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THE HYPER-ZAGREB INDEX OF SOME DERIVED GRAPHS

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ABSTRACT

Let $G = (V, E)$ be a connected graph. The hyper-Zagreb index is defined as $HZ(G) = \sum_{uv\in E(G)}[d_G(u) + d_G(v)]^2$. In this paper, comparison of the hyper-Zagreb index and other degree based topological indices like the Forgotten index, Zagreb and Banhatti indices of some derived graphs such as line graph, subdivision graph, vertex-semitotal graph, edge-semitotal graph and total graph are obtained. In addition, exact values of some standard graphs of above derived graphs are presented.

***Key words and phrases**: Hyper-Zagreb index; Line graph, Subdivision graph; Vertex-semitotal graph, Edge-semitotal graph, Total graph.*

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1. INTRODUCTION

All graphs considered in this paper are finite, connected, undirected without loops and multiple edges. Any undefined term in this paper may be found in Kulli [17]. Let G be a simple connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v will be denoted by uv. Let $d_G(e)$ denotes the degree of an edge e in G, which is defined by $d_G(e) = d_G(u)+d_G(v) - 2$ with $e = uv$.

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. In Chemical Science, the physico-chemical properties of chemical compounds are often modelled by means of molecular graph based structure descriptors, which are also referred to as topological indices, see [21].

The first two Zagreb indices was introduced by Gutman and Trinajstic [13] to take account of the contributions of pairs of adjacent vertices. For their history, applications, and mathematical properties, see [3], [9], [11], [12] and the references cited therein. The first and second Zagreb indices of G are defined as $M_1(G) = \sum_{v\in V(G)} d_G(v)^2$ or $M_1(G) = \sum_{uv\in E(G)} d_G(u) + d_G(v)$ and $M_2(G) = \sum_{uv\in E(G)}[d_G(u).d_G(v)]$. Followed by the First Zagreb index of a graph G, Shirdel *et al.* [8] was introduced the hyper-Zagreb index of G defined as $\mathrm{HZ}(G) = \sum_{uv\in E(G)}[d_G(u) + d_G(v)]^2$. In [5], Furtula and Gutman was introduced the so-called forgotten topological index F, defined as $\mathrm{F}(G) = \sum_{v\in V(G)} d_G(v)^3 = \sum_{uv\in E(G)} (d_G(u))^2 + (d(_G(v))^2$.

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In [18], Kulli introduced the first and second K Banhatti indices, intending to take into account the contributions of pairs of incident elements. The first and second K Banhatti indices of a graph G are defined as $B_1(G) = \sum_{ue} d_G(u) + d_G(e)$ and $B_2(G) = \sum_{ue} d_G(u) \times d_G(e)$, respectively, where ue means that the vertex u and edge e are incident in G. The Banhatti and Zagreb indices are closely related, see [10]. Recently many other indices were studied, for example, in [4] and [19].

2. EXISTING RESULTS OF DEGREE BASED INDICES

To prove our main results, we make use of the following results in sequel.

Theorem 2.1: [10] For any graph G, the first Banhatti index and second Banhatti indices are related to the first Zagreb index and Hyper Zagreb indices as

(i) $B_1(G) = 3M_1(G) - 4\,|E(G)|$

(ii) $B_2(G) = HZ(G) - 2M_1(G)$.

Let G be a standard graph of path P_n ; $n \geq 2$ and complete bipartite graph $K_{r,,s}$; $1 \leq r \leq s$. Also, in r-regular graph G, if r =2, then G is a cycle C_n ; $n \geq 3$ and if $r = n-1$, then G is a complete graph K_n; $n \geq 3$vertices.

Proposition 2.2: [3, 12] Let G be some standard class of graphs. Then

(i) $M_1(P_n) = 4n - 6$ for $n \geq 2$ vertices

(ii) $M_1(C_n) = 4n$ for $n \geq 3$ vertices

(iii) $M_1(K_n) = n(n-1)^2$ for $n \geq 3$ vertices

(iv) $M_1(G) = nr^2$, where G is a r-regular graph

(v) $M_1(K_{r,s}) = rs(r+s)$ for $1 \leq r \leq s$ vertices.

