (3, (2c_1, 2c_2))-regular bipolar fuzzy graph.

If $(c_{1,}, c_2\ ) > (c_{3,}, c_4)$, then $d_3(v)$ = $(2c_{3,}, 2c_4)$, for all v∈ V. So G is (3, $(2c_{3,}, 2c_4)$-regular bipolar fuzzy graph.

Remark 8.5: The theorem 8.4 does not hold for totally (3, $(c_{1,}, c_2)$)-regular bipolar fuzzy graphs

For example, Consider G= (A, B) be fuzzy graph such that G^*: (V, E) is even cycle of length ≥ 5.

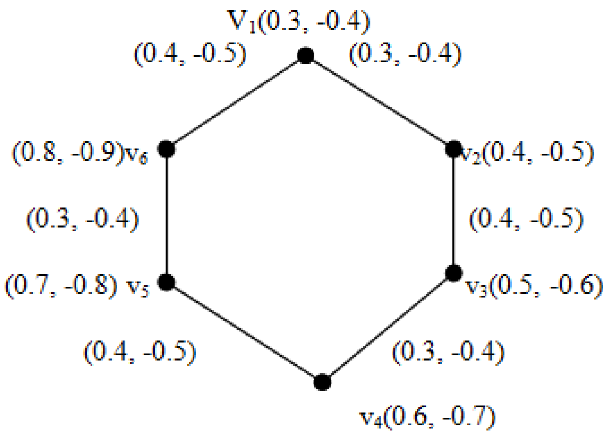

Figure-22

Here, $d_3(v_1)$ = (0.6,-0.8), $d_3(v_2)$ =(0.6,-0.8), $d_3(v_3)$ = (0.6,-0.8), $d_3(v_4)$ = (0.6,-0.8), $d_3(v_5)$ = (0.6,-0.8), $d_3(v_6)$ = (0.6,-0.8) and $td_3(v_1)$ = (0.9, -1.2), $td_3(v_2)$ = (1.0, -1.3), $td_3(v_3)$ = (1.1, -1.4), $td_3(v_4)$ = (1.2, -1.5), $td_3(v_5)$ = (1.3, -1.6), $td_3(v_6)$ = (1.4, -1.7). So G is not totally (3, (c_1, c_2))-regular bipolar fuzzy graph.

Remark 8.6: Let G= (A, B) be bipolar fuzzy graph such that G^*:(V, E) is odd cycle of length ≥ 5. If alternate edges have same membership values, then G is (3, (c_1, c_2))-regular bipolar fuzzy graph.

For example, Consider G:(A, B) be bipolar fuzzy graph such that G^*:(V, E) is odd cycle of length 5.

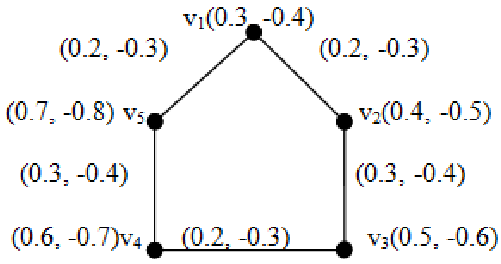

Figure-23

Here $d_3(v_1)$ = (0.4, -0.6), $d_3(v_2)$ = (0.4, -0.6), $d_3(v_3)$ = (0.4, -0.6), $d_3(v_4)$ = (0.4, -0.6), $d_3(v_5)$ = (0.4, -0.6). G is (3, (0.4, -0.6))-regular bipolar fuzzy graph.

Remark 8.7: The remark 8.6 does not hold for totally (3, (c_1, c_2))-regular bipolar fuzzy graphs. For example, Consider G= (A, B) be bipolar fuzzy graph such that G^*: (V, E) is odd cycle of length 5.

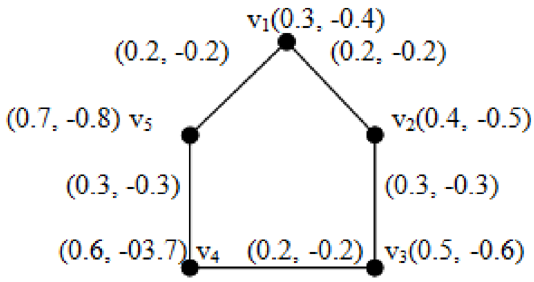

Figure-24

Here $d_3(v_1)$ = (0.4, -0.4), $d_3(v_2)$ =(0.4, -0.4), $d_3(v_3)$ = (0.4, -0.4), $d_3(v_4)$ = (0.4, -0.4), $d_3(v_5)$ =(0.4, -0.4) and $td_3(v_1)$ = (0.7, -0.8), $td_3(v_2)$ = (0.8,-0.9), $td_3(v_3)$ = (0.9, 1.0), $td_3(v_4)$ = (1.0, 1.1), $td_3(v_5)$ = (1.1, 1.2). Hence G is not totally (3, (c_1, c_2)) – regular bipolar fuzzy graph.

