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BIANCHI TYPE-V WET DARK FLUID COSMOLOGICAL MODEL IN SCALAR TENSOR THEORY OF GRAVITATION

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ABSTRACT

In this paper, Bianchi type-V metric is considered in the presence of wet dark fluid in scalar-tensor theory of gravitation formulated by Saez and Ballester (Phys. Lett. A 113:467, 1986). The exact solutions are obtained from the corresponding field equations in the form of spacial volume V. Some physical properties like expansion factor θ, shear scalar σ and the deceleration parameter q are also discussed.

Keywords: *Bianchi type-V space-time; Saez-Ballester theory of gravitation; Wet dark fluid.*

1. INTRODUCTION

General theory of Relativity proposed by Einstein is one of the most beautiful structures of theoretical physics which is also known as the most successful theory of gravitation in term of geometry. In the last decade, several alternative theories of gravitation have been proposed as alternatives to the theory of General Relativity. The most important among them are scalar tensor theories of gravitation proposed by Saez and Ballester [1] and Brans and Dicke [2]. Brans-Dicke theory introduces an additional scalar field ϕ besides the metric tensor g_{ij} and a dimensionless coupling constant ω while in Saez-Ballester scalar-tensor theory, the metric is coupled with a dimensionless scalar field ϕ in a simple manner. This coupling gives a satisfactory description of the weak fields. In spite of the dimensionless character of the scalar field, an antigravity regime appears. This theory also suggests a possible way to solve missing matter problem in non-flat FRW cosmologies. Saez [3], Shri Ram and Singh [4], Reddy and Rao [5] have investigated several aspects of this theory. Reddy *et al*. [6], Rao *et al*. [7] Singh and Choubey [8], Katore *et al.* [9]**,** Reddy *et al.* [10] are some of the authors who have investigated several aspects of the cosmological models in Saez–Ballester scalar–tensor theory. Recently Reddy *et al*. [11] had studied a spatially homogeneous and anisotropic Bianchi type-V cosmological model in a scalar-tensor theory of gravitation proposed by Saez and Ballester. Furthermore there has been considerable interest in cosmological models with dark energy because of the fact that our universe is currently undergoing an accelerated expansion supposedly, driven by an exotic dark energy which has been confirmed by a host of observations, such as type 1a supernovae [12]. Based on these observations, cosmologists have accepted the idea of dark energy, which is a fluid with negative pressure making up around 70% of the present universe energy content to be responsible for this acceleration due to repulsive gravitation. These observational data also suggest that the universe is dominated by two dark components containing dark matter and dark energy. Dark matter, a matter without pressure, is mainly used to explain galastic curves and large scale structure formation, while dark energy, an exotic energy with negative pressure is used to explain the present cosmic accelerating expansion. The most interesting problem in modern astrophysics and cosmology is to know the behaviour of dark energy. Here we have used an empirical equation of state proposed by Tait [13] and Hayward [14] to treat water an aqueous solution. Several authors such as Ray *et al*.[15], Saha [16], Sahni, V[17] Singh and Chaubey [18], Chaubey [19], Tade and Sambhe [20], Jain *et al.* [21], Samanta e*t al.*[22], Chirde and Rahate [23], Vinutha *et al*.[24], Rao and Divya Prasanthi [25] have studied dark energy models in recent years.

The equation of state for wet dark fluid is

$$P_{WDF} = \gamma(\rho_{WDF} - \rho_*), \tag{1}$$

and is motivated by the fact that it is good approximation for many fluids, including water, in which initial attraction of the molecules makes negative pressure possible. Here p_{WDF}, ρ_{WDF} is pressure and energy density of wet dark fluid respectively.