Proposition 2.3: [8] Let G be some standard class of graphs. Then

(i) $HZ(P_n) = 16n - 30$ for $n \geq 2$ vertices

(ii) $HZ(C_n) = 16n$ for $n \geq 3$ vertices

(iii) $HZ(K_n) = 2n(n-1)^3$ for $n \geq 3$ vertices

(iv) $HZ(G) = 2nr^3$, where G is a r-regular graph

(v) $HZ(K_{r,s}) = rs(r+s)^2$ for $1 \leq r \leq s$ vertices.

3. SOME DERIVED GRAPHS

The Line graph $L(G)$ is the graph with vertex set $V(L(G)) = E(G)$ and whose vertices correspond to the edges of G with two vertices being adjacent if and only if the corresponding edges in G have a vertex in common to, see [15].

The Subdivision graph $S(G)$ is the graph obtained from G by replacing each of its edges by a path of length two, or equivalently, by inserting an additional vertex into each edge of a graph G, see [14].

The Vertex-Semitotal graph $T_1(G)$ with vertex set $V(G) \cup E(G)$ and edge set $E(S(G)) \cup E(G)$ is the graph obtained from G by adding a new vertex corresponding to each edge of G and by joining each new vertex to the end vertices of the edge corresponding to it, see [6].

The Edge-Semitotal graph $T_2(G)$ with vertex set $V(G) \cup E(G)$ and edge set $E(S(G)) \cup E(L(G))$ is the graph obtained from G by inserting a new vertex into each edge of G and by joining with edges those pairs of these new vertices which lie on adjacent edges of G, see [7].

The Total graph of a graph G denoted by $T(G)$ with vertex set V(G) ∪ E(G) and any two vertices of $T(G)$ are adjacent if and only if they are either incident or adjacent in G, see [2].

Different Topological indices of some derived graphs have been studied by Basavanagoud *et al.* [2], Khalifeh *et al.* [16] and Nilanjan De [20]. In view of these references, some of the existing results as follows.

Proposition 3.1:

1) Let $L(G)$ be the line graph of a graph G. Then

(i) $V(L(G)) = E(G)$

(ii) $|E(L(G))| = \frac{1}{2}M_1(G) - |E(G)|$.

2) Let $S(G)$ be the subdivision graph of a graph G. Then

(i) $M_1(S(G)) = M_1(G) + 4|E(G)|$

(ii) $|E(S(G))| = 2|E(G)|$.

3) Let $T_1(G)$ be the vertex semi-total graph of a graph G. Then
   (i) $M_1(T_1(G)) = 4\, M_1(G) + 4|E(G)|$
   (ii) $|E(T_1(G))| = 3|E(G)|$.
4) Let $T_1(G)$ be the edge semi-total graph of a graph G. Then
   (i) $M_1(T_2(G)) = M_1(G) + M_1\big(L(G)\big) + 8\big|E\big(L(G)\big)\big| + 8|E(G)|$
   (ii) $|E(T_2(G))| = |E(G)| + \frac{1}{2}M_1(G)$.
5) Let $T(G)$ be the total graph of a graph G. Then
   (i) $M_1(T(G)) = 4M_1(G) + M_1\big(L(G)\big) + 8\big|E\big(L(G)\big)\big| + 4|E(G)|$
   (ii) $|E(T_2(G))| = 2|E(G)| + \frac{1}{2}M_1(G)$.

Proposition 3.2: Let G be a graph of order n and size m. Then
(i) $HZ(G) = F(G) + 2\,M_2(G)$
(ii) $F(S(G)) = F(G) + 8|E(G)|$
(iii) $M_2(S(G)) = 2M_1(G)$.

4. MAIN RESULTS

Proposition 4.1:
1) Let $S(G)$ be the subdivision graph of a graph G. Then
   (i) $M_1(S(G)) = nr(r+2)$, where G is a r-regular graph
   (ii) $M_1(S(P_n)) = 8n - 10$
   (iii) $M_1\big(S(K_{r,s})\big) = rs(r+s+4)$ for $1 \le r \le s$ vertices.
2) Let $T_1(G)$ be the vertex semi-total graph of a graph G. Then
   (i) $M_1(T_1(G)) = 2nr(2r+1)$, where G is a r-regular graph
   (ii) $M_1(T_1(P_n)) = 20n - 28$
   (iii) $M_1\big(T_1(K_{r,s})\big) = 4rs(r+s+1)$ for $1 \le r \le s$ vertices.
3) Let $T_2(G)$ be the edge semi-total graph of a graph G. Then
   (i) $M_1(T_2(G)) = 2nr^3 + nr^2$, where G is a r-regular graph
   (ii) $M_1(T_2(P_n)) = 20n - 36$
   (iii) $M_1\big(T_2(K_{r,s})\big) = rs(r^2 + s^2 + r + s + 2rs)$ for $1 \le r \le s$ vertices.
4) Let $T(G)$ be a total graph of a graph G. Then
   (i) $M_1(T(G)) = 2nr^2(r+2)$, where G is a r-regular graph
   (ii) $M_1(T(P_n)) = 32n - 54$
   (iii) $M_1\big(T(K_{r,s})\big) = rs(r^2 + s^2 + 4r + 4s + 2rs)$ for $1 \le r \le s$ vertices.