Theorem 8.8: Let G= (A, B) be bipolar fuzzy graph such that G^*: (V, E) is any odd cycle of length ≥ 5.

Let $B(e_i) = \begin{Bmatrix} (c_1, c_2), & \text{if } i \text{ is odd.} \\ (c_1, c_2) \geq (c_3, c_4), & \text{if } i \text{ is even} \end{Bmatrix}$, then G is (3, ($2c_1$, $2c_2$))-regular bipolar fuzzy graph.

Proof: Let $B(e_i) = \begin{Bmatrix} (c_1, c_2), & \text{if } i \text{ is odd.} \\ (c_1, c_2) \geq (c_3, c_4), & \text{if } i \text{ is even} \end{Bmatrix}$

Case-1: Let G= (A, B) be bipolar fuzzy graph such that G^*:(V,E) is even cycle of length ≥ 5.
$d_3(v_i)$= ($c_1 \wedge c_3 \wedge c_1$, $c_2 \vee c_4 \vee c_2$) + ($c_3 \wedge c_1 \wedge c_3$, $c_4 \vee c_2 \vee c_4$) = (c_1, c_2) + (c_1, c_2)= $(2c_1, 2c_2)$, for all v∈V. So, G is (3, $(2c_1, 2c_2)$)-regular bipolar fuzzy graph.

Case-2: Let G= (A, B) be fuzzy graph such that G^*:(V, E) is an odd cycle of length ≥ 5. Let e_1, e_2, e_3…..e_{2n+1} be the edges of the odd cycle of G* in that order.

$d_3(v_1) = (m_2^+(e_{2n+1}) \wedge m_2^+(e_{2n}) \wedge m_2^+(e_{2n-1}),\ m_2^-(e_{2n+1}) \vee m_2^-(e_{2n}) \vee m_2^-(e_{2n-1})) + (m_2^+(e_1) \wedge m_2^+(e_2) \wedge m_2^+(e_3),$
$m_2^-(e_1) \vee m_2^-(e_2) \vee m_2^-(e_3))$
$= (c_1 \wedge c_3 \wedge c_1, c_2 \vee c_4 \vee c_2) + \{c_1 \wedge c_3 \wedge c_1, c_2 \vee c_4 \vee c_2\} = (c_1, c_2) + (c_1, c_2) = (2c_1, 2c_2).$

$d_3(v_2) = \{m_2^+(e_1) \wedge m_2^+(e_{2n+1}) \wedge m_2^+(e_{2n}),\ m_2^-(e_1) \vee m_2^-(e_{2n+1}) \vee m_2^-(e_{2n})\} + \{m_2^+(e_2) \wedge m_2^+(e_3) \wedge m_2^+ e_4,\ m_2^-$
$(e_2) \vee m_2^-(e_3) \vee m_2^- e_4)\}$
$= \{c_1 \wedge c_1 \wedge c_3, c_2 \vee c_2 \vee c_4\} + \{c_3 \wedge c_1 \wedge c_3, c_4 \vee c_2 \vee c_4\} = (c_1, c_2) + (c_1, c_2) = (2c_1, 2c_2).$

$d_3(v_3) = \{m_2^+(e_2) \wedge m_2^+(e_1) \wedge m_2^+(e_{2n+1}),\ m_2^-(e_2) \vee m_2^-(e_1) \vee m_2^-(e_{2n+1})\} + \{m_2^+(e_3) \wedge m_2^+(e_4) \wedge m_2^+(e_5),\ m_2^-(e_3)$
$\vee m_2^-(e_4) \vee m_2^-(e_5)\}$
$= \{c_3 \wedge c_1 \wedge c_1, c_4 \vee c_2 \vee c_2\} + \{c_1 \wedge c_3 \wedge c_1, c_2 \vee c_4 \vee c_2\} = (c_1, c_2) + (c_1, c_2) = (2c_1, 2c_2).$

For i= 4, 5, 6,….,2n.

