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ALGEBRAIC PROPERTIES OF INTUITIONISTIC FUZZY SET OPERATORS

SANTHOSH KUMAR. S

**Assistant Professor & Head, Department of Mathematics,
KG College of Arts and Science, Coimbatore, India.**

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ABSTRACT

In this paper, some results are proved for establishing the algebraic properties of intuitionistic operators with respect to intuitionistic fuzzy sets.

Keywords: *Intuitionistic Fuzzy Set, Intuitionistic Fuzzy Operators.*

INTRODUCTION

Crisp sets [5] which has a membership function only 0 and 1 is applied in a lot of branches beside mathematics. L.A.Zadeh [6] introduced the notion of fuzzy sub set μ of a set X is a function from X to [0,1], To get a wider application of the set theory. The fuzzy concept has been introduced in almost all branches of mathematics, After the introduction of fuzzy sets by L.A.Zadeh [6]. Then the concept intuitionistic fuzzy sets (IFS) was introduced by K.T.Atanassov [1] as a generalization of notation of a fuzzy set. Here, we discuss the algebraic properties of intuitionistic fuzzy operators and proved some theorems for the same.

1. PRELIMINARIES

For any two IFSs A and B the following relation and operations can be defined [2,3,4] as follows

Definition 1.1-Crisp Sets: The crisp set is defined in such a way to classify the individuals the universe in two groups: Members and Non-Members.

Definition 1.2-Fuzzy Sets: A Fuzzy set is a class of object with a continuum of grades of membership. Such a set is chacterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one.

Definition 1.3-Fuzzy Sub Sets: Let S be any non empty set, A mapping μ from S to [0, 1] is called a fuzzy sub set of S.

Definition 1.4-Intuitionistic Fuzzy Set: Intuitionistic fuzzy sets are sets whose elements have degrees of membership and non-membership. Intuitionistic fuzzy sets have been introduced by Krassimir Atanassov (1983) as a extention of Lotfi Zadeh's notion of fuzzy sets, which itself extends the classical notion of a set. An intuitionistic fuzzy set A is a non-empty set X is an object having the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \varepsilon E\}$ where the function $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote the degrees of membership and non-membership of the element $x \varepsilon X$ to A respectively and satisfy $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for all $x \varepsilon X$. The family of all intuitionistic fuzzy set in X denoted by IFS(X).

***Corresponding Author: Santhosh Kumar. S
Assistant Professor & Head, Department of Mathematics,
KG College of Arts and Science, Coimbatore, India.***

Definition 1.5- Operators of Intuitionistic fuzzy sets: For every two IFSs A and B the following operation and relations can be defined as

$$A \cap B \Leftrightarrow (\forall x \varepsilon E)(\mu_A(x) \le \mu_{\mathbf{B}}(x) \,\&\, \gamma_A(x) \ge \gamma_{\mathbf{B}}(x))$$
$$A = B \Leftrightarrow (\forall x \varepsilon E)(\mu_A(x) = \mu_{\mathbf{B}}(x) \,\&\, \gamma_A(x) = \gamma_{\mathbf{B}}(x))$$
$$A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle / x\varepsilon E\}$$
$$A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle / x\varepsilon E\}$$
$$A + B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \gamma_A(x)\gamma_B(x)\rangle / x\varepsilon E\}$$
$$A \bullet B = \{\langle x, \mu_A(x)\mu_B(x), \gamma_A(x) + \gamma_B(x) - \gamma_A(x)\gamma_B(x)\rangle / x\varepsilon E\}$$
$$A @ A = \left\{\langle x, \frac{\mu_A(x) + \mu_A(x)}{2}, \frac{\gamma_A(x) + \gamma_A(x)}{2}\rangle \Big/ x\varepsilon E\right\}$$

2. PROOF OF IMPORTANT RESULTS

Idempotent Law with Respect to Union &Intersection & @

2.1 $A \cap A = \{\langle x, \min(\mu_A(x), \mu_A(x)), \max(\gamma_A(x), \gamma_A(x))\rangle / x\varepsilon E\}$ By definition of Intersection

$= \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x\varepsilon E\}$

= A

2.2 $A \cup A = \{\langle x, \max(\mu_A(x), \mu_A(x)), \min(\gamma_A(x), \gamma_A(x))\rangle / x\varepsilon E\}$ By definition of Union

$= \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x\varepsilon E\}$

= A

2.3 $A @ A = \left\{\langle x, \frac{\mu_A(x) + \mu_A(x)}{2}, \frac{\gamma_A(x) + \gamma_A(x)}{2}\rangle \Big/ x\varepsilon E\right\}$

$= \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x\varepsilon E\}$

= A

3. DEMORGAN'S LAW WITH RESPECT TO UNION & INTERSECTION

3.1 $\overline{\overline{A} \cap \overline{B}} = A \bigcup B$

L.H.S = $\overline{\overline{A} \cap \overline{B}}$

$= \overline{\{\langle x, \gamma_A(x), \mu_A(x)\rangle / x\varepsilon E\} \cap \{\langle x, \gamma_B(x), \mu_B(x)\rangle / x\varepsilon E\}}$