Proof: From Propositions 2. 2 and 3.1, the results are immediate.

Proposition 4.2:
(i) $HZ\big(L(G)\big) = 8nr(r-1)^3$, where G is a r-regular graph
(ii) $HZ\Big(L\big(K_{r,s}\big)\Big) = 2rs(r+s-2)^3$ for $1 \le r \le s$ vertices.
(iii) $HZ\big(S(G)\big) = nr(r+2)^2$, where G is a r-regular graph.
(iv) $HZ\big(S(P_n)\big) = 32n - 46$
(v) $HZ\Big(S\big(K_{r,s}\big)\Big) = rs(r^2 + s^2 + 4r + 4s + 8)$ for $1 \le r \le s$ vertices.
(vi) $HZ\big(T_1(G)\big) = 4nr(r+1)^2 + 8nr^3$, where G is a r-regular graph.
(vii) $HZ\big(T_1(P_n)\big) = 136n - 232$
(viii) $HZ\Big(T_1\big(K_{r,s}\big)\Big) = 4rs(2r^2 + 2s^2 + 2r + 2s + 2rs + 2)$ for $1 \le r \le s$ vertices
(ix) $HZ\big(T_2(G)\big) = 9nr^3 + 8nr^3(r-1)$, where G is a r-regular graph
(x) $HZ\big(T_2(P_n)\big) = 136n - 292$
(xi) $HZ\Big(T_2\big(K_{r,s}\big)\Big) = rs(2r^3 + 2s^3 + 2r^2s + 2rs^2 + r^2 + s^2)$ for $1 \le r \le s$ vertices
(xii) $HZ\big(T(G)\big) = 8nr^3(r+2)$, where G is a r-regular graph.
(xiii) $HZ\big(T(P_n)\big) = 256n - 514$
(xiv) $HZ\Big(T\big(K_{r,s}\big)\Big) = rs(2r^3 + 2s^3 + 6r^2s + 6rs^2 + 10s^2 + 10r^2 + 12rs)$ for $1 \le r \le s$ vertices.

Proof:
(i) We have, $HZ\big(L(G)\big) = \sum_{uv\in E(L(G))} [d_{L(G)}(u) + d_{L(G)}(v)]^2$. Since the line graph of a r - regular graph is $(2r-2)$ – regular. Hence $HZ\big(L(G)\big) = \frac{nr}{2}(r-1)(4r-4)^2 = 8nr(r-1)^3$.

(ii) Since the line graph of complete bipartite graph $K_{r,\ s}$ is a $(r+s-2)$-regular graph and $\left|E(L(K_{r,s})\right| = \frac{rs}{2}\,(r+s-2)$. Hence the result follows.

(iii) We have, $HZ\big(S(G)\big) = \sum_{uv\in E(S(G))}\,[d_{S(G)}(u) + d_{S(G)}(v)]^2$. In $S(G)$, there is an edge partition $E_1 = \{uv \in E\big(S(G)\big): d_{S(G)}(u) = 2, d_{S(G)}(v) = r\}; |E_1| = nr$.
Therefore $HZ\big(S(G)\big) = nr(r+2)^2$.

(iv) In $S(P_n)$, there are two edge partitions,
$E_1 = \{uv \in E\big(S(P_n)\big): d_{S(P_n)}(u) = 1, d_{S(P_n)}(v) = 2\}; |E_1| = 2$
$E_2 = \{uv \in E\big(S(P_n)\big): d_{S(P_n)}(u) = 2, d_{S(P_n)}(v) = 2\}; |E_2| = 2(n-2)$.
Therefore, $HZ\big(S(P_n)\big) = \sum_{uv\in E(S(P_n))}\,\left[d_{S(P_n)}(u) + d_{S(P_n)}(v)\right]^2$
$= 2(1+2)^2 + 2(n-2)(2+2)^2 = 32n - 46$.

