ALGEBRAIC PROPERTIES OF INTUITIONISTIC FUZZY SET OPERATORS

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ABSTRACT

In this paper, some results are proved for establishing the algebraic properties of intuitionistic operators with respect to intuitionistic fuzzy sets.

Keywords: Intuitionistic Fuzzy Set, Intuitionistic Fuzzy Operators.

INTRODUCTION

Crisp sets [5] which has a membership function only 0 and 1 is applied in a lot of branches beside mathematics. L.A.Zadeh [6] introduced the notion of fuzzy sub set \( \mu \) of a set \( X \) is a function from \( X \) to \([0,1]\). To get a wider application of the set theory. The fuzzy concept has been introduced in almost all branches of mathematics. After the introduction of fuzzy sets by L.A. Zadeh [6], then the concept intuitionistic fuzzy sets (IFS) was introduced by K.T. Atanassov [1] as a generalization of notation of a fuzzy set. Here, we discuss the algebraic properties of intuitionistic fuzzy operators and proved some theorems for the same.

1. PRELIMINARIES

For any two IFSs A and B the following relation and operations can be defined [2,3,4] as follows

**Definition 1.1-Crisp Sets:** The crisp set is defined in such a way to classify the individuals the universe in two groups: Members and Non-Members.

**Definition 1.2-Fuzzy Sets:** A Fuzzy set is a class of object with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one.

**Definition 1.3-Fuzzy Sub Sets:** Let \( S \) be any non empty set, A mapping \( \mu \) from \( S \) to \([0, 1]\) is called a fuzzy sub set of \( S \).

**Definition 1.4-Intuitionistic Fuzzy Set:** Intuitionistic fuzzy sets are sets whose elements have degrees of membership and non-membership. Intuitionistic fuzzy sets have been introduced by Krassimir Atanassov (1983) as an extention of Lotfi Zadeh’s notion of fuzzy sets, which itself extends the classical notion of a set. An intuitionistic fuzzy set \( A \) is a non-empty set \( X \) is an object having the form \( A = \{ (x, \mu_A(x), \gamma_A(x)) \mid x \in X \} \) where the function \( \mu_A : X \to [0,1] \) and \( \gamma_A : X \to [0,1] \) denote the degrees of membership and non-membership of the element \( x \in X \) to \( A \) respectively and satisfy \( 0 \leq \mu_A(x) + \gamma_A(x) \leq 1 \) for all \( x \in X \). The family of all intuitionistic fuzzy set in \( X \) denoted by IFS(X).

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Definition 1.5- Operators of Intuitionistic fuzzy sets: For every two IFSs A and B the following operation and relations can be defined as

\[ A \cap B \Leftrightarrow \left( \forall x \in E \right) \left( \mu_A(x) \leq \mu_B(x) \& \gamma_A(x) \geq \gamma_B(x) \right) \]
\[ A = B \Leftrightarrow \left( \forall x \in E \right) \left( \mu_A(x) = \mu_B(x) \& \gamma_A(x) = \gamma_B(x) \right) \]
\[ A \cap B = \left\{ \left( x, \min \{ \mu_A(x), \mu_B(x) \}, \max \{ \gamma_A(x), \gamma_B(x) \} / x \in E \right) \right\} \]
\[ A \cup B = \left\{ \left( x, \max \{ \mu_A(x), \mu_B(x) \}, \min \{ \gamma_A(x), \gamma_B(x) \} / x \in E \right) \right\} \]
\[ A + B = \left\{ \left( x, \mu_A(x) + \mu_B(x) - \mu_A(x) \mu_B(x), \gamma_A(x) \gamma_B(x) / x \in E \right) \right\} \]
\[ A \cdot B = \left\{ \left( x, \mu_A(x) \mu_B(x), \gamma_A(x) + \gamma_B(x) - \gamma_A(x) \gamma_B(x) / x \in E \right) \right\} \]
\[ A@A = \left\{ \left( x, \frac{\mu_A(x) + \mu_A(x)}{2}, \frac{\gamma_A(x) + \gamma_A(x)}{2} / x \in E \right) \right\} \]

2. PROOF OF IMPORTANT RESULTS

Idempotent Law with Respect to Union & Intersection & @

2.1 \[ A \cap A = \left\{ \left( x, \min \{ \mu_A(x), \mu_A(x) \}, \max \{ \gamma_A(x), \gamma_A(x) \} / x \in E \right) \right\} \]
By definition of Intersection
\[ = \left\{ \left( x, \mu_A(x), \gamma_A(x) \right) / x \in E \right\} \]
\[ = A \]

