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BIVARIATE NEW BETTER THAN USED CLASS OF LIFE DISTRIBUTIONS

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ABSTRACT

Some new results on Bivariate New Better than Used of order 3 (BNBU (3)) class of life distributions are obtained. Closure of the BNBU(3) class under convolution is given. Shock models where shocks are arriving according to homogeneous Poisson Process are established.

Key Words: *BNBU(3), convolution, shock models, life distributions.*

AMS Subject Classification: *60K10.*

1. INTRODUCTION

Several classes of life distributions have been introduced in reliability. The applications of these classes of life distribution can be seen in engineering, medical, maintenance, biometrics, etc. Therefore, statisticians and reliability analysts have shown a growing interest in modeling survival data using classifications of life distributions based on some aspects of aging. Among the most well known families of life distributions are the classes of increasing failure rate (IFR), increasing failure rate average (IFRA), new better than used (NBU), new better than used in expectation (NBUE), and harmonic new better than used in expectation (HNBUE). For more details, one may refer to Barlow and Proschan (1981), Hendi and Mashhour (1993), Chen (1994), Li *et al.* (2000), Belzunce *et al.* (2001), Li and Kochar (2001), Nanda et al.(2003), Li and Zou (2004), and Ahmad et al.(2006) among others.

A complete study on bivariate ageing classes of life distributions was carried out by Rizwan and Syed Tahir Hussainy (2016). The authors have investigated the relationship among the bivariate ageing classes of life distributions. They have also presented the closure and preservation properties of some of the bivariate ageing classes of life distributions.

The rest of the paper is organized as follows. In section 2, we give some preliminaries and the definition of the BNBU (3) class. Closure properties under convolution and preservation under homogeneous Poisson shock model of this ageing class is given in Section 3. Finally, in Section 4, Conclusion is given.

2. BASIC TOOLS

In this section, we give the definition of the BNBU (3) ageing class and its discrete version is also given.

Definition 2.1: A bivariate life distribution F is called a bivariate new better than used in a third order BNBU (3), if

$$\int_0^\infty\int_0^\infty\int_0^u\int_0^v \overline{F}(x+t,y+s)\,ds\,dt\,dv\,du \le \overline{F}(x,y)\int_0^\infty\int_0^\infty\int_0^u\int_0^v \overline{F}(t,s)\,ds\,dt\,dv\,du.$$

Remark 1: The above definition is a very strong version of the BNBU (3) ageing class. This class may be referred as BNBU (3)-S. A some what weaker version of the BNBU (3) ageing class is given below.

- *weak Bivariate New Better than Used (BNBU (3)-W)* :

$$\int_0^\infty\int_0^\infty\int_0^u\int_0^v \overline{F}(x+t,x+s)\,ds\,dt\,dv\,du \le \overline{F}(x,x)\int_0^\infty\int_0^\infty\int_0^u\int_0^v \overline{F}(t,s)\,ds\,dt\,dv\,du$$

for $x,t,s \ge 0$.

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Definition 2.2: A discrete bivariate distribution $\{\overline{P}(i,j)\}_{i,j=0}^{\infty}$ is said to be discrete bivariate new better than used of third order, BNBU (3), if

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{m}\sum_{l=0}^{n}\overline{P}(i+k,\, j+l) \le \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{m}\sum_{l=0}^{n}\overline{P}(i,\, j)\ \overline{P}(k,l).$$

3. CLOSURE PROPERTIES CONVOLUTION

As an important reliability operation, convolution of life distribution of a certain class is often paid much attention. The closure properties of BIFR, BNBU, BNBUE and BIFRA can be found in Rizwan and Syed Tahir Hussainy (2016). In the next theorem, we establish the closure property of the BNBU (3) class under the convolution operation.