(v) In $S\big(K_{r,\ s}\big)$, there are two edge partitions,
$E_1 = \{uv \in E\left(S\big(K_{r,s}\big)\right): d_{S(K_{r,s})}(u) = r, d_{S(K_{r,s})}(v) = 2\right\}; |E_1| = rs$
$E_2 = \{uv \in E\left(S\big(K_{r,s}\big)\right): d_{S(K_{r,s})}(u) = s, d_{S(K_{r,s})}(v) = 2\right\}; |E_2| = rs$.
Therefore $HZ\left(S\big(K_{r,s}\big)\right) = \sum_{uv\in E\left(S(K_{r,s})\right)}\,\left[d_{S(K_{r,s})}(u) + d_{S(K_{r,s})}(v)\right]^2$
$= rs(r+2)^2 + rs(s+2)^2$
$= rs(r^2 + s^2 + 4r + 4s + 8)$.

(vi) In $T_1(G)$, there are two edge partitions,
$E_1 = \{uv \in E\big(T_1(G)\big): d_{T_1(G)}(u) = 2, d_{ST_1(G)}(v) = 2r\}; |E_1| = nr$
$E_2 = \{uv \in E\big(T_1(G)\big): d_{T_1(G)}(u) = d_{T_1(G)}(v) = 2r\}; |E_2| = \frac{nr}{2}$.
Therefore $HZ\big(T_1(G)\big) = \sum_{uv\in E(T_1(G))}\,\left[d_{T_1(G)}(u) + d_{T_1(G)}(v)\right]^2$
$= nr(2+2r)^2 + \frac{nr}{2}\,(2r+2r)^2$
$= 4nr(r+1)^2 + 8nr^3$.

(vii) In $T_1(P_n)$, there are three edge partitions,
$E_1 = \{uv \in E\big(T_1(P_n)\big): d_{T_1(P_n)}(u) = 2, d_{T_1(P_n)}(v) = 2\}; |E_1| = 2$
$E_2 = \{uv \in E\big(T_1(P_n)\big): d_{T_1(P_n)}(u) = 2, d_{T_1(P_n)}(v) = 4\}; |E_2| = 2(n-1)$
$E_3 = \{uv \in E\big(T_1(P_n)\big): d_{T_1(P_n)}(u) = d_{T_1(P_n)}(v) = 4\}; |E_3| = n-3$.
Therefore $HZ\big(T_1(P_n)\big) = \sum_{uv\in E(T_1(P_n))}\,\left[d_{T_1(P_n)}(u) + d_{T_1(P_n)}(v)\right]^2$
$= 2(2+2)^2 + 2(n-1)(2+4)^2 + (n-3)(4+4)^2$
$= 136n - 232$.

(viii) In $T_1\big(K_{r,s}\big)$, there are three edge partitions,
$E_1 = \{uv \in E\left(T_1\big(K_{r,s}\big)\right): d_{T_1(K_{r,s})}(u) = 2r, d_{T_1(K_{r,s})}(v) = 2\right\}; |E_1| = rs$
$E_2 = \{uv \in E\left(T_1\big(K_{r,s}\big)\right): d_{T_1(K_{r,s})}(u) = 2s, d_{T_1(K_{r,s})}(v) = 2\right\}; |E_2| = rs$
$E_3 = \{uv \in E\left(T_1\big(K_{r,s}\big)\right): d_{T_1(K_{r,s})}(u) = 2r, d_{T_1(K_{r,s})}(v) = 2s\right\}; |E_3| = rs$.
Hence, $HZ\left(T_1\big(K_{r,s}\big)\right) = \sum_{uv\in E\left(T_1(K_{r,s})\right)}\,\left[d_{T_1(K_{r,s})}(u) + d_{T_1(K_{r,s})}(v)\right]^2$
$= rs(2r+2)^2 + rs(2s+2)^2 + rs(2s+2s)^2$
$= 4rs(2r^2 + 2s^2 + 2r + 2s + 2rs + 2)$.