$d_3(v_i) = \{ m_2^+(e_{i-1}) \wedge m_2^+(e_{i-2}) \wedge m_2^+(e_{i-3}), m_2^-(e_{i-1}) \vee m_2^-(e_{i-2}) \vee m_2^-(e_{i-3})\} + \{ m_2^+(e_i) \wedge m_2^+(e_{i+1})\ m_2^+(e_{i+2}), m_2^-(e_i) \vee m_2^-(e_{i+1}) \vee m_2^-(e_{i+2})\}$

$= \{c_1 \wedge c_3 \wedge c_1, c_2 \vee c \vee \vee c_2\} + \{c_3 \wedge c_1 \wedge c_3, c_4 \vee c_2 \vee c_4\} = (c_{1,}, c_2) + (c_{1,}, c_2) = (2c_{1,}, 2c_2)$.

$d_3(v_{2n+1}) = \{ m_2^+(e_{2n+1}) \wedge m_2^+(e_1) \wedge m_2^+(e_2), m_2^-(e_{2n+1}) \vee m_2^-(e_1) \vee m_2^-(e_2)\} + \{m_2^+(e_{2n}) \wedge m_2^+(e_{2n-1}) \wedge m_2^+(e_{2n-2}), m_2^-(e_{2n}) \vee m_2^-(e_{2n-1}) \vee m_2^-(e_{2n-2}\}$

$= \{c_1 \wedge c_3 \wedge c_1, c_2 \vee c_4 \vee c_2\} + \{c_3 \wedge c_1 \wedge c_3, c_4 \vee c_2 \vee c_4\} = (c_{1,}, c_2) + (c_{1,}, c_2) = (2c_{1,}, 2c_2)$.

Hence, $d_3(v_i) = (2c_{1,}, 2c_2)$, for all v ϵ V.

So G is (3, ($2c_1$, $2c_{2)}$)) - regular bipolar fuzzy graph.

Remark 8.9: The theorem 8.8 does not hold for totally (3, (c_1, $c_{2)}$) - regular bipolar fuzzy graph.

1. Consider G= (A, B) be bipolar fuzzy graph such that G^*:(V, E) is even cycle of length six.

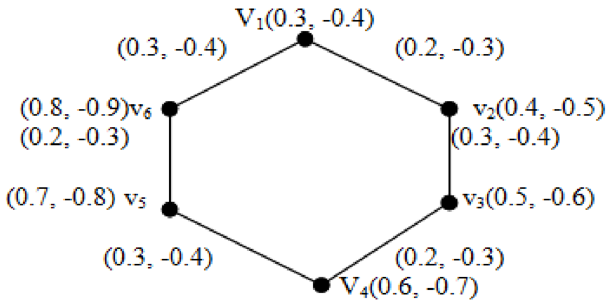

Figure-25

Here, $d_3(v_1)$ = (0.4, -0.6), $d_3(v_2)$ = (0.4, -0.6), $d_3(v_3)$ = (0.4, -0.6), $d_3(v_4)$ = (0.4, -0.6), $d_3(v_5)$ = (0.4, -0.6), $d_3(v_6)$ = (0.4, -0.6). So G is (3, k)-regular fuzzy graph. But $td_3(v_1)$ = (0.7, -1.0) $td_3(v_2)$ = (0.8, -1.1), $td_3(v_3)$ = (0.9, -1.2), $td_3(v_4)$ = (1.0, -1.3), $td_3(v_5)$ = (1.1, -1.4), $td_3(v_6)$ = (1.2, -1.5). So G is not totally (3, (c_1, $c_{2)}$))-regular bipolar fuzzy graph.

