International Journal of Mathematical Archive-9(1), 2018, 65-68
Available online through www.ijma.info ISSN 2229 – 5046

SPLITTING AND ADMISSIBLE TOPOLOGIES DEFINED ON THE SET OF CONTINUOUS FUNCTIONS BETWEEN BITOPOLOGICAL SPACES

N. E. MUTURI*, J. M. KHALAGAI AND G. P. POKHARIYAL

School of Mathematics, University of Nairobi, P. O. Box 00100-30197, Nairobi, Kenya.

(Received On: 09-12-17; Revised & Accepted On: 02-01-18)

ABSTRACT

In this paper, p-splitting, p-admissible, s-splitting and s-admissible topologies on the sets p−C(Y, Z) and s−C(Y, Z) are defined and their properties explored. exponential functions are introduced in function spaces and s-splitting and s-admissible topologies defined on s-C(Y, Z) compared using these mappings.

2010 AMS Classification: *54A10, 54C35, 54E55.*

Keywords: *p-splitting, p-admissible, s-splitting and s-admissible topologies.*

1. INTRODUCTION

Let X, Y and Z be topological spaces, the set of all continuous functions from Y to Z is denoted by $C(Y, Z)$. This set when given a topology τ forms the function space $C_\tau(Y,Z)$. For any function $h : X \times Y \to Z$ which is continuous in Y for each fixed $x \in X$, there is an associated map $h^* : X \to C_\tau(Y,Z)$. The function h^* is defined as follows, $h^*(x) = h_x$, where $h_x(y) = h(x, y)$ for every $y \in Y$ (Fox [3]). Arens and Dugundji [1] defines a topology τ defined on $C(Y, Z)$ to be splitting, if the continuity of the mapping h implies the continuity of the mapping h^*. Topology τ defined on $C(Y, Z)$ is said to be admissible, if the continuity of the mapping h^* implies the continuity of the mapping h. The latter is also defined, if the evaluation mapping $e: C_\tau(Y, Z) \times Y \to Z$ defined by $e(f, y) = f(y)$ is continuous. For the bitopological spaces (Y, τ_1, τ_2) and (Z, δ_1, δ_2) introduced by Kelly [4], the following sets of continuous functions have been defined. The set $i{-}C(Y, Z)$ of all i-continuous functions for i=1,2, the set $p{-}C(Y,Z)$ of all pairwise continuous functions and the set $s{-}C(Y, Z)$ of all supremum continuous functions (Muturi *et.al* [6] and Dvalishvili [2]). In this paper, we generalize bitopological concepts to function spaces defined on bitopological space and introduce p-splitting, p-admissible, s-splitting and s-admissible topologies on the set $p{-}C(Y, Z)$ and $s{-}C(Y, Z)$. exponential functions are also defined on function spaces and and s-splitting and s-admissible topologies defined on the set $s{-}C(Y, Z)$ compared.

2. PRELIMINARIES

The following definition are important in this work.

Definition 2.1: *(Pervin* [5]*). A function* $f : (Y,\tau_1,\tau_2) \to (Z,\delta_1,\delta_2)$, *is said to be pairwise continuous (p-continuous) if the induced functions* $f : (Y,\tau_1) \to (Z,\delta_1)$ *and* $f : (Y,\tau_2) \to (Z,\delta_2)$ *are continuous.*

Definition 2.2: *(Muturi et al.* [6]*). A subset A of a bitopological space* $(Y,\tau_1\vee\tau_2)$ *is called a supremum-open set or simply s-open set if* $A = U_1 \cup U_2$, *where* $U_1 \in \tau_1$ *and* $U_2 \in \tau_2$.