$= \overline{\{\langle x, \min\{\gamma_A(x), \gamma_B(x)\}, \max\{\mu_A(x), \mu_B(x)\rangle\} / x\varepsilon E\}} \rightarrow I$

R.H.S = $A \bigcup B$

$= \{\langle x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\gamma_A(x), \gamma_B(x)\rangle\} / x\varepsilon E\} \rightarrow II$

Case (i) Let $\mu_A(x) \rangle \mu_{\mathbf{B}}(x) \,\&\, \gamma_A(x) \langle \gamma_{\mathbf{B}}(x)$

L.H.S = $\overline{\overline{A} \cap \overline{B}}$ (Using I)

$= \overline{\{\langle x, \gamma_A(x), \mu_A(x)\rangle / x\varepsilon E\}}$

$= \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x\varepsilon E\}$

= A (1)

R.H.S = $A \bigcup B$ (Using II)

$= \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x\varepsilon E\}$

= A (2)

$\therefore 1 = 2$

Case-(ii): Let $\mu_A(x) \rangle \mu_{\mathbf{B}}(x) \,\&\, \gamma_A(x) \rangle \gamma_{\mathbf{B}}(x)$

L.H.S $= \overline{\overline{A} \cap \overline{B}}$ (Using I)

$= \overline{\{\langle x, \gamma_B(x), \mu_A(x)\rangle / x\varepsilon E\}}$

$= \{\langle x, \mu_A(x), \gamma_B(x)\rangle / x\varepsilon E\}$ (3)

R.H.S $= A \cup B$ (Using II)

$= \{\langle x, \mu_A(x), \gamma_B(x)\rangle / x\varepsilon E\}$ (4)

$\therefore 3 = 4$

Case (iii) Let $\mu_A(x) \langle \mu_{\mathbf{B}}(x) \,\&\, \gamma_A(x) \langle \gamma_{\mathbf{B}}(x)$

L.H.S $= \overline{\overline{A} \cap \overline{B}}$ (Using I)

$= \overline{\{\langle x, \gamma_A(x), \mu_B(x)\rangle / x\varepsilon E\}}$

$= \{\langle x, \mu_B(x), \gamma_A(x)\rangle / x\varepsilon E\}$ (5)

R.H.S $= A \cup B$ (Using II)

$= \{\langle x, \mu_B(x), \gamma_A(x)\rangle / x\varepsilon E\}$ (6)

$\therefore 5 = 6$

Case-(iv): Let $\mu_A(x) \langle \mu_{\mathbf{B}}(x) \,\&\, \gamma_A(x) \rangle \gamma_{\mathbf{B}}(x)$

L.H.S $= \overline{\overline{A} \cap \overline{B}}$ (Using I)

$= \overline{\{\langle x, \gamma_B(x), \mu_B(x)\rangle / x\varepsilon E\}}$

$= \{\langle x, \mu_B(x), \gamma_B(x)\rangle / x\varepsilon E\}$

= B (7)

R.H.S $= A \cup B$ (Using II)

$= \{\langle x, \mu_B(x), \gamma_B(x)\rangle / x\varepsilon E\}$

= B (8)

$\therefore 7 = 8$

Hence in all the cases the results are verified

3.2 $\overline{\overline{A} \cup \overline{B}} = A \cap B$

L.H.S $= \overline{\overline{A} \cup \overline{B}}$

$= \overline{\{\langle x, \gamma_A(x), \mu_A(x)\rangle / x\varepsilon E\} \cup \{\langle x, \gamma_B(x), \mu_B(x)\rangle / x\varepsilon E\}}$

$= \overline{\{\langle x, \max\{\gamma_A(x), \gamma_B(x)\}, \min\{\mu_A(x), \mu_B(x)\rangle\} / x\varepsilon E\}} \rightarrow III$

R.H.S $= A \cap B$

$= \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x\varepsilon E\} \cap \{\langle x, \mu_B(x), \gamma_B(x)\rangle / x\varepsilon E\}$

$= \{\langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\gamma_A(x), \gamma_B(x)\rangle\} / x\varepsilon E\} \rightarrow IV$

Case-(i): Let $\mu_A(x) \rangle \mu_{\mathbf{B}}(x) \,\&\, \gamma_A(x) \rangle \gamma_{\mathbf{B}}(x)$

L.H.S $= \overline{\overline{A} \cup \overline{B}}$ (Using III)

$= \overline{\{\langle x, \gamma_A(x), \mu_B(x)\rangle / x\varepsilon E\}}$

$= \{\langle x, \mu_B(x), \gamma_A(x)\rangle / x\varepsilon E\}$ (9)

R.H.S $= A \cap B$ (Using IV)

$= \{\langle x, \mu_B(x), \gamma_A(x)\rangle / x\varepsilon E\}$ (10)

