

THE HYPER-ZAGREB INDEX OF SOME DERIVED GRAPHS

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ABSTRACT

Let $G = (V, E)$ be a connected graph. The hyper-Zagreb index is defined as $HZ(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]^2$. In this paper, comparison of the hyper-Zagreb index and other degree based topological indices like the Forgotten index, Zagreb and Banhatti indices of some derived graphs such as line graph, subdivision graph, vertex-semitotal graph, edge-semitotal graph and total graph are obtained. In addition, exact values of some standard graphs of above derived graphs are presented.

Key words and phrases: Hyper-Zagreb index; Line graph, Subdivision graph; Vertex-semitotal graph, Edge-semitotal graph, Total graph.

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1. INTRODUCTION

All graphs considered in this paper are finite, connected, undirected without loops and multiple edges. Any undefined term in this paper may be found in Kulli [17]. Let G be a simple connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v . The edge connecting the vertices u and v will be denoted by uv . Let $d_G(e)$ denotes the degree of an edge e in G , which is defined by $d_G(e) = d_G(u) + d_G(v) - 2$ with $e = uv$.

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. In Chemical Science, the physico-chemical properties of chemical compounds are often modelled by means of molecular graph based structure descriptors, which are also referred to as topological indices, see [21].

The first two Zagreb indices was introduced by Gutman and Trinajstic [13] to take account of the contributions of pairs of adjacent vertices. For their history, applications, and mathematical properties, see [3], [9], [11], [12] and the references cited therein. The first and second Zagreb indices of G are defined as $M_1(G) = \sum_{v \in V(G)} d_G(v)^2$ or $M_1(G) = \sum_{uv \in E(G)} d_G(u) + d_G(v)$ and $M_2(G) = \sum_{uv \in E(G)} [d_G(u) \cdot d_G(v)]$. Followed by the First Zagreb index of a graph G , Shirdel *et al.* [8] was introduced the hyper-Zagreb index of G defined as $HZ(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]^2$. In [5], Furtula and Gutman was introduced the so-called forgotten topological index F , defined as $F(G) = \sum_{v \in V(G)} d_G(v)^3 = \sum_{uv \in E(G)} (d_G(u))^2 + (d_G(v))^2$.

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In [18], Kulli introduced the first and second K Banhatti indices, intending to take into account the contributions of pairs of incident elements. The first and second K Banhatti indices of a graph G are defined as $B_1(G) = \sum_{ue} d_G(u) + d_G(e)$ and $B_2(G) = \sum_{ue} d_G(u) \times d_G(e)$, respectively, where ue means that the vertex u and edge e are incident in G . The Banhatti and Zagreb indices are closely related, see [10]. Recently many other indices were studied, for example, in [4] and [19].

2. EXISTING RESULTS OF DEGREE BASED INDICES

To prove our main results, we make use of the following results in sequel.

Theorem 2.1: [10] For any graph G , the first Banhatti index and second Banhatti indices are related to the first Zagreb index and Hyper Zagreb indices as

- (i) $B_1(G) = 3M_1(G) - 4|E(G)|$
- (ii) $B_2(G) = HZ(G) - 2M_1(G)$.

Let G be a standard graph of path P_n ; $n \geq 2$ and complete bipartite graph $K_{r,s}$; $1 \leq r \leq s$. Also, in r -regular graph G , if $r=2$, then G is a cycle C_n ; $n \geq 3$ and if $r = n-1$, then G is a complete graph K_n ; $n \geq 3$ vertices.

Proposition 2.2: [3, 12] Let G be some standard class of graphs. Then

- (i) $M_1(P_n) = 4n - 6$ for $n \geq 2$ vertices
- (ii) $M_1(C_n) = 4n$ for $n \geq 3$ vertices
- (iii) $M_1(K_n) = n(n-1)^2$ for $n \geq 3$ vertices
- (iv) $M_1(G) = nr^2$, where G is a r -regular graph
- (v) $M_1(K_{r,s}) = rs(r+s)$ for $1 \leq r \leq s$ vertices.

Proposition 2.3: [8] Let G be some standard class of graphs. Then

- (i) $HZ(P_n) = 16n - 30$ for $n \geq 2$ vertices
- (ii) $HZ(C_n) = 16n$ for $n \geq 3$ vertices
- (iii) $HZ(K_n) = 2n(n-1)^3$ for $n \geq 3$ vertices
- (iv) $HZ(G) = 2nr^3$, where G is a r -regular graph
- (v) $HZ(K_{r,s}) = rs(r+s)^2$ for $1 \leq r \leq s$ vertices.