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The parameters γ and ρ_* are taken to be positive and we restrict ourselves to $0 \leq \gamma \leq 1$. To find the wet dark fluid energy density, we use the energy conservation equation

$$\dot{p}_{WDF} + 3H(P_{WDF} + \rho_{WDF}) = 0. \tag{2}$$

Using the equation (1) and relation $3H = \frac{\dot{V}}{V}$ in above equation (2), we get

$$\rho_{WDF} = \frac{\gamma}{\gamma+1}\rho_* + \frac{D}{V^{(1+\gamma)}}, \tag{3}$$

where D is the constant of integration and V is volume expansion.

Wet dark fluid naturally include two components: one component behaves as cosmological constant and other component act as the standard fluid with an equation of state

$$P = \gamma\rho .$$

We can show that, if $D > 0$, this fluid will not violet the strong condition p+$\rho \geq 0$.
Thus we get

$$P_{WDF} + \rho_{WDF} = (1+\gamma)\rho_{WDF} - \gamma\rho_* = (1+\gamma)\frac{D}{V^{(1+\gamma)}} \geq 0. \tag{4}$$

In this paper, we consider Bianchi type-V cosmological model in Saez-Ballester theory of gravitation with dark energy as a fluid satisfying equation of state (1). The solution has been obtained in the quadrature form. We also discuss the models for $\gamma = 0$ and $\gamma = 1$.

2. METRIC AND FIELD EQUATIONS

We consider Bianchi type-V space-time

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}[B^2(t)dy^2 + C^2(t)dz^2], \tag{5}$$

where *A, B, C* are functions of t and m is a constant.

The field equations for combined scalar and tensor field proposed by Saez-Ballester are

$$G_{ij} - \omega\varphi^n\left(\varphi_{,i}\varphi_{,j} - \frac{1}{2}g_{ij}\varphi_{,k}\varphi^{,k}\right) = -T_{ij}\,, \tag{6}$$

where $G_{ij} = \left(R_{ij} - \frac{1}{2}g_{ij}R\right)$ and $8\pi G = C = 1$.

The scalar field ϕ satisfies the equation

$$2\phi^n\phi^{;j}_{;i} + n\phi^{n-1}\phi^{,k}_{,k} = 0 \quad . \tag{7}$$

Also the energy momentum tensor of the source is given by

$$T_i^j = (p_{WDF} + \rho_{WDF})u_i\, u^j - p_{WDF}\delta_i^j, \tag{8}$$

where u^j is four velocity vector of the fluid satisfying $g_{ij}u_iu^j = 1$.

In co-moving system of co-ordinates, we find

$$T_0^0 = \rho_{WDF}, T_1^1 = T_2^2 = T_3^3 = -P_{WDF}\,. \tag{9}$$

Now with the help of equation (9), the field equations (6) for metric (5) are

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - 3\frac{m^2}{A^2} = \rho_{WDF} - \frac{\omega}{2}\varphi^n\dot{\varphi}^2, \tag{10}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -p_{WDF} + \frac{\omega}{2}\varphi^n\dot{\varphi}^2, \tag{11}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{m^2}{A^2} = -p_{WDF} + \frac{\omega}{2}\varphi^n\dot{\varphi}^2, \tag{12}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -p_{WDF} + \frac{\omega}{2}\varphi^n\dot{\varphi}^2, \tag{13}$$

Also $$\ddot{\varphi}+\dot{\varphi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\frac{n}{2\varphi}\dot{\varphi}^2=0, \tag{14}$$

where over dot denotes differentiation with respect to *t* and *m* is constant.

Again from (11), (12) and (13) we get

$$\frac{\ddot{A}}{A}-\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}-\frac{\dot{B}}{B}\frac{\dot{C}}{C}=0\cdot \tag{15}$$

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}-\frac{\dot{A}}{A}\frac{\dot{C}}{C}=0\cdot \tag{16}$$

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}-\frac{\dot{B}}{B}\frac{\dot{C}}{C}=0\cdot \tag{17}$$