Definition 2.3: *(Muturi et al.* [6]*). A function* $f : (Y,\tau_1\vee\tau_2) \to (Z,\delta_1\vee \delta_2)$, *is said to be s-continuous, if the inverse image of each s-open subset of Z is s-open in Y .*

Definition 2.4: *The set of all pairwise continuous functions from the bitopological space* (Y,τ_1,τ_2) *to the bitopological space* (Z,δ_1,δ_2) *is denoted by* $p{-}C(Y,Z)$, *and the set of all supremum continuous function from the bitopological space* $(Y,\tau_1 \vee \tau_2)$ *to the bitopological space* $(Z,\delta_1 \vee \delta_2)$ *is denoted by* $s{-}C(Y,Z)$.

Definition 2.5: *The sets of the form* $S((U,V),(A,B))_p = \{f \in p{-}C(Y,Z) : f(U) \subset V \text{ and } f(A) \subset B\}$ *for U open in* τ_1, *V open in* δ_1, *A open in* τ_2 *and B open in* δ_2, *defines the subbasis for the open-open topology on the set* $p{-}C(Y,Z)$.

Corresponding Author: N. E. Muturi*,
School of Mathematics, University of Nairobi, P. O. Box 00100-30197, Nairobi, Kenya.

3. PAIRWISE SPLITTING AND PAIRWISE ADMISSIBLE TOPOLOGIES DEFINED ON THE SET $p-C(Y, Z)$

In this section, we explore pairwise splitting and pairwise admissible topologies defined on the set $p-C(Y, Z)$.

Proposition 3.1: *The function $h : (X, \sigma) \times (Y, \tau_1, \tau_2) \to (Z, \delta_1, \delta_2)$ is pairwise continuous in Y for each fixed $x \in X$, if the functions $h : (X, \sigma) \times (Y, \tau_1) \to (Z, \delta_1)$ and $h : (X, \sigma) \times (Y, \tau_2) \to (Z, \delta_2)$ are continuous in Y for each fixed $x \in X$.*

Proof: Let $h : (X, \sigma) \times (Y, \tau_1) \to (Z, \delta_1)$ and $h : (X, \sigma) \times (Y, \tau_2) \to (Z, \delta_2)$ be continuous functions in Y for each fixed $x \in X$, then the functions $h_x : (Y, \tau_1) \to (Z, \delta_1)$ and $h_x : (Y, \tau_2) \to (Z, \delta_2)$ are continuous. By definition of pairwise continuity, the function $h_x : (Y, \tau_1, \tau_2) \to (Z, \delta_1, \delta_2)$ is continuous for each $x \in X$. Since $h_x(y) = h(x, y)$ and $h(x)(y) = h(x, y)$, then $h_x(y) = h(x)(y)$, implying that the function $h : (X, \sigma) \times (Y, \tau_1, \tau_2) \to (Z, \delta_1, \delta_2)$ is continuous in Y for each fixed $x \in X$.

Proposition 3.2: *The function $h^* : (X, \sigma) \to p-C_\omega(Y, Z)$ is pairwise continuous, if the functions $h^* : (X, \sigma) \to 1-C_\varsigma(Y, Z)$ and $h^* : (X, \sigma) \to 2-C_\zeta(Y, Z)$ are continuous, where $h : (X, \sigma)\times(Y, \tau_i) \to (Z, \delta_i)$ for $i = 1,2$.*

Proof: Let $h^* : (X, \sigma) \to 1-C_\varsigma(Y, Z)$ and $h^* : (X, \sigma) \to 2-C_\zeta(Y, Z)$ be continuous functions. Then for each fixed $x \in X$, the functions $h_x : (Y, \tau_1) \to (Z, \delta_1)$ and $h_x : (Y, \tau_2) \to (Z, \delta_2)$ are continuous. By definition of pairwise continuity, the function $h_x : (Y, \tau_1, \tau_2) \to (Z, \delta_1, \delta_2)$ is continuous for each $x \in X$. Since $h_x = h^*(x)$, then the function $h^* : (X, \sigma) \to p-C_\omega(Y, Z)$ is continuous.

From the above propositions, we introduce the following definitions.