3. SOME DERIVED GRAPHS

The Line graph $L(G)$ is the graph with vertex set $V(L(G)) = E(G)$ and whose vertices correspond to the edges of G with two vertices being adjacent if and only if the corresponding edges in G have a vertex in common to, see [15].

The Subdivision graph $S(G)$ is the graph obtained from G by replacing each of its edges by a path of length two, or equivalently, by inserting an additional vertex into each edge of a graph G , see [14].

The Vertex-Semitotal graph $T_1(G)$ with vertex set $V(G) \cup E(G)$ and edge set $E(S(G)) \cup E(G)$ is the graph obtained from G by adding a new vertex corresponding to each edge of G and by joining each new vertex to the end vertices of the edge corresponding to it, see [6].

The Edge-Semitotal graph $T_2(G)$ with vertex set $V(G) \cup E(G)$ and edge set $E(S(G)) \cup E(L(G))$ is the graph obtained from G by inserting a new vertex into each edge of G and by joining with edges those pairs of these new vertices which lie on adjacent edges of G , see [7].

The Total graph of a graph G denoted by $T(G)$ with vertex set $V(G) \cup E(G)$ and any two vertices of $T(G)$ are adjacent if and only if they are either incident or adjacent in G , see [2].

Different Topological indices of some derived graphs have been studied by Basavanagoud *et al.* [2], Khalifeh *et al.* [16] and Nilanjan De [20]. In view of these references, some of the existing results as follows.

Proposition 3.1:

- 1) Let $L(G)$ be the line graph of a graph G . Then
 - (i) $V(L(G)) = E(G)$
 - (ii) $|E(L(G))| = \frac{1}{2}M_1(G) - |E(G)|$.
- 2) Let $S(G)$ be the subdivision graph of a graph G . Then
 - (i) $M_1(S(G)) = M_1(G) + 4|E(G)|$
 - (ii) $|E(S(G))| = 2|E(G)|$.

- 3) Let $T_1(G)$ be the vertex semi-total graph of a graph G . Then
 - (i) $M_1(T_1(G)) = 4M_1(G) + 4|E(G)|$
 - (ii) $|E(T_1(G))| = 3|E(G)|$.
- 4) Let $T_1(G)$ be the edge semi-total graph of a graph G . Then
 - (i) $M_1(T_2(G)) = M_1(G) + M_1(L(G)) + 8|E(L(G))| + 8|E(G)|$
 - (ii) $|E(T_2(G))| = |E(G)| + \frac{1}{2}M_1(G)$.
- 5) Let $T(G)$ be the total graph of a graph G . Then
 - (i) $M_1(T(G)) = 4M_1(G) + M_1(L(G)) + 8|E(L(G))| + 4|E(G)|$
 - (ii) $|E(T_2(G))| = 2|E(G)| + \frac{1}{2}M_1(G)$.

Proposition 3.2: Let G be a graph of order n and size m . Then

- (i) $HZ(G) = F(G) + 2M_2(G)$
- (ii) $F(S(G)) = F(G) + 8|E(G)|$
- (iii) $M_2(S(G)) = 2M_1(G)$.

4. MAIN RESULTS

Proposition 4.1:

- 1) Let $S(G)$ be the subdivision graph of a graph G . Then
 - (i) $M_1(S(G)) = nr(r + 2)$, where G is a r -regular graph
 - (ii) $M_1(S(P_n)) = 8n - 10$
 - (iii) $M_1(S(K_{r,s})) = rs(r + s + 4)$ for $1 \leq r \leq s$ vertices.
- 2) Let $T_1(G)$ be the vertex semi-total graph of a graph G . Then
 - (i) $M_1(T_1(G)) = 2nr(2r + 1)$, where G is a r -regular graph
 - (ii) $M_1(T_1(P_n)) = 20n - 28$
 - (iii) $M_1(T_1(K_{r,s})) = 4rs(r + s + 1)$ for $1 \leq r \leq s$ vertices.
- 3) Let $T_2(G)$ be the edge semi-total graph of a graph G . Then
 - (i) $M_1(T_2(G)) = 2nr^3 + nr^2$, where G is a r -regular graph
 - (ii) $M_1(T_2(P_n)) = 20n - 36$
 - (iii) $M_1(T_2(K_{r,s})) = rs(r^2 + s^2 + r + s + 2rs)$ for $1 \leq r \leq s$ vertices.
- 4) Let $T(G)$ be a total graph of a graph G . Then
 - (i) $M_1(T(G)) = 2nr^2(r + 2)$, where G is a r -regular graph
 - (ii) $M_1(T(P_n)) = 32n - 54$
 - (iii) $M_1(T(K_{r,s})) = rs(r^2 + s^2 + 4r + 4s + 2rs)$ for $1 \leq r \leq s$ vertices.