2. Consider G= (A, B) be bipolar fuzzy graph such that G^*:(V, E) is an odd cycle of length five.

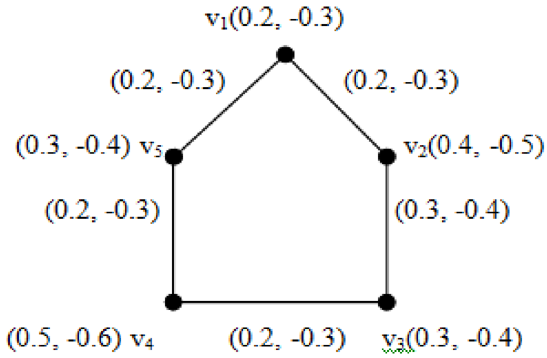

Figure-26

Here, $d_3(v_1)$ = (0.4, -0.6), $d_3(v_2)$ = (0.4, -0.6), $d_3(v_3)$ = (0.4, -0.6), $d_3(v_4)$ = (0.4, -0.6), $d_3(v_5)$ = (0.4, -0.6). So G is (3, (0.4, -0.6))-regular bipolar fuzzy graph. But $td_3(v_1)$ = (0.6, -0.9), $td_3(v_2)$ = (0.8, -1.1), $td_3(v_3)$ = (0.7, -1.0), $td_3(v_4)$ = (0.9, -1.2), $td_3(v_5)$ = (0.7, -1.0). So G is not totally (3, (c_1, $c_{2)}$)-regular bipolar fuzzy graph.

9. CONCLUSION AND FUTURE STUDIES

In this paper (3, (c_1, $c_{2)}$) - regular bipolar fuzzy graphs and totally (3, (c_1, $c_{2)}$)-regular bipolar fuzzy graphs are compared through various examples. A necessary and sufficient condition under which they are equivalent is provided. Also we provide (3, (c_1, $c_{2)}$)-regular bipolar fuzzy graphs and totally (3, (c_1, $c_{2)}$)-regular bipolar fuzzy graphs in which underlying crisp graphs are a path on six vertices, a corona graph, a Wagner graph and a cycle of length ≥ 5 is studied with some specific membership function. Some properties of (3, (c_1, $c_{2)}$)-regular bipolar fuzzy graphs studied and they are examined for totally (3, k)-regular bipolar fuzzy graphs. The results discussed may be used to study about various fuzzy graphs invariants. For further investigation, the following open problem is suggested.

"(r, (c_1, $c_{2)}$)-regular bipolar fuzzy graph and totally (r, (c_1, $c_{2)}$)-regular bipolar fuzzy graph for r > 3 may be investigated".

REFERENCES

1. Akram and W. Dudek, *Regular bipolar fuzzy graphs*, Neural Computing and Application, 1007/s00521-011-0772-6.
2. G.S. Bloom, J.K. Kennedy and L.V. Quintas, *Distance degree regular graphs*, The Theory and Applications of Graphs, Wiley, New York, (1981), 95-108.
3. J. A. Bondy and U.S.R. Murty, *Graph Theory with Application*, MacMillan, London (1979).
4. P. Bhattacharya, *Some remarks on Fuzzy Graphs*, Pattern Recognition Lett, 6(1987), 297-302.
5. G. Chartrand, P. Erdos and O. R. Oellermann, *How to define an irregular graph*, College Math. J., 19 (1988), 36-42.
6. F. Harary, *Graph Theory*, Addison - Wesley, (1969).
7. S.Meena Devi and M.Andal, *On* (3, *k*) *- regular fuzzy graph and totally* (3, *k*) *- regular fuzzy graph* International Journal of Science and Research.
8. John N.Moderson and Premchand S. Nair, *Fuzzy graphs and Fuzzy hypergraphs* Physica verlag, eidelberg(2000).
9. A.Nagoor Gani and K. Radha, *On Regular Fuzzy Graphs*, Journal of Physical Sciences, Volume 12(2008), 33-44.
10. N.R.Santhi Maheswari and C.Sekar, *On* (2, *k*) *- regular fuzzy graph and totally* (2, *k*)*- regular fuzzy graph* International Journal of Mathematics and Soft computing 4(2)(2014)59-69.
11. N. R. Santhi Maheswari and C. Sekar (r, 2, r (r-1))- regular graphs, International Journal of Mathematics and Soft Computing, Vol. 2. No.2 (2012), 25-33.
12. N.R. Santhi Maheswari and C.Sekar On d_2 of a vertex in Product of Graphs. (ICODIMA 2013) December 3rd, 2013.Periyar Maniammai University, Thanjavur.
13. S. Ravi Narayanan and N.R. Santhi Maheswari, On (2,(c_1, c_2))- regular bipolar fuzzy graph International Journal of Mathematics and Soft computing 5(2015), 91-103.
14. A.Rosenfeld, *Fuzzy graphs, in: Fuzzy sets and their applications to Cognetive and Deci-sion Processes* (L.A. Zadeh, K.S. Fu, Tanaka and M. Shimura, EDs), Academic Press, Newyork (1975), 77-95.

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