Definition 3.3: *A topology ω on $p-C(Y, Z)$ is said to be pairwise splitting (p-splitting) if the continuity of the functions $h : (X, \sigma) \times (Y, \tau_1) \to (Z, \delta_1)$ and $h : (X, \sigma) \times (Y, \tau_2) \to (Z, \delta_2)$ in Y for each fixed $x \in X$, implies that of $h^* : (X,\sigma) \to p-C_\omega(Y,Z)$.*

Definition 3.4: *A topology ω on $p-C(Y, Z)$ is said to be pairwise admissible (p-admissible) if the continuity of the functions $h^* : (X, \sigma) \to 1-C_\varsigma(Y, Z)$ and $h^* : (X, \sigma) \to 2-C_\zeta(Y, Z)$ implies that of $h : (X, \sigma) \times (Y, \tau_1, \tau_2) \to (Z, \delta_1, \delta_2)$ in Y for each fixed $x \in X$.*

Theorem 3.5: *Let $h : (X, \sigma) \times (Y, \tau_1) \to (Z, \delta_1)$ and $h : (X, \sigma) \times (Y, \tau_2) \to (Z, \delta_2)$ be continuous functions, then the compact open topology ω defined on $p-C(Y, Z)$ is pairwise splitting.*

Proof: Let $h : (X, \sigma)\times(Y, \tau_1) \to (Z, \delta_1)$ and $h : (X, \sigma)\times(Y, \tau_2) \to (Z, \delta_2)$ be continuous functions in Y for each fixed $x \in X$, and let $x_0 \in X$ such that $h^*(x_0) \in S((U, V)(A, B))_p$, where $S((U, V)(A,B))_p$ is open in $p-C(Y, Z)$. Then $h^*(x_0) \in S(U, V)_1$ and $h^*(x_0) \in S(A, B)_2$, implying that $x_0 \times U \subset h^{-1}(V)$ and $x_0 \times A \subset h^{-1}(B)$. Since U and A are compact, then by tube lemma there exist an open set W neighbourhood of x_0 such that $W \times U \subset h^{-1}(V)$ and $W \times A \subset h^{-1}(B)$, this implies that $h^*(W) \subset S(U, V)_1$ and $h^*(W) \subset S(A, B)_2$, implying further that $h^* : (X, \sigma) \to 1-C_\varsigma(Y, Z)$ and $h^* : (X, \sigma) \to 2-C_\zeta(Y, Z)$ are continuous functions. By proposition 3.2, the function $h^* : (X, \sigma) \to p-C_\omega(Y, Z)$ is continuous and by definition 3.3, topology ω is pairwise splitting on $p-C(Y, Z)$.

Theorem 3.6: *Let $h^* : (X, \sigma) \to 1-C_\varsigma(Y, Z)$ and $h^* : (X, \sigma) \to 2-C_\zeta(Y, Z)$ be continuous functions, then the compact open topology ω defined on $p-C(Y, Z)$ is pairwise admissible for locally compact spaces (Y, τ_1) and (Y, τ_2).*

Proof: Let ς and ζ be compact open topologies on $1-C(Y,Z)$ and $2-C(Y,Z)$ respectively such that the evaluation functions $e:1-C_\varsigma(Y,Z)\times Y \to Z$ and $e:2-C_\varsigma(Y,Z)\times Y \to Z$ are continuous. Let $h^* : (X, \sigma) \to 1-C_\varsigma(Y, Z)$ and $h^* : (X, \sigma) \to 2-C_\zeta(Y, Z)$ be continuous functions and $i : (Y, \tau_1) \to (Y, \tau_1)$ and $i : (Y, \tau_2) \to (Y, \tau_2)$ be identity functions, then $e \circ (h^* \times i) : (X, \sigma) \times (Y, \tau_1) \to (Z, \delta_1)$ and $e \circ (h^* \times i) : (X, \sigma) \times (Y, \tau_2) \to (Z, \delta_2)$ are continuous functions. By proposition 3.1, the function $e\circ(h^*\times i) : (X, \sigma)\times(Y, \tau_1, \tau_2) \to (Z, \delta_1, \delta_2)$ is continuous in Y for each fixed $x \in X$ and by definition 3.4, topology ω defined on $p-C(Y, Z)$ is pairwise admissible.