Theorem 3.1: *Suppose that F_1 and F_2 are two independent BNBU (3) life distributions. Then their convolution is also BNBU(3).*

Proof: The convolution of F_1 *and* F_2 is

$$\overline{F}(t_1,t_2) = \int_0^{\infty} \overline{F}_1(t_1-u,t_2-v)\, d\, F_2(u,v).$$

Consider

$$\begin{aligned}
\int_0^{\infty}\int_0^{\infty}\int_0^{u}\int_0^{v}\overline{F}(x+t,y+s)\,ds\,dt\,dv\,du &= \int_0^{\infty}\int_0^{\infty}\int_0^{u}\int_0^{v}\left[\int_0^{\infty}\overline{F}_1(x+t-z,y+s-w)\,dF_2(z,w)\right]ds\,dt\,dv\,du\\
&= \int_0^{\infty}\left[\int_0^{\infty}\int_0^{u}\int_0^{v}\int_0^{\infty}\overline{F}_1(x+t-z,y+s-w)\,ds\,dt\,dv\,du\right]dF_2(z,w)\\
&\le \int_0^{\infty}\left[\overline{F}_1(x,y)\int_0^{\infty}\int_0^{\infty}\int_0^{u}\int_0^{v}\overline{F}_1(t-z,s-w)\,ds\,dt\,dv\,du\right]dF_2(z,w)\\
&= \overline{F}_1(x,y)\int_0^{\infty}\int_0^{\infty}\int_0^{u}\int_0^{v}\left[\int_0^{\infty}\overline{F}_1(t-z,s-w)\,dF_2(z,w)\right]ds\,dt\,dv\,du\\
&\le \overline{F}(x,y)\int_0^{\infty}\int_0^{\infty}\int_0^{u}\int_0^{v}\left[\int_0^{\infty}\overline{F}_1(t-z,s-w)\,d\,F_2(z,w)\right]ds\,dt\,dv\,du\\
&= \overline{F}(x,y)\int_0^{\infty}\int_0^{\infty}\int_0^{u}\int_0^{v}\overline{F}(t,s)\,ds\,dt\,dv\,du.
\end{aligned}$$

The first inequality is because F_1 is BNBU(3) and the next inequality is because $\overline{F}_1 \le \overline{F}$. It then follows that F is BNBU(3).

Theorem 3.2: *Let F_1 and F_2 be two independent discrete BNBU(3) life distributions. Then their convolution is BNBU (3).*

Proof: Clearly

$$\overline{P}(i+k,\, j+l) = \sum_{\alpha=0}^{\infty}\sum_{\beta=0}^{\infty}\overline{P}_1(i+k-\alpha,\, j+l-\beta)\, p_2(\alpha,\beta)$$

For $i, j, k, l = 0,1,2,\ldots$. Clearly,

$$\begin{aligned}
\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{m}\sum_{l=0}^{n}\overline{P}(i+k,\, j+l) &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{m}\sum_{l=0}^{n}\left[\sum_{\alpha=0}^{\infty}\sum_{\beta=0}^{\infty}\overline{P}_1(i+k-\alpha,\, j+l-\beta)\, p_2(\alpha,\beta)\right]\\
&= \sum_{\alpha=0}^{\infty}\sum_{\beta=0}^{\infty}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{m}\sum_{l=0}^{n}\overline{P}_1(i+k-\alpha,\, j+l-\beta)\, p_2(\alpha,\beta)\\
&= \sum_{\alpha=0}^{\infty}\sum_{\beta=0}^{\infty}p_2(\alpha,\beta)\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{m}\sum_{l=0}^{n}\overline{P}_1(i+k-\alpha,\, j+l-\beta)
\end{aligned}$$

$$\leq \sum_{\alpha=0}^{\infty}\sum_{\beta=0}^{\infty} p_2(\alpha,\beta) \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{m}\sum_{l=0}^{n} \overline{P}_1(i,j)\ \overline{P}_1(k-\alpha,l-\beta)$$

$$\leq \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{m}\sum_{l=0}^{n} \overline{P}(i,j) \sum_{\alpha=0}^{\infty}\sum_{\beta=0}^{\infty} \overline{P}_1(k-\alpha,l-\beta)\ p_2(\alpha,\beta)$$

$$= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{m}\sum_{l=0}^{n} \overline{P}(i,j)\ \overline{P}(k,l).$$

The first inequality is because $\overline{P}_1$ is BNBU(3) and the next inequality is because $\overline{P}_1 \leq \overline{P}$. It then follows that *F* discrete is BNBU (3).

SHOCK MODELS

Suppose that a device with two components is subjected to shocks occurring randomly as events in Poisson process with constant intensity λ. Suppose further that the device has the probability $P(i, j)$ of surviving *i* shocks on the first component and *j* shocks on the second component. The survival function of the device is then given by

$$\overline{H}(t_1,t_2) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} e^{-\lambda t_1} \frac{(\lambda t_1)^k}{k!}\ e^{-\lambda(t_2-t_1)} \frac{[\lambda(t_2-t_1)]^l}{l!}\ \overline{P}(k,l), \tag{1}$$

for $0 \leq t_1 \leq t_2$.