Let the special volume *V* be the function of *t* defined by

$$V=a^3=ABC\ . \tag{18}$$

Then $$\frac{\dot{V}}{V}=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C} \tag{19}$$

and $$\frac{\ddot{V}}{V}=\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+2\frac{\dot{A}}{A}\frac{\dot{B}}{B}+2\frac{\dot{B}}{B}\frac{\dot{C}}{C}+2\frac{\dot{A}}{A}\frac{\dot{C}}{C}. \tag{20}$$

From field equations (15) to (17) we obtained

$$\frac{A}{C}=d_1\exp(x_1\int\frac{1}{V}dt), \tag{21}$$

$$\frac{B}{C}=d_2\exp(x_2\int\frac{1}{V}dt)\ , \tag{22}$$

and $$\frac{A}{B}=d_3\exp(x_3\int\frac{1}{V}dt). \tag{23}$$

Here $d_1, d_{2,}d_3$ and x_1, x_2, x_3 are constants.

In view of (18), equations (21), (22) and (23) lead to

$$A=D_1V^{\frac{1}{3}}\exp(X_1\int\frac{1}{V}dt), \tag{24}$$

$$B=D_2V^{\frac{1}{3}}\exp(X_2\int\frac{1}{V}dt), \tag{25}$$

$$C=D_3V^{\frac{1}{3}}\exp(X_3\int\frac{1}{V}dt), \tag{26}$$

where $D_i(i=1,2,3)$ and $X_i=(i=1,2,3)$ satisfies the relation

$$X_1+X_2+X_3=0 \text{ and } D_1D_2D_3=1. \tag{27}$$

The dimensionless scalar field ϕ, from equation (14) is founded as

$$\phi=\left[\frac{\phi_0(n+2)}{2}\int\frac{1}{V}dt\right]^{2/_{n+2}}, \tag{28}$$

where ϕ_0 is constant of integration .

From equations (10), (11), (12) and (13) we obtained

$$\frac{\ddot{V}}{V}-6\frac{m^2}{A^2}=\frac{3}{2}(\rho_{WDF}-P_{WDF})\cdot \tag{29}$$

The conservation law for the energy momentum tensor gives

$$\dot{\rho}=-\frac{\dot{V}}{V}(P_{WDF}+\rho_{WDF}), \tag{30}$$

Case-I: When BC=V

In this case, equation (29) reduces to

$$\frac{\ddot{V}}{V} - 6m^2 = \frac{3}{2}(\rho_{WDF} - P_{WDF}) \quad . \tag{31}$$

From equations (30) and (31), on simplification,

$$\dot{V} = \pm\sqrt{C_1 + 3V^2(\rho_{WDF} + 2m^2)} \quad . \tag{32}$$

where C_1 being an integration constant.

Rewriting equation (30) in the form

$$\frac{\dot{\rho}}{\rho_{WDF} + P_{WDF}} = -\frac{\dot{V}}{V}, \tag{33}$$

and taking into account that pressure and energy density obeying an equation of state of the type $p_{WDF} = f(\rho_{WDF})$, we conclude that p_{WDF} and ρ_{WDF} are both function of V and hence the right hand side of equation (31) is the function of V only.

$$\ddot{V} = \frac{3}{2}(\rho_{WDF} - P_{WDF})V + 6m^2 V = F(V) . \tag{34}$$

From mechanical point of view, equation (32) can be interpreted as equation of motion of single particle with unit mass under force F (*V*) then

$$\dot{V} = \sqrt{2(\in - \mu(V))} . \tag{35}$$

Here ε can be viewed as energy and $U(V)$as the potential of the force F.

Comparing (32) and (35), we find

$$\in = \frac{C_1}{2} \text{ and } \mu(V) = -\frac{3}{2}V^2(\rho_{WDF} + 2m^2) . \tag{36}$$

Finally, we write the solution to equation (32) in quadrature form

$$\frac{dv}{\sqrt{C_1 + 3V^2(\rho_{WDF} + 2m^2)}} = t + t_0 , \tag{37}$$

where integration constant t_0 can be zero, since it only gives shift in time.