Remark 3.7: *From theorem 3.5 and theorem 3.6, we conclude that τ on $p-C(Y, Z)$ is p-splitting or p-admissible topology if ς and ζ are splitting or admissible topologies on $1-C(Y, Z)$ and $2-C(Y, Z)$ respectively.*

4. SUPREMUM SPLITTING AND SUPREMUM ADMISSIBLE TOPOLOGIES DEFINED ON THE SET s-C(Y, Z)

In this section, supremum splitting and supremum admissible topologies are introduced on the set $s-C(Y, Z)$.

Definition 4.1: *A topology τ on $s-C(Y, Z)$ is said to be supremum splitting (s-splitting) if the continuity of the functions $f : (X, \sigma) \times (Y, \tau_1) \to (Z, \delta_1)$ and $f : (X, \sigma) \times (Y, \tau_2) \to (Z, \delta_2)$ in Y for each fixed $x \in X$, implies that of $f^* : (X, \sigma) \to s-C_\tau(Y, Z)$.*

Definition 4.2: *A topology τ on $s-C(Y, Z)$ is said to be supremum admissible (s-admissible) if the continuity of the functions $f^* : (X, \sigma) \to 1-C_\varsigma(Y, Z)$ and $f^* : (X, \sigma) \to 2-C_\zeta(Y, Z)$, implies that of $f : (X, \sigma) \times (Y, \tau_1 \vee \tau_2) \to (Z, \delta_1 \vee \delta_2)$ in Y for each fixed $x \in X$.*

Proposition 4.3: *The function $f : (X, \sigma) \times (Y, \tau_1 \vee \tau_2) \to (Z, \delta_1 \vee \delta_2)$ is continuous if the functions $f : (X, \sigma) \times (Y, \tau_1) \to (Z, \delta_1)$ and $f : (X, \sigma) \times (Y, \tau_2) \to (Z, \delta_2)$ are continuous.*

Proof: Let the functions $f : (X, \sigma) \times (Y, \tau_1) \to (Z, \delta_1)$ and $f : (X, \sigma) \times (Y, \tau_2) \to (Z, \delta_2)$ be continuous in Y for each fixed $x \in X$. then the associated functions $f_x : (Y, \tau_1) \to (Z, \delta_1)$ and $f_x : (Y, \tau_2) \to (Z, \delta_2)$ defined by $f_x(y) = f(x, y)$, are continuous $\forall x \in X$. From theorem 3.1 [6], it follows that the function $f_x : (Y, \tau_1 \vee \tau_2) \to (Z, \delta_1 \vee \delta_2)$ is s-continuous $\forall x \in X$. Since $f_x(y) = f(x, y)$ and $f(x)(y) = f(x, y)$, then $f_x(y) = f(x)(y)$ and hence $f : (X, \sigma) \times (Y, \tau_1 \vee \tau_2) \to (Z, \delta_1 \vee \delta_2)$ is continuous in Y for each fixed $x \in X$.

Proposition 4.4: *The function $f^* : (X, \sigma) \to s-C_\tau(Y, Z)$ is continuous if the functions $f^* : (X, \sigma) \to 1-C_\varsigma(Y, Z)$ and $f^* : (X, \sigma) \to 2-C_\zeta(Y, Z)$ are continuous.*