(ix) In $T_2(G)$, there are two edge partitions,
$E_1 = \{uv \in E\big(T_2(G)\big): d_{T_2(G)}(u) = r, d_{T_2(G)}(v) = 2r\}; |E_1| = nr$
$E_2 = \{uv \in E\big(T_1(G)\big): d_{T_2(G)}(u) = d_{T_2(G)}(v) = 2r\}; |E_2| = \frac{nr}{2}(r-1)$.
Therefore $HZ\big(T_2(G)\big) = \sum_{uv\in E(T_2(G))}\,\left[d_{T_2(G)}(u) + d_{T_2(G)}(v)\right]^2$
$= nr(r+2r)^2 + \frac{nr}{2}(r-1)(2r+2r)^2$
$= 9nr^3 + 8nr^3(r-1)$.

(x) In $T_2(P_n)$, there are five edge partitions,
$E_1 = \{uv \in E\big(T_2(P_n)\big): d_{T_2(P_n)}(u) = 1, d_{T_2(P_n)}(v) = 3\}; |E_1| = 2$
$E_2 = \{uv \in E\big(T_2(P_n)\big): d_{T_2(P_n)}(u) = 2, d_{T_2(P_n)}(v) = 3\}; |E_2| = 2$
$E_3 = \{uv \in E\big(T_2(P_n)\big): d_{T_2(P_n)}(u) = 2, d_{T_2(P_n)}(v) = 4\}; |E_3| = 2(n-3)$
$E_4 = \{uv \in E\big(T_2(P_n)\big): d_{T_2(P_n)}(u) = 3, d_{T_2(P_n)}(v) = 4\}; |E_4| = 2$
$E_5 = \{uv \in E\big(T_2(P_n)\big): d_{T_2(P_n)}(u) = 4, d_{T_2(P_n)}(v) = 4\}; |E_5| = n-4$.

Hence, $HZ(T_2(P_n)) = \sum_{uv\in E(T_2(P_n))} \left[d_{T_2(P_n)}(u) + d_{T_2(P_n)}(v)\right]^2$
$= 2(1+3)^2 + 2(2+3)^2 + 2(n-3)(2+4)^2 + 2(3+4)^2 + (n-4)(4+4)^2$
$= 136n - 292.$

(xi) In $T_2(K_{r,s})$, there are three edge partitions,

$E_1 = \{uv \in E\left(T_2(K_{r,s})\right): d_{T_2(K_{r,s})}(u) = r, d_{T_2(K_{r,s})}(v) = r+s\}; |E_1| = rs$

$E_2 = \{uv \in E\left(T_2(K_{r,s})\right): d_{T_2(K_{r,s})}(u) = s, d_{T_2(K_{r,s})}(v) = r+s\}; |E_2| = rs$

$E_3 = \{uv \in E\left(T_2(K_{r,s})\right): d_{T_2(K_{r,s})}(u) = d_{T_2(K_{r,s})}(v) = r+s\}; |E_3| = \frac{rs}{2}(r+s-2).$

Hence, $HZ\left(T_2(K_{r,s})\right) = \sum_{uv\in E(T_2(K_{r,s}))} \left[d_{T_2(K_{r,s})}(u) + d_{T_2(K_{r,s})}(v)\right]^2$
$= rs(r+r+s)^2 + rs(s+r+s)^2 + \frac{rs}{2}(r+s-2)(2r+2s)^2$
$= rs(2r^3 + 2s^3 + 2r^2s + 2rs^2 + r^2 + s^2).$

(xii) In $T(G)$, there is one edge partition,

$E_1 = \{uv \in E(T(G)): d_{T(G)}(u) = 2r, d_{T(G)}(v) = 2r\}; |E_1| = \frac{nr^2}{2} + nr$

Hence, $HZ(T(G)) = \sum_{uv\in E(T(G))} \left[d_{T(G)}(u) + d_{T(G)}(v)\right]^2$
$= \left(\frac{nr^2}{2} + nr\right)(2r+2r)^2$
$= 8nr^3(r+2).$

(xiii) In $T(P_n)$, there are four edge partitions,

$E_1 = \{uv \in E(T(P_n)): d_{T(P_n)}(u) = 2, d_{T(P_n)}(v) = 3\}; |E_1| = 2$

$E_2 = \{uv \in E(T(P_n)): d_{T(P_n)}(u) = 2, d_{T(P_n)}(v) = 4\}; |E_2| = 2$

$E_3 = \{uv \in E(T(P_n)): d_{T(P_n)}(u) = 3, d_{T(P_n)}(v) = 4\}; |E_3| = 4$

$E_4 = \{uv \in E(T(P_n)): d_{T(P_n)}(u) = 4, d_{T(P_n)}(v) = 4\}; |E_4| = 4n - 13.$