Theorem 3.3: *The survival function* $\overline{H}(t)$ *in (1) is BNBU(3) if and only if* $\{\overline{P}(i,j)\}_{i,j=0}^{\infty}$ *has the discrete BNBU (3) property.*

Proof: Consider

$$\int_0^\infty\int_0^\infty\int_0^u\int_0^v \overline{H}(x+t,y+s)\,ds\,dt\,dv\,du = \int_0^\infty\int_0^\infty\int_0^u\int_0^v \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} e^{-\lambda(x+t)} \frac{[\lambda(x+t)]^i}{i!}\ e^{-\lambda[(y+s)-(x+t)]} \frac{[\lambda[(y+s)-(x+t)]]^j}{j!}\ \overline{P}(i,j)\,ds\,dt\,dv\,du$$

$$= \int_0^\infty\int_0^\infty\int_0^u\int_0^v \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left(\sum_{k=0}^{i}\binom{i}{k}(\lambda x)^{i-k}(\lambda t)^k\right)\frac{e^{-\lambda(x+t)}}{i!} \left(\sum_{l=0}^{j}\binom{j}{l}(\lambda(y-x))^{j-l}(\lambda(s-t))^l\right)\frac{e^{-\lambda[(y-x)+(s-t)]}}{j!}\ \overline{P}(i,j)\,ds\,dt\,dv\,du$$

$$= \int_0^\infty\int_0^\infty\int_0^u\int_0^v \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{i}\frac{i!}{k!(i-k)!}(\lambda t)^k(\lambda x)^{i-k}\ \frac{e^{-\lambda x}e^{-\lambda t}}{i!} \sum_{l=0}^{j}\frac{j!}{l!(j-l)!}(\lambda(s-t))^l(\lambda(y-x))^{j-l}\ \frac{e^{-\lambda(y-x)}e^{-\lambda(s-t)}}{j!}\ \overline{P}(i,j)\,ds\,dt\,dv\,du$$

$$= \int_0^\infty\int_0^\infty\int_0^u\int_0^v \sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{i=k}^{\infty}\frac{(\lambda t)^k(\lambda x)^{i-k}}{k!(i-k)!}\ e^{-\lambda x}e^{-\lambda t} \sum_{l=0}^{j}\frac{[\lambda(s-t)]^l[\lambda(y-x)]^{j-l}}{l!(j-l)!}\ e^{-\lambda(y-x)}e^{-\lambda(s-t)}\ \overline{P}(i,j)\,ds\,dt\,dv\,du$$

$$= \int_0^\infty\int_0^\infty\int_0^u\int_0^v \sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{\alpha=0}^{\infty}\sum_{\beta=0}^{\infty}\frac{(\lambda t)^k(\lambda x)^{\alpha}}{k!(\alpha)!}\ e^{-\lambda x}e^{-\lambda t}\ \frac{[\lambda(s-t)]^l[\lambda(y-x)]^{\beta}}{l!(\beta)!}\ e^{-\lambda(y-x)}e^{-\lambda(s-t)}\ \overline{P}(k+\alpha,l+\beta)\,ds\,dt\,dv\,du$$

$$= \int_0^\infty\int_0^\infty\int_0^u\int_0^v \sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{\alpha=0}^{\infty}\sum_{\beta=0}^{\infty}\frac{e^{-\lambda x}(\lambda x)^{\alpha}}{\alpha!}\ \frac{e^{-\lambda t}(\lambda t)^k}{k!}\ \frac{e^{-\lambda(y-x)}[\lambda(y-x)]^{\beta}}{\beta!}\ \frac{e^{-\lambda(s-t)}[\lambda(s-t)]^l}{l!}\ \overline{P}(k,l)\ \overline{P}(\alpha,\beta)\ ds\,dt\,dv\,du$$

The inequality is because $\{\overline{P}(i, j)\}_{i,j=0}^{\infty}$ is discrete-BNBU (3). It follows that $\overline{H}$ is BNBU (3) and this completes the proof.

4. CONCLUSION

In this paper, the discrete version of the bivariate ageing classes BNBU (3) of life distribution have been introduced. The closure property of these bivariate ageing class of life distributions under convolution and the Poisson shock models have been studied.

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