Proof: Let $f^* : (X, \sigma) \to 1-C_\varsigma(Y, Z)$ and $f^* : (X, \sigma) \to 2-C_\zeta(Y, Z)$ be a continuous functions, then for the functions $f : (X, \sigma) \times (Y, \tau_1) \to (Z, \delta_1)$ and $f : (X, \sigma) \times (Y, \tau_2) \to (Z, \delta_2)$, the associated functions $f_x : (Y, \tau_1) \to (Z, \delta_1)$ and $f_x : (Y, \tau_2) \to (Z, \delta_2)$ defined by $f_x = f^*(x)$, $\forall x \in X$ are continuous. From theorem 3.1 [6], it follows that the function $f_x : (Y, \tau_1 \vee \tau_2) \to (Z, \delta_1 \vee \delta_2)$ is s-continuous $\forall x \in X$. Since $f_x = f^*(x)$, then the function $f^* : (X, \sigma) \to s-C(Y,Z)$ is continuous.

Theorem 4.5: *A compact open topology τ is s-splitting if the continuity of the functions $f : (X, \sigma) \times (Y, \tau_1) \to (Z, \delta_1)$ and $f : (X, \sigma) \times (Y, \tau_2) \to (Z, \delta_2)$ implies continuity of the function $f^* : (X, \delta) \to s-C_\tau(Y, Z)$.*

Proof: Let $f : (X, \sigma) \times (Y, \tau_1) \to (Z, \delta_1)$ and $f : (X, \sigma) \times (Y, \tau_2) \to (Z, \delta_2)$ be continuous functions in Y for each fixed $x \in X$. Then from proposition 4.3, the function $f : (X, \sigma) \times (Y, \tau_1 \vee \tau_2) \to (Z, \delta_1 \vee \delta_2)$ is continuous. Let $x_0 \in X$ and $S(U,V)_s$ be open in $s-C_\tau(Y, Z)$, then $f^*(x_0) \in S(U, V)_s$, implying that $x_0 \times U \subset f^{-1}(V)$. Since U is compact, then by tube lemma, there exist an open set W neighbourhood of x_0 such that $W \times U \subset f^{-1}(V)$. This implies that $f^*(W) \subset S(U,V)_s$, implying further that $f^* : (X, \sigma) \to s-C_\tau(Y, Z)$ is continuous functions. By definition 4.1, topology τ is s-splitting on $s-C(Y, Z)$.

Theorem 4.6: *Let $f^* : (X, \sigma) \to 1-C_\varsigma(Y, Z)$ and $f^* : (X, \sigma) \to 2-C_\zeta(Y, Z)$ be continuous functions, then the compact open topology τ defined on $s-C(Y, Z)$ is s-admissible for locally compact spaces (Y, τ_1) and (Y, τ_2).*

Proof: Let ς and ζ be compact open topologies on $1-C(Y, Z)$ and $2-C(Y, Z)$ respectively, and let $f^* : (X, \sigma) \to 1-C_\varsigma(Y,Z)$ and $f^* : (X, \sigma) \to 2-C_\zeta(Y, Z)$ be a continuous functions, then by proposition 4.4, the function $f^* : (X, \sigma) \to s-C_\tau(Y, Z)$ is continuous. Let $i : (Y, \tau_1 \vee \tau_2) \to (Y, \tau_1 \vee \tau_2)$ be an identity function and let $e : s-C_\tau(Y, Z) \times (Y, \tau_1 \vee \tau_2) \to (Z, \delta_1 \vee \delta_2)$ be an evaluation mapping. Since τ is compact open topology, then the evaluation mapping e is continuous and the composite mapping $e \circ (f^* \times i) : (X, \sigma) \times (Y, \tau_1 \vee \tau_2) \to (Z, \delta_1 \vee \delta_2)$ is also continuous in Y for each fixed $x \in X$. By definition 4.2, topology τ is s-admissible.

Remark 4.7: *From theorem 4.5 and theorem 4.6, we note that if ς and ζ are splitting or admissible topologies on $1-C(Y,Z)$ and $2-C(Y,Z)$ respectively, then τ on $s-C(Y,Z)$ is s-splitting or s-admissible topology.*

5. EXPONENTIAL MAPPINGS DEFINED ON FUNCTION SPACES

Let (X, σ), $(Z, \delta_1 \vee \delta_2)$ be arbitrary spaces and let $(Y, \tau_1 \vee \tau_2)$ be locally compact Hausdorff space.