Proof: From Propositions 2. 2 and 3.1, the results are immediate.

Proposition 4.2:

- (i) $HZ(L(G)) = 8nr(r - 1)^3$, where G is a r -regular graph
- (ii) $HZ(L(K_{r,s})) = 2rs(r + s - 2)^3$ for $1 \leq r \leq s$ vertices.
- (iii) $HZ(S(G)) = nr(r + 2)^2$, where G is a r -regular graph.
- (iv) $HZ(S(P_n)) = 32n - 46$
- (v) $HZ(S(K_{r,s})) = rs(r^2 + s^2 + 4r + 4s + 8)$ for $1 \leq r \leq s$ vertices.
- (vi) $HZ(T_1(G)) = 4nr(r + 1)^2 + 8nr^3$, where G is a r -regular graph.
- (vii) $HZ(T_1(P_n)) = 136n - 232$
- (viii) $HZ(T_1(K_{r,s})) = 4rs(2r^2 + 2s^2 + 2r + 2s + 2rs + 2)$ for $1 \leq r \leq s$ vertices
- (ix) $HZ(T_2(G)) = 9nr^3 + 8nr^3(r - 1)$, where G is a r -regular graph
- (x) $HZ(T_2(P_n)) = 136n - 292$
- (xi) $HZ(T_2(K_{r,s})) = rs(2r^3 + 2s^3 + 2r^2s + 2rs^2 + r^2 + s^2)$ for $1 \leq r \leq s$ vertices
- (xii) $HZ(T(G)) = 8nr^3(r + 2)$, where G is a r -regular graph.
- (xiii) $HZ(T(P_n)) = 256n - 514$
- (xiv) $HZ(T(K_{r,s})) = rs(2r^3 + 2s^3 + 6r^2s + 6rs^2 + 10s^2 + 10r^2 + 12rs)$ for $1 \leq r \leq s$ vertices.

Proof:

- (i) We have, $HZ(L(G)) = \sum_{uv \in E(L(G))} [d_{L(G)}(u) + d_{L(G)}(v)]^2$. Since the line graph of a r -regular graph is $(2r - 2)$ -regular. Hence $HZ(L(G)) = \frac{nr}{2}(r - 1)(4r - 4)^2 = 8nr(r - 1)^3$.