$\therefore 9 = 10$

Case-(ii): Let $\mu_A(x)\rangle \mu_{\mathbf{B}}(x)\ \&\ \gamma_A(x)\langle \gamma_{\mathbf{B}}(x)$

L.H.S $= \overline{\overline{A}\cup\overline{B}}$ (Using III)

$= \overline{\{\langle x, \gamma_B(x), \mu_B(x)\rangle / x\varepsilon E\}}$

$= \{\langle x, \mu_B(x), \gamma_B(x)\rangle / x\varepsilon E\}$

=B (11)

R.H.S $= A\cap B$ (Using IV)

$= \{\langle x, \mu_B(x), \gamma_B(x)\rangle / x\varepsilon E\}$

=B (12)

$\therefore 11 = 12$

Case-(iii): Let $\mu_A(x)\langle \mu_{\mathbf{B}}(x)\ \&\ \gamma_A(x)\rangle \gamma_{\mathbf{B}}(x)$

L.H.S $= \overline{\overline{A}\cup\overline{B}}$ (Using III)

$= \overline{\{\langle x, \gamma_A(x), \mu_A(x)\rangle / x\varepsilon E\}}$

$= \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x\varepsilon E\}$ (13)

R.H.S $= A\cap B$ (Using IV)

$= \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x\varepsilon E\}$

=A (14)

$\therefore 13 = 14$

Case-(iv): Let $\mu_A(x)\langle \mu_{\mathbf{B}}(x)\ \&\ \gamma_A(x)\langle \gamma_{\mathbf{B}}(x)$

L.H.S $= \overline{\overline{A}\cup\overline{B}}$ (Using III)

$= \overline{\{\langle x, \gamma_B(x), \mu_A(x)\rangle / x\varepsilon E\}}$

$= \{\langle x, \mu_A(x), \gamma_B(x)\rangle / x\varepsilon E\}$ (15)

R.H.S $= A\cap B$ (Using IV)

$= \{\langle x, \mu_A(x), \gamma_B(x)\rangle / x\varepsilon E\}$ (16)

$\therefore 15 = 16$

Hence in all the cases the results are verified

3.3 $\overline{\overline{A}+\overline{B}} = A\bullet B$

L.H.S $= \overline{\overline{A}+\overline{B}}$

$= \overline{\{\langle x, \gamma_A(x), \mu_A(x)\rangle / x\varepsilon E\} + \{\langle x, \gamma_B(x), \mu_B(x)\rangle / x\varepsilon E\}}$

$= \overline{\{\langle x, \gamma_A(x)+\gamma_B(x)-\gamma_A(x)\gamma_B(x), \mu_A(x)\mu_B(x)\rangle / x\varepsilon E\}}$ By definition of $A+B$

$= \{\langle x, \mu_A(x)\mu_B(x), \gamma_A(x)+\gamma_B(x)-\gamma_A(x)\gamma_B(x)\rangle / x\varepsilon E\}$

$= A\bullet B$ By definition of $A\bullet B$

= R.H.S

3.4 $\overline{\overline{A}\bullet\overline{B}} = A+B$

L.H.S $= \overline{\overline{A}\bullet\overline{B}}$

$= \overline{\{\langle x, \gamma_A(x), \mu_A(x)\rangle / x\varepsilon E\} \bullet \{\langle x, \gamma_B(x), \mu_B(x)\rangle / x\varepsilon E\}}$

$= \overline{\{\langle x, \gamma_A(x)\gamma_B(x), \mu_A(x)+\mu_B(x)-\mu_A(x)\mu_B(x)\rangle / x\varepsilon E\}}$ By definition of $A\bullet B$

$= \{\langle x, \mu_A(x)+\mu_B(x)-\mu_A(x)\mu_B(x), \gamma_A(x)\gamma_B(x)\rangle / x\varepsilon E\}$

$= A+B$ By definition of $A+B$

= R.H.S

3.5 $\overline{\overline{A}@\overline{B}} = A\bullet B$

L.H.S $= \overline{\overline{A}@\overline{B}}$

$$= \overline{\{\langle x, \gamma_A(x), \mu_A(x)\rangle / x\varepsilon E\} @ \{\langle x, \gamma_B(x), \mu_B(x)\rangle / x\varepsilon E\}}$$

$$= \overline{\left\{\langle x, \frac{\gamma_A(x)+\gamma_B(x)}{2}, \frac{\mu_A(x)+\mu_B(x)}{2} \Big/ x\varepsilon E\right\}}$$

$$= \left\{\langle x, \frac{\mu_A(x)+\mu_{\mathbf{B}}(x)}{2}, \frac{\gamma_A(x)+\gamma_{\mathbf{B}}(x)}{2} \Big/ x\varepsilon E\right\}$$ By definition of $\overline{A}$

$= A @ B$ By definition of @

= R.H.S

CONCLUSION

We have defined the different operations of intuitionistic fuzzy sets. Using this, we have proved the different algebraic relation between this operators in the intuitionistic fuzzy sets.

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