Definition 5.1: *Consider the exponential mapping $\Lambda : C(X \times Y,Z) \to C(X, s - C_\phi(Y, Z))$, defined by $\Lambda(f)(x)(y) = f(x, y)$ for each $f \in C(X \times Y,Z)$, $x \in X$ and $y \in Y$. A topology ϕ on $s-C(Y, Z)$ is called s-splitting topology if Λ is a continuous function with respect to ϕ.*

Definition 5.2: *Consider the exponential mapping $\Lambda^{-1} : C(X, s-C_\phi(Y, Z)) \to C(X \times Y,Z)$, defined by $\Lambda^{-1}((g)(x, y)) = g(x)(y)$ where $g \in C(X, s-C_\phi(Y, Z))$ for each $(x, y) \in X \times Y$. A topology ϕ on $s-C(Y, Z)$ is called s-admissible topology if the function Λ^{-1} is continuous with respect to ϕ.*

Proposition 5.3: *The function* $\Lambda^{-1}\circ\Lambda : C(X\times Y, Z)\rightarrow C(X\times Y, Z)$ *is continuous.*

Proof: Observe that $(\Lambda^{-1}\circ\Lambda(f))(x, y) = \Lambda^{-1}(\Lambda(f))(x, y) = \Lambda(f)(x)(y) = f(x, y)$. Implying that $\Lambda^{-1}\circ\Lambda(f) = f$, hence $\Lambda^{-1}\circ\Lambda$ is an identity function.

Proposition 5.4: *The function* $\Lambda\circ\Lambda^{-1} : C(X, s{-}C_\phi(Y, Z))\rightarrow C(X, s{-}C_\phi(Y, Z))$ *is continuous.*

Proof: Observe that $(\Lambda\circ\Lambda-1(f))(x)(y) = \Lambda(\Lambda-1(f))(x)(y) = \Lambda-1(f)(x, y) = f(x)(y)$. Implying that $\Lambda\circ\Lambda-1(f) = f$, hence $\Lambda\circ\Lambda^{-1}$ is an identity function.

Remark 5.5: *From proposition 5.3 and proposition 5.4, it follows that* Λ *is a homeomorphism.*

Proposition 5.6: *The function* $i : C(X, s{-}C_{\phi1}(Y, Z))\rightarrow C(X, s{-}C_{\phi2}(Y, Z))$ *is continuous if and only if* $\phi_2\subset\phi_1$.

Proof: The function i is continuous if and only if $S(W, S(U,V)) \in \phi_2$ implies that $i^{-1}(S(W, S(U,V))) \in \phi_1$, but i is an identity function, therefore $i^{-1}(S(W, S(U, V))) = S(W, S(U, V))$. Hence i is continuous if and only if $S(W, S(U, V)) \in \phi_2$ implies $S(W, S(U, V)) \in \phi_1$.

Theorem 5.7: *The following statements are true;*

(i) Let ϕ_1 *be s-splitting topology on* $s{-}C(Y, Z)$ *and let* $\phi_2\subset\phi_1$, *then* ϕ_2 *is also s-splitting topology on* $s{-}C(Y, Z)$.
(ii) Let ϕ_1 *be s-admissible topology on* $s{-}C(Y, Z)$ *and let* $\phi_1\subset\phi_2$, *then* ϕ_2 *is also s-admissible topology on* $s{-}C(Y, Z)$.
(iii) Let ϕ_1 *be s-splitting topology on* $s{-}C(Y, Z)$ *and let* ϕ_2 *be admissible topology on* $s{-}C(Y, Z)$, *then* $\phi_1\subset\phi_2$.