- (ii) Since the line graph of complete bipartite graph $K_{r, s}$ is a $(r + s - 2)$ -regular graph and $|E(L(K_{r,s}))| = \frac{rs}{2} (r + s - 2)$. Hence the result follows.
- (iii) We have, $HZ(S(G)) = \sum_{uv \in E(S(G))} [d_{S(G)}(u) + d_{S(G)}(v)]^2$. In $S(G)$, there is an edge partition $E_1 = \{uv \in E(S(G)): d_{S(G)}(u) = 2, d_{S(G)}(v) = r\}; |E_1| = nr$.
Therefore $HZ(S(G)) = nr(r + 2)^2$.
- (iv) In $S(P_n)$, there are two edge partitions,
 $E_1 = \{uv \in E(S(P_n)): d_{S(P_n)}(u) = 1, d_{S(P_n)}(v) = 2\}; |E_1| = 2$
 $E_2 = \{uv \in E(S(P_n)): d_{S(P_n)}(u) = 2, d_{S(P_n)}(v) = 2\}; |E_2| = 2(n - 2)$.
Therefore, $HZ(S(P_n)) = \sum_{uv \in E(S(P_n))} [d_{S(P_n)}(u) + d_{S(P_n)}(v)]^2$
 $= 2(1 + 2)^2 + 2(n - 2)(2 + 2)^2 = 32n - 46$.
- (v) In $S(K_{r, s})$, there are two edge partitions,
 $E_1 = \{uv \in E(S(K_{r,s})): d_{S(K_{r,s})}(u) = r, d_{S(K_{r,s})}(v) = 2\}; |E_1| = rs$
 $E_2 = \{uv \in E(S(K_{r,s})): d_{S(K_{r,s})}(u) = s, d_{S(K_{r,s})}(v) = 2\}; |E_2| = rs$.
Therefore $HZ(S(K_{r,s})) = \sum_{uv \in E(S(K_{r,s}))} [d_{S(K_{r,s})}(u) + d_{S(K_{r,s})}(v)]^2$
 $= rs(r + 2)^2 + rs(s + 2)^2$
 $= rs(r^2 + s^2 + 4r + 4s + 8)$.
- (vi) In $T_1(G)$, there are two edge partitions,
 $E_1 = \{uv \in E(T_1(G)): d_{T_1(G)}(u) = 2, d_{T_1(G)}(v) = 2r\}; |E_1| = nr$
 $E_2 = \{uv \in E(T_1(G)): d_{T_1(G)}(u) = d_{T_1(G)}(v) = 2r\}; |E_2| = \frac{nr}{2}$.
Therefore $HZ(T_1(G)) = \sum_{uv \in E(T_1(G))} [d_{T_1(G)}(u) + d_{T_1(G)}(v)]^2$
 $= nr(2 + 2r)^2 + \frac{nr}{2} (2r + 2r)^2$
 $= 4nr(r + 1)^2 + 8nr^3$.
- (vii) In $T_1(P_n)$, there are three edge partitions,
 $E_1 = \{uv \in E(T_1(P_n)): d_{T_1(P_n)}(u) = 2, d_{T_1(P_n)}(v) = 2\}; |E_1| = 2$
 $E_2 = \{uv \in E(T_1(P_n)): d_{T_1(P_n)}(u) = 2, d_{T_1(P_n)}(v) = 4\}; |E_2| = 2(n - 1)$
 $E_3 = \{uv \in E(T_1(P_n)): d_{T_1(P_n)}(u) = d_{T_1(P_n)}(v) = 4\}; |E_3| = n - 3$.
Therefore $HZ(T_1(P_n)) = \sum_{uv \in E(T_1(P_n))} [d_{T_1(P_n)}(u) + d_{T_1(P_n)}(v)]^2$
 $= 2(2 + 2)^2 + 2(n - 1)(2 + 4)^2 + (n - 3)(4 + 4)^2$
 $= 136n - 232$.
- (viii) In $T_1(K_{r,s})$, there are three edge partitions,
 $E_1 = \{uv \in E(T_1(K_{r,s})): d_{T_1(K_{r,s})}(u) = 2r, d_{T_1(K_{r,s})}(v) = 2\}; |E_1| = rs$
 $E_2 = \{uv \in E(T_1(K_{r,s})): d_{T_1(K_{r,s})}(u) = 2s, d_{T_1(K_{r,s})}(v) = 2\}; |E_2| = rs$
 $E_3 = \{uv \in E(T_1(K_{r,s})): d_{T_1(K_{r,s})}(u) = 2r, d_{T_1(K_{r,s})}(v) = 2s\}; |E_3| = rs$.
Hence, $HZ(T_1(K_{r,s})) = \sum_{uv \in E(T_1(K_{r,s}))} [d_{T_1(K_{r,s})}(u) + d_{T_1(K_{r,s})}(v)]^2$
 $= rs(2r + 2)^2 + rs(2s + 2)^2 + rs(2s + 2s)^2$
 $= 4rs(2r^2 + 2s^2 + 2r + 2s + 2rs + 2)$.
- (ix) In $T_2(G)$, there are two edge partitions,
 $E_1 = \{uv \in E(T_2(G)): d_{T_2(G)}(u) = r, d_{T_2(G)}(v) = 2r\}; |E_1| = nr$
 $E_2 = \{uv \in E(T_2(G)): d_{T_2(G)}(u) = d_{T_2(G)}(v) = 2r\}; |E_2| = \frac{nr}{2} (r - 1)$.
Therefore $HZ(T_2(G)) = \sum_{uv \in E(T_2(G))} [d_{T_2(G)}(u) + d_{T_2(G)}(v)]^2$
 $= nr(r + 2r)^2 + \frac{nr}{2} (r - 1)(2r + 2r)^2$
 $= 9nr^3 + 8nr^3(r - 1)$.
- (x) In $T_2(P_n)$, there are five edge partitions,
 $E_1 = \{uv \in E(T_2(P_n)): d_{T_2(P_n)}(u) = 1, d_{T_2(P_n)}(v) = 3\}; |E_1| = 2$
 $E_2 = \{uv \in E(T_2(P_n)): d_{T_2(P_n)}(u) = 2, d_{T_2(P_n)}(v) = 3\}; |E_2| = 2$
 $E_3 = \{uv \in E(T_2(P_n)): d_{T_2(P_n)}(u) = 2, d_{T_2(P_n)}(v) = 4\}; |E_3| = 2(n - 3)$
 $E_4 = \{uv \in E(T_2(P_n)): d_{T_2(P_n)}(u) = 3, d_{T_2(P_n)}(v) = 4\}; |E_4| = 2$
 $E_5 = \{uv \in E(T_2(P_n)): d_{T_2(P_n)}(u) = 4, d_{T_2(P_n)}(v) = 4\}; |E_5| = n - 4$.