2.2 \[ A \cup A = \left\{ \left( x, \max \{ \mu_A(x), \mu_A(x) \}, \min \{ \gamma_A(x), \gamma_A(x) \} / x \in E \right) \right\} \]
By definition of Union
\[ = \left\{ \left( x, \mu_A(x), \gamma_A(x) \right) / x \in E \right\} \]
\[ = A \]

2.3 \[ A@A = \left\{ \left( x, \frac{\mu_A(x) + \mu_A(x)}{2}, \frac{\gamma_A(x) + \gamma_A(x)}{2} / x \in E \right) \right\} \]
\[ = \left\{ \left( x, \mu_A(x), \gamma_A(x) \right) / x \in E \right\} \]
\[ = A \]

3. DEMORGAN’S LAW WITH RESPECT TO UNION & INTERSECTION

3.1 \[ \overline{A \cap B} = A \cup B \]
L.H.S. = \[ \overline{A \cap B} \]
\[ = \left\{ \left( x, \frac{\mu_A(x) + \mu_A(x)}{2}, \frac{\gamma_A(x) + \gamma_A(x)}{2} / x \in E \right) \right\} \]
\[ = \left\{ \left( x, \mu_A(x), \gamma_A(x) \right) / x \in E \right\} \]
\[ = A \]

R.H.S. = \[ A \cup B \]
\[ = \left\{ \left( x, \max \{ \mu_A(x), \mu_B(x) \}, \min \{ \gamma_A(x), \gamma_B(x) \} / x \in E \right) \right\} \rightarrow II \]

Case (i) Let \( \mu_A(x) \mu_B(x) \& \gamma_A(x) \gamma_B(x) \)

L.H.S. = \[ \overline{A \cap B} \]
(Using I)
\[ = \left\{ \left( x, \frac{\mu_A(x) + \mu_A(x)}{2}, \frac{\gamma_A(x) + \gamma_A(x)}{2} / x \in E \right) \right\} \]
\[ = \left\{ \left( x, \mu_A(x), \gamma_A(x) \right) / x \in E \right\} \]
\[ = A \] (1)

R.H.S. = \[ A \cup B \]
(Using II)
\[ = \left\{ \left( x, \max \{ \mu_A(x), \mu_B(x) \}, \min \{ \gamma_A(x), \gamma_B(x) \} / x \in E \right) \right\} \]
\[ = A \] (2)

\[ \therefore 1 = 2 \]
Case-(ii): Let \( \mu_A(x) \mu_B(x) \& \gamma_A(x) \gamma_B(x) \)

L.H.S = \( A \cap B \) (Using I)
= \( \{ (x, \gamma_A(x), \mu_B(x)) / x \in E \} \)
= \( \{ (x, \mu_A(x), \gamma_B(x)) / x \in E \} \) \( (3) \)

R.H.S = \( A \cup B \) (Using II)
= \( \{ (x, \mu_A(x), \gamma_B(x)) / x \in E \} \) \( (4) \)
\[ \therefore 3 = 4 \]

Case (iii) Let \( \mu_A(x) \mu_B(x) \& \gamma_A(x) \gamma_B(x) \)

L.H.S = \( A \cap B \) (Using I)
= \( \{ (x, \gamma_A(x), \mu_B(x)) / x \in E \} \)
= \( \{ (x, \mu_B(x), \gamma_A(x)) / x \in E \} \) \( (5) \)

R.H.S = \( A \cup B \) (Using II)
= \( \{ (x, \mu_B(x), \gamma_A(x)) / x \in E \} \) \( (6) \)
\[ \therefore 5 = 6 \]

Case-(iv): Let \( \mu_A(x) \mu_B(x) \& \gamma_A(x) \gamma_B(x) \)

L.H.S = \( A \cap B \) (Using I)
= \( \{ (x, \gamma_A(x), \mu_B(x)) / x \in E \} \)
= \( \{ (x, \mu_B(x), \gamma_A(x)) / x \in E \} \)
= \( B \) \( (7) \)

R.H.S = \( A \cup B \) (Using II)
= \( \{ (x, \mu_B(x), \gamma_A(x)) / x \in E \} \)
= \( B \) \( (8) \)
\[ \therefore 7 = 8 \]

Hence in all the cases the results are verified

3.2 \( \overline{A \cup B} = A \cap B \)

L.H.S = \( \overline{A \cup B} \)
= \( \{ (x, \gamma_A(x), \mu_B(x)) / x \in E \} \cup \{ (x, \gamma_B(x), \mu_A(x)) / x \in E \} \)
= \( \{ x, \max \{ \gamma_A(x), \gamma_B(x) \}, \min \{ \mu_A(x), \mu_B(x) \} / x \in E \} \rightarrow III \)