Proof:

(i) Let ϕ_1 be *s*-splitting topology, then by definition 5.1 the function $\Lambda : C(X\times Y, Z)\rightarrow C(X,s{-}C_{\phi1}(Y, Z))$, defined by $\Lambda(f)(x)(y) = f(x, y)$ for each $f\in C(X\times Y, Z)$, $x\in X$ and $y\in Y$, is continuous with respect to ϕ_1. Let ϕ_2 be any other topology such that $\phi_2\subset\phi_1$, then by proposition 5.6, the function $i : C(X, s{-}C_{\phi1}(Y, Z))\rightarrow C(X, s{-}C_{\phi2}(Y, Z))$ is continuous. Now the composite function $i\circ\Lambda : C(X\times Y, Z)\rightarrow C(X, s{-}C_{\phi2}(Y, Z))$ is continuous with respect to ϕ_2, implying that ϕ_2 is also *s*-splitting topology.

(ii) Let ϕ_1 be *s*-admissible topology, then by definition 5.2 the function $\Lambda^{-1} : C(X, s{-}C_{\phi1}(Y, Z))\rightarrow C(X\times Y, Z)$ defined by $\Lambda^{-1}((g)(x, y)) = g(x)(y)$ where $g\in C(X, s{-}C_\phi(Y, Z))$ for each $(x, y)\in X\times Y$, is continuous with respect to ϕ_1. Let $\phi_1\subset\phi_2$, then by proposition 5.6, the function $i : C(X, s{-}C_{\phi2}(Y, Z))\rightarrow C(X, s{-}C_{\phi1}(Y, Z))$ is continuous. Now the composite function $\Lambda^{-1}\circ i : C(X, s{-}C_{\phi2}(Y, Z))\rightarrow C(X\times Y, Z)$ is continuous with respect to ϕ_2. Hence ϕ_2 is also *s*-admissible topology.

(iii) Let ϕ_1 be *s*-splitting topology, then by definition 5.1 the function $\Lambda : C(X\times Y, Z)\rightarrow C(X, s{-}C_{\phi1}(Y, Z))$, defined by $\Lambda(f)(x)(y) = f(x, y)$ for each $f\in C(X\times Y, Z)$, $x\in X$ and $y\in Y$, is continuous with respect to ϕ_1. Let ϕ_2 be *s*-admissible topology, then by definition 5.2 the function $\Lambda^{-1} : C(X, s{-}C_{\phi1}(Y, Z))\rightarrow C(X\times Y, Z)$ defined by $\Lambda^{-1}((g)(x, y)) = g(x)(y)$ where $g\in C(X, s{-}C_\phi(Y, Z))$ for each $(x, y)\in X\times Y$, is continuous with respect to ϕ_1. Now the composite function $\Lambda\circ\Lambda^{-1} : C((X, \sigma), s{-}C_{\phi2}(Y, Z))\rightarrow C((X, \sigma), s{-}C_{\phi1}(Y, Z))$ is continuous by proposition 5.6, implying that $\phi_1\subset\phi_2$.

REFERENCES

[1] Arens, R. F. and Dugundji J., *Topologies for function space*, Pacific J. Math. **1** (1951), 5-31.
[2] Dvalishvili B. P., *Bitopological Spaces, Theory, Relations with Generalized Algebraic Structures, and Applications*, Elsevier Ltd, San Diego, USA, 2005.
[3] Fox R. H., *On topologies for function spaces*, American Mathematical Society **27** (1945), 427-432.
[4] Kelly J. C., *Bitopological Spaces*., Proc. London Math Soc. **13** (1963), 71- 89.
[5] Pervin W. J., *Connectedness in Bitopological Spaces*, Nederl. Akad. Wetensch. Proc. Ser. A70, Indag. Math. **29** (1967) 369-372.
[6] Muturi N. E., Pokhariyal G.P. and Khalaghai J. M., *Continuity of functions on function spaces defined on bitopological spaces.* Journal of Advanced Studies in Topology **8 (2)** (2017), 130-134.

Source of support: Nil, Conflict of interest: None Declared.