Hence, $HZ(T(P_n)) = \sum_{uv\in E(T(P_n))} \left[d_{T(P_n)}(u) + d_{T(P_n)}(v)\right]^2$
$= 2(2+3)^2 + 2(2+4)^2 + 4(3+4)^2 + (4n-13)(4+4)^2$
$= 256n - 514.$

(xiv) In $(K_{r,s})$, there are three edge partitions,

$E_1 = \{uv \in E\left(T(K_{r,s})\right): d_{T(K_{r,s})}(u) = 2s, d_{T(K_{r,s})}(v) = 2r\}; |E_1| = rs$

$E_2 = \{uv \in E\left(T(K_{r,s})\right): d_{T(K_{r,s})}(u) = 2s, d_{T(K_{r,s})}(v) = r+s\}; |E_2| = rs$

$E_3 = \{uv \in E\left(T(K_{r,s})\right): d_{T(K_{r,s})}(u) = 2r, d_{T(K_{r,s})}(v) = r+s\}; |E_3| = rs$

$E_4 = \{uv \in E\left(T(K_{r,s})\right): d_{T(K_{r,s})}(u) = d_{T(K_{r,s})}(v) = r+s\}; |E_4| = \frac{rs}{2}(r+s-2).$

Hence, $HZ\left(T(K_{r,s})\right) = \sum_{uv\in E(T(K_{r,s}))} \left[d_{T(K_{r,s})}(u) + d_{T(K_{r,s})}(v)\right]^2$
$= rs(2r+2s)^2 + rs(2s+r+s)^2 + rs(2r+r+s)^2 + \frac{rs}{2}(r+s-2)(2r+2s)^2$
$= rs(2r^3 + 2s^3 + 6r^2s + 6rs^2 + 10s^2 + 10r^2 + 12rs).$

Theorem 4.1: For any graph G with n vertices and m edges,

(i) $B_1(L(G)) = 3M_1(L(G)) - 2M_1(G) + 4\,|E(G)|$
(ii) $B_2(L(G)) = HZ(L(G)) - 2M_1(L(G))$
(iii) $2B_1(L(G)) + 3B_2(L(G)) = 3HZ(L(G)) - 4M_1(G) + 8\,|E(G)|$.

Proof: (i) From Theorem 2.1, $B_1(G) = 3M_1(G) - 4\,|E(G)|$

$B_1(L(G)) = 3M_1(L(G)) - 4\,|E(L(G))|$
$= 3M_1(L(G)) - 4 \times \frac{1}{2}\{M_1(G) - 2\,|E(G)|\}$
$B_1(L(G)) = 3M_1(L(G)) - 2M_1(G) + 4\,|E(G)|$ ………….(1)

(ii) From Theorem 2.1, $B_2(G) = HZ(G) - 2M_1(G)$

$B_2(L(G)) = HZ(L(G)) - 2M_1(L(G))$ ……………………….(2)

From (1) and (2), we have $2B_1(L(G)) + 3B_2(L(G)) = 3HZ(L(G)) - 4M_1(G) + 8\,|E(G)|$.

From Theorem 2.1, Propositions 3.1, 3.2, and Theorem 4.1 with their respective sections, the following results are obtained.

Theorem 4.2: Let G be any connected graph with $n \geq 2$ vertices. Then

(i) $B_1(S(G)) = 3\, M_1(G) + 4|E(G)|$

(ii) $B_2(S(G)) = HZ\big(S(G)\big) - 2|M_1(S(G))|$.

Theorem 4.3: Let G be any connected graph with $n \geq 2$ vertices. Then

(i) $B_1(T_1(G)) = 12M_1(G)$

(ii) $B_2(T_1(G)) = HZ\big(T_1(G)\big) - 8\, M_1(G) - 8\, |E(G)|$.

Theorem 4.4: Let G be any connected graph with $n \geq 2$ vertices, then

(i) $B_1(T_2(G)) = 13M_1(G) + 3M_1(L(G)) - 16|E(G)|$

(ii) $B_2(T_2(G)) = HZ\big(T_2(G)\big) - 2\, M_1(T_2(G))$.

Theorem 4.4: Let G be any connected graph with $n \geq 2$ vertices, then

(i) $B_1(T(G)) = 10M_1(G) + 3M_1(L(G)) + 24|E(L(G))| + 4|E(G)|$

(ii) $B_2(T_2(G)) = HZ\big(T(G)\big) - 2\, M_1(T(G))$.

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