$$\begin{aligned} \text{Hence, } HZ(T_2(P_n)) &= \sum_{uv \in E(T_2(P_n))} [d_{T_2(P_n)}(u) + d_{T_2(P_n)}(v)]^2 \\ &= 2(1+3)^2 + 2(2+3)^2 + 2(n-3)(2+4)^2 + 2(3+4)^2 + (n-4)(4+4)^2 \\ &= 136n - 292. \end{aligned}$$

(xi) In $T_2(K_{r,s})$, there are three edge partitions,

$$\begin{aligned} E_1 &= \{uv \in E(T_2(K_{r,s})) : d_{T_2(K_{r,s})}(u) = r, d_{T_2(K_{r,s})}(v) = r+s\}; |E_1| = rs \\ E_2 &= \{uv \in E(T_2(K_{r,s})) : d_{T_2(K_{r,s})}(u) = s, d_{T_2(K_{r,s})}(v) = r+s\}; |E_2| = rs \\ E_3 &= \{uv \in E(T_2(K_{r,s})) : d_{T_2(K_{r,s})}(u) = d_{T_2(K_{r,s})}(v) = r+s\}; |E_3| = \frac{rs}{2}(r+s-2). \end{aligned}$$

$$\begin{aligned} \text{Hence, } HZ(T_2(K_{r,s})) &= \sum_{uv \in E(T_2(K_{r,s}))} [d_{T_2(K_{r,s})}(u) + d_{T_2(K_{r,s})}(v)]^2 \\ &= rs(r+r+s)^2 + rs(s+r+s)^2 + \frac{rs}{2}(r+s-2)(2r+2s)^2 \\ &= rs(2r^3 + 2s^3 + 2r^2s + 2rs^2 + r^2 + s^2). \end{aligned}$$

(xii) In $T(G)$, there is one edge partition,

$$E_1 = \{uv \in E(T(G)) : d_{T(G)}(u) = 2r, d_{T(G)}(v) = 2r\}; |E_1| = \frac{nr^2}{2} + nr$$

$$\begin{aligned} \text{Hence, } HZ(T(G)) &= \sum_{uv \in E(T(G))} [d_{T(G)}(u) + d_{T(G)}(v)]^2 \\ &= \left(\frac{nr^2}{2} + nr\right)(2r+2r)^2 \\ &= 8nr^3(r+2). \end{aligned}$$

(xiii) In $T(P_n)$, there are four edge partitions,

$$\begin{aligned} E_1 &= \{uv \in E(T(P_n)) : d_{T(P_n)}(u) = 2, d_{T(P_n)}(v) = 3\}; |E_1| = 2 \\ E_2 &= \{uv \in E(T(P_n)) : d_{T(P_n)}(u) = 2, d_{T(P_n)}(v) = 4\}; |E_2| = 2 \\ E_3 &= \{uv \in E(T(P_n)) : d_{T(P_n)}(u) = 3, d_{T(P_n)}(v) = 4\}; |E_3| = 4 \\ E_4 &= \{uv \in E(T(P_n)) : d_{T(P_n)}(u) = 4, d_{T(P_n)}(v) = 4\}; |E_4| = 4n - 13. \end{aligned}$$

$$\begin{aligned} \text{Hence, } HZ(T(P_n)) &= \sum_{uv \in E(T(P_n))} [d_{T(P_n)}(u) + d_{T(P_n)}(v)]^2 \\ &= 2(2+3)^2 + 2(2+4)^2 + 4(3+4)^2 + (4n-13)(4+4)^2 \\ &= 256n - 514. \end{aligned}$$