Now from equation (3) and (37), we obtain

$$\frac{dv}{\sqrt{\left(\frac{3\gamma\rho_*}{\gamma+1} + 6m^2\right)V^2 + 3DV^{1-\gamma} + C_1}} = t + t_0 . \tag{38}$$

3. SOME PARTICULAR CASES

Case-(i): $\gamma = 0$ (dust universe)

In this case, equation (38) reduces to

$$\frac{dv}{\sqrt{6m^2V^2 + 3DV + C_1}} = t \tag{39}$$

which after integration and simplification gives

$$V = \alpha \sinh t - \beta . \tag{40}$$

where $\alpha = \sqrt{C_1 - \frac{3D^2}{8m^2}}$ and $\beta = \frac{D}{4m^2}$ are constants.

Here again arise two sub cases:

Case-(a): when $\alpha > 0$

Then for small *t* (i.e. near singularity *t=0*) $\sinh t \approx t$.

So, equation (40) reduces to

$$V = \alpha t - \beta \ . \tag{41}$$

Using (41) and after integration and simplification equations (24), (25) and (26) becomes

$$A = D_1(\alpha t - \beta)^{\frac{\alpha+3X_1}{3\alpha}} e^{aX_1}, \tag{42}$$

$$B = D_2(\alpha t - \beta)^{\frac{\alpha+3X_2}{3\alpha}} e^{bX_2}, \tag{43}$$

$$C = D_3(\alpha t - \beta)^{\frac{\alpha+3X_3}{3\alpha}} e^{cX_3}, \tag{44}$$

where *a*, *b*, *c* are constants of integration and X_i (i=1, 2, 3) and D_2 (i=1, 2, 3) satisfies the relation

$$X_1 + X_2 + X_3 = 0 \quad \text{and} \quad D_1 D_2 D_3 = 1 \ .$$

Also, using equation (41) in equation (28), after integration and simplification we obtained

$$\phi = \left[\frac{\phi_0 (n+2)}{2} \log(\alpha t - \beta)^{\frac{1}{\alpha}} + \phi_1 \right]^{2/n+2} . \tag{45}$$

where $\emptyset_1$ are constants of integration.

Similarly, using equation (41) and (3) we obtained

$$\rho_{WDF} = 4\beta m^2 (\alpha t - \beta)^{-1}. \tag{46}$$

Also, for γ = 0 equation (1) reduces to

$$P_{WDF} = 0 \ . \tag{47}$$

Now we can express the physical quantities such as expansion factor *θ*, shear scalar *σ* and the deceleration parameter *q* by using equation (41) as fallows

$$\theta = \frac{\alpha}{(\alpha t - \beta)}, \tag{48}$$

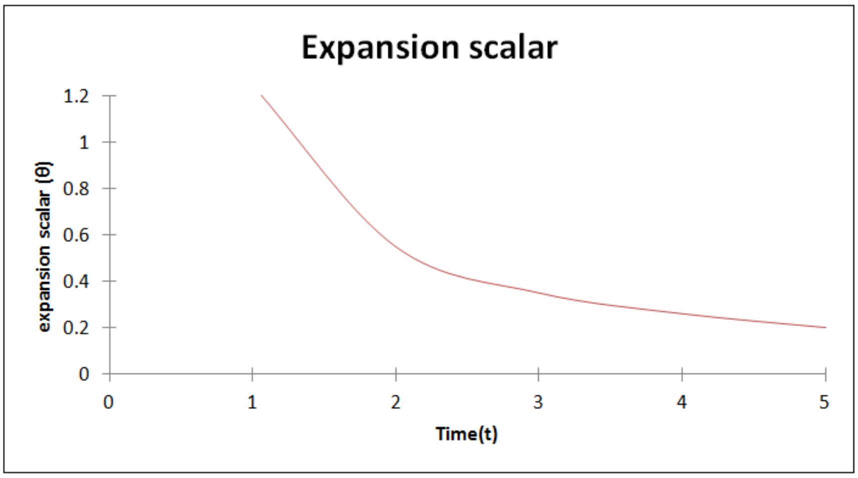

Figure-1: The plot of scalar expansion (*θ*) verses time (*t*), *α=5, β=1*.

From the figure 1, it is observed that, at an initial time, scalar expansion diverges which shows that it possess initial singularity. Further as cosmic time *t* increases gradually, expansion scalar decreases and finally vanishes as $t \to \infty$.

$$\sigma^2 = \frac{\alpha}{3(\alpha t - \beta)^2}, \tag{49}$$

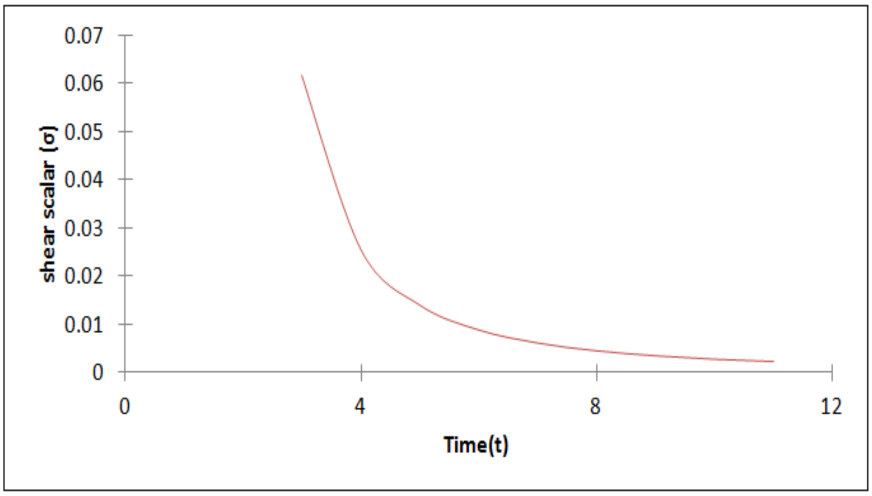

Figure-2: The plot of shear scalar (σ) verses time (t), $\alpha=5,\ \beta=1$.

From figure 2, it can be observed that for large value of t, shear dies out.

$$q = 2 \quad . \tag{50}$$

Positive constant value of deceleration parameter implies that model decelerated in standard ways.

Case-(b): when $\alpha < 0$

In this case, equation (38) reduces to

$$V = \alpha \cosh t - \beta \quad . \tag{51}$$

Then for small value of t (near singularity t =0), $\cosh t = 1 + t^2$.

So, (51) reduces to

$$V = \alpha(1+t^2) - \beta . \tag{52}$$

Using equation (52), equations (24), (25), (26) after integration and simplification becomes

$$A = D_1\left[\alpha(1+t^2)-\beta\right]^{\frac{1}{3}} \exp\left(X_1 \sqrt{\frac{1}{\alpha(\alpha-\beta)}} \tan^{-1}\sqrt{\frac{\alpha}{\alpha-\beta}}t + a \right), \tag{53}$$

$$B = D_2\left[\alpha(1+t^2)-\beta\right]^{\frac{1}{3}} \exp\left(X_2 \sqrt{\frac{1}{\alpha(\alpha-\beta)}} \tan^{-1}\sqrt{\frac{\alpha}{\alpha-\beta}}t + b \right), \tag{54}$$

$$C = D_3\left[\alpha(1+t^2)-\beta\right]^{\frac{1}{3}} \exp\left(X_3 \sqrt{\frac{1}{\alpha(\alpha-\beta)}} \tan^{-1}\sqrt{\frac{\alpha}{\alpha-\beta}}t + c \right), \tag{55}$$

where X_i (i=1, 2, 3) and D_i (i=1, 2, 3) satisfies the relation

$$X_1 + X_2 + X_3 = 0 \quad \text{and} \quad D_1 D_2 D_3 = 1 .$$

Also, using Equation (52) in equation (28), after integration and simplification we obtained

$$\phi = \left[\frac{\phi_0 (n+2)}{2\sqrt{\alpha(\alpha-\beta)}} \tan^{-1}\left(\sqrt{\frac{\alpha}{\alpha-\beta}} \right) t + \phi_1 \right]^{2/_{n+2}} . \tag{56}$$