(xiv) In $(K_{r,s})$, there are three edge partitions,

$$\begin{aligned} E_1 &= \{uv \in E(T(K_{r,s})) : d_{T(K_{r,s})}(u) = 2s, d_{T(K_{r,s})}(v) = 2r\}; |E_1| = rs \\ E_2 &= \{uv \in E(T(K_{r,s})) : d_{T(K_{r,s})}(u) = 2s, d_{T(K_{r,s})}(v) = r+s\}; |E_2| = rs \\ E_3 &= \{uv \in E(T(K_{r,s})) : d_{T(K_{r,s})}(u) = 2r, d_{T(K_{r,s})}(v) = r+s\}; |E_3| = rs \\ E_4 &= \{uv \in E(T(K_{r,s})) : d_{T(K_{r,s})}(u) = d_{T(K_{r,s})}(v) = r+s\}; |E_4| = \frac{rs}{2}(r+s-2). \end{aligned}$$

$$\begin{aligned} \text{Hence, } HZ(T(K_{r,s})) &= \sum_{uv \in E(T(K_{r,s}))} [d_{T(K_{r,s})}(u) + d_{T(K_{r,s})}(v)]^2 \\ &= rs(2r+2s)^2 + rs(2s+r+s)^2 + rs(2r+r+s)^2 + \frac{rs}{2}(r+s-2)(2r+2s)^2 \\ &= rs(2r^3 + 2s^3 + 6r^2s + 6rs^2 + 10s^2 + 10r^2 + 12rs). \end{aligned}$$

Theorem 4.1: For any graph G with n vertices and m edges,

- (i) $B_1(L(G)) = 3M_1(L(G)) - 2M_1(G) + 4|E(G)|$
- (ii) $B_2(L(G)) = HZ(L(G)) - 2M_1(L(G))$
- (iii) $2B_1(L(G)) + 3B_2(L(G)) = 3HZ(L(G)) - 4M_1(G) + 8|E(G)|$.

Proof: (i) From Theorem 2.1, $B_1(G) = 3M_1(G) - 4|E(G)|$

$$\begin{aligned} B_1(L(G)) &= 3M_1(L(G)) - 4|E(L(G))| \\ &= 3M_1(L(G)) - 4 \times \frac{1}{2}\{M_1(G) - 2|E(G)|\} \\ B_1(L(G)) &= 3M_1(L(G)) - 2M_1(G) + 4|E(G)| \quad \dots \dots \dots (1) \end{aligned}$$

(ii) From Theorem 2.1, $B_2(G) = HZ(G) - 2M_1(G)$

$$B_2(L(G)) = HZ(L(G)) - 2M_1(L(G)) \quad \dots \dots \dots (2)$$

From (1) and (2), we have $2B_1(L(G)) + 3B_2(L(G)) = 3HZ(L(G)) - 4M_1(G) + 8|E(G)|$.

From Theorem 2.1, Propositions 3.1, 3.2, and Theorem 4.1 with their respective sections, the following results are obtained.

Theorem 4.2: Let G be any connected graph with $n \geq 2$ vertices. Then

- (i) $B_1(S(G)) = 3M_1(G) + 4|E(G)|$
- (ii) $B_2(S(G)) = HZ(S(G)) - 2|M_1(S(G))|$.

Theorem 4.3: Let G be any connected graph with $n \geq 2$ vertices. Then

- (i) $B_1(T_1(G)) = 12M_1(G)$
- (ii) $B_2(T_1(G)) = HZ(T_1(G)) - 8M_1(G) - 8|E(G)|$.

Theorem 4.4: Let G be any connected graph with $n \geq 2$ vertices, then

- (i) $B_1(T_2(G)) = 13M_1(G) + 3M_1(L(G)) - 16|E(G)|$
- (ii) $B_2(T_2(G)) = HZ(T_2(G)) - 2M_1(T_2(G))$.

Theorem 4.4: Let G be any connected graph with $n \geq 2$ vertices, then

- (i) $B_1(T(G)) = 10M_1(G) + 3M_1(L(G)) + 24|E(L(G))| + 4|E(G)|$
- (ii) $B_2(T_2(G)) = HZ(T(G)) - 2M_1(T(G))$.

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