Similarly, using equation (52) in (3)

$$\rho_{WDF} = 4\beta m^2 \left[\alpha(1+t^2)-\beta\right]^{-1} . \tag{57}$$

Also, for $\gamma = 0$ equation (1) reduces to

$$P_{WDF} = 0 \quad . \tag{58}$$

Now we can express the physical quantities such as expansion factor θ, shear scalar $\sigma^2$and the declaration parameter q by using equation (52) as fallows

$$\theta = \frac{2\alpha t}{\alpha(1+t^2)-\beta} \quad , \tag{59}$$

$$\sigma^2 = \frac{4\alpha^2 t^2}{3\left[\alpha(1+t^2)-\beta\right]^2}, \tag{60}$$

$$q = \frac{\alpha\left(t^2+3\right)+3\beta}{2\alpha t^2} \quad , \tag{61}$$

where α is taken to be negative.

Case-(ii): $\gamma = 1$ (Zeldovich universe)

In this case, equation (38) reduces to

$$\int \frac{dv}{\sqrt{\left(\frac{3\rho_*}{2}+6m^2\right)V^2+3D+C_1}} = t\,. \tag{62}$$

which after integration and simplification gives

$$V = \eta\, t \quad \text{where} \quad \eta = \sqrt{3D+C_1} \quad . \tag{63}$$

Using equation (63), equations (24), (25), (26) after integration and simplification becomes

$$A = D_1 \eta^{\frac{1}{3}} t^{\frac{\eta+3X_1}{3\eta}} e^{\frac{k_1X_1}{\eta}} \,, \tag{64}$$

$$B = D_2 \eta^{\frac{1}{3}} t^{\frac{\eta+3X_2}{3\eta}} e^{\frac{k_2X_2}{\eta}} \,, \tag{65}$$

$$C = D_3 \eta^{\frac{1}{3}} t^{\frac{\eta+3X_3}{3\eta}} e^{\frac{k_3X_3}{\eta}} \,, \tag{66}$$

where k_1, k_2, k_3 are constants of integration and X_i (i=1, 2, 3) and D_i (i=1, 2, 3) satisfies the relation

$$X_1 + X_2 + X_3 = 0 \quad \text{and} \quad D_1 D_2 D_3 = 1.$$

Also, using equation (63) in equation (28) after integration and simplification we obtained

$$\phi = \left[\frac{\phi_0(n+2)}{2\eta}\log t + \phi_1\right]^{2/n+2} \quad . \tag{67}$$

Similarly using equation (63) in equation (3)

$$\rho_{WDF} = \frac{1}{2}\rho_* + \frac{\eta^2 - C_1}{3\eta^2 t^2}\,. \tag{68}$$

Also, for $\gamma = 1$ with the help of (68) equation (1) reduces to

$$\rho_{WDF} = \frac{\eta^2 - C_1}{3\eta^2 t^2} - \frac{1}{2}\rho_* \quad . \tag{69}$$

Now, we can express the physical quantities such as expansion factor θ, shear scalar σ^2 and the declaration parameter q by using equation (63) as follows:

$$\theta = \frac{1}{t} \tag{70}$$

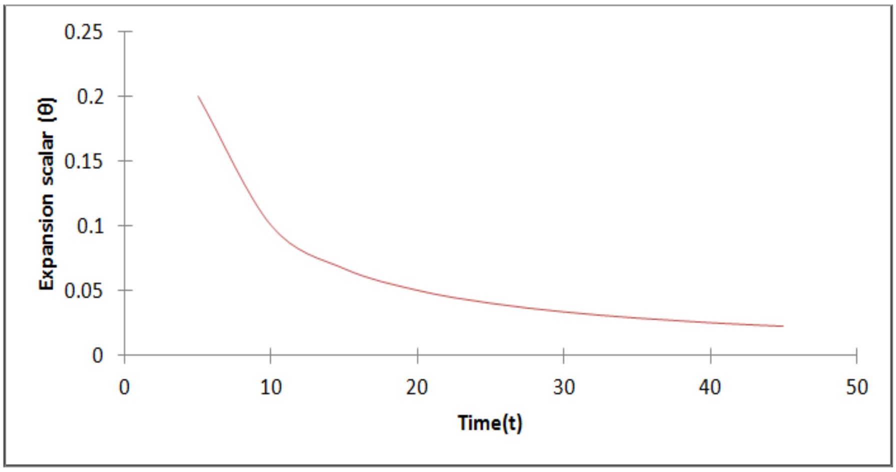

Figure-3: The plot of scalar expansion (θ) verses time (t), $\alpha=5, \beta=1$.

From the figure, it is observed that, model has nonzero expansion rate and is decreases with increase in time t.

$$\sigma^2 = \frac{1}{3t^2} \tag{71}$$

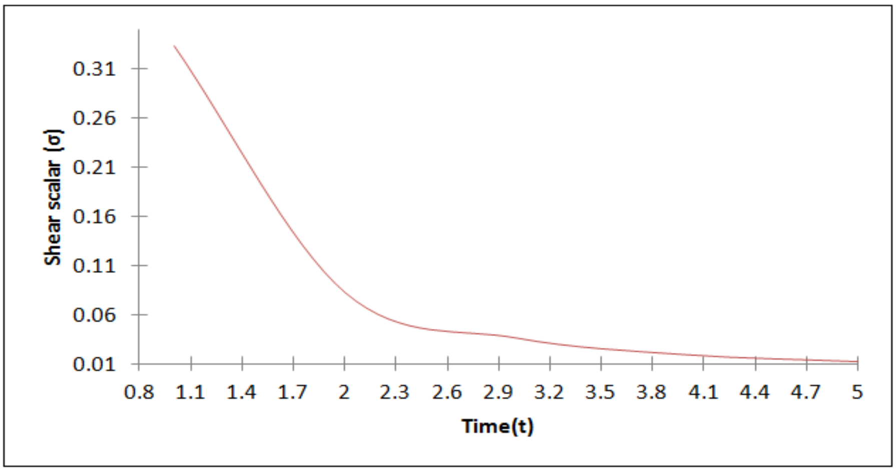

Figure-4: The plot of shear scalar (σ) verses time (t), $\alpha=5, \beta=1$.

From the figure, it is clear that at an initial, shear possess singularity while for large value of t, it dies out.

$$q = 2 \tag{72}$$

Model decelerated in standard ways. Further it is observed that $\frac{\sigma^2}{\theta^2} \neq 0$ i.e. model does not approaches to isotropy for large value of t.

4. CONCLUSION

In this paper, we have studied Bianchi type-V space-time in presence of wet dark fluid in scalar tensor theory of gravitation formulated by Seaz and Ballester. The solution has been obtained in quadrature form with the help of special volume V. We have also discussed some physical properties like expansion factor θ, shear scalar σ and the deceleration parameter q for two cases i.e. dust universe and Zeldovich universe. The behaviour of the model for large time has been analyzed. From the above result, it is observed that in both cases, universe is evolving with zero volume and expanding with time t. i.e. universe is expanding with time. The scalar expansion θ and the shear scalar σ diverges at time $t = 0$ while they become zero when $t \to \infty$. Also in both cases, model do not approaches to isotropy as $\frac{\sigma^2}{\theta^2} \neq 0$.

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