R.H.S = \( A \cap B \)
= \( \{ (x, \mu_A(x), \gamma_B(x)) / x \in E \} \cap \{ (x, \mu_B(x), \gamma_A(x)) / x \in E \} \)
= \( \{ x, \min \{ \mu_A(x), \mu_B(x) \}, \max \{ \gamma_A(x), \gamma_B(x) \} / x \in E \} \rightarrow IV \)

Case-(i): Let \( \mu_A(x) \mu_B(x) \& \gamma_A(x) \gamma_B(x) \)

L.H.S = \( \overline{A \cup B} \) (Using III)
= \( \{ (x, \gamma_A(x), \mu_B(x)) / x \in E \} \)
= \( \{ (x, \mu_B(x), \gamma_A(x)) / x \in E \} \) \( (9) \)

R.H.S = \( A \cap B \) (Using IV)
= \( \{ (x, \mu_B(x), \gamma_A(x)) / x \in E \} \) \( (10) \)
\[ \therefore 9 = 10 \]
Case-(ii): Let $\mu_A(x)\mu_B(x)\gamma_A(x)\gamma_B(x)$

L.H.S = $A \cup B$ (Using III)
= $\{ (x, \gamma_B(x), \mu_B(x), x \in E) \}$
= $\{ (x, \mu_B(x), \gamma_B(x), x \in E) \}$
= $B$

R.H.S = $A \cap B$ (Using IV)
= $\{ (x, \mu_B(x), \gamma_B(x), x \in E) \}$
= $B$

$\therefore 11 = 12$

Case-(iii): Let $\mu_A(x)\mu_B(x)\gamma_A(x)\gamma_B(x)$

L.H.S = $A \cup B$ (Using III)
= $\{ (x, \gamma_A(x), \mu_A(x), x \in E) \}$
= $\{ (x, \mu_A(x), \gamma_A(x), x \in E) \}$

R.H.S = $A \cap B$ (Using IV)
= $\{ (x, \mu_A(x), \gamma_A(x), x \in E) \}$
= $A$

$\therefore 13 = 14$

Case-(iv): Let $\mu_A(x)\mu_B(x)\gamma_A(x)\gamma_B(x)$

L.H.S = $A \cup B$ (Using III)
= $\{ (x, \gamma_A(x), \mu_A(x), x \in E) \}$
= $\{ (x, \mu_A(x), \gamma_A(x), x \in E) \}$

R.H.S = $A \cap B$ (Using IV)
= $\{ (x, \mu_A(x), \gamma_A(x), x \in E) \}$

$\therefore 15 = 16$

Hence in all the cases the results are verified

3.3

$L.H.S = \overline{A + B} = A \bullet B$

L.H.S = $\overline{A + B}$
= $\{ (x, \gamma_A(x), \mu_A(x), x \in E) + \{ (x, \gamma_B(x), \mu_B(x), x \in E) \}$
= $\{ (x, \mu_A(x), \gamma_A(x) + \gamma_B(x) - \gamma_A(x) \gamma_B(x), \mu_A(x) \mu_B(x), x \in E) \}$ By definition of $A + B$
= $\{ (x, \mu_A(x) \mu_B(x), \gamma_A(x) + \gamma_B(x) - \gamma_A(x) \gamma_B(x), x \in E) \}$ By definition of $A \bullet B$
= $A \bullet B$
= R.H.S

3.4

$L.H.S = \overline{A \bullet B} = A + B$

L.H.S = $\overline{A \bullet B}$
= $\{ (x, \gamma_A(x), \mu_A(x), x \in E) \bullet \{ (x, \gamma_B(x), \mu_B(x), x \in E) \}$
= $\{ (x, \gamma_A(x) \gamma_B(x), \mu_A(x) + \mu_B(x) - \mu_A(x) \mu_B(x), x \in E) \}$ By definition of $A \bullet B$
= $\{ (x, \mu_A(x) + \mu_B(x) - \mu_A(x) \mu_B(x), \gamma_A(x) \gamma_B(x), x \in E) \}$ By definition of $A + B$
= $A + B$
= R.H.S

3.5

$L.H.S = \overline{A @ B} = A \bullet B$

L.H.S = $\overline{A @ B}$
\[ \frac{\langle x, \gamma_A(x), \mu_A(x) \rangle}{x \in E} @ \frac{\langle x, \gamma_B(x), \mu_B(x) \rangle}{x \in E} = \frac{\langle x, \frac{1}{2} (\gamma_A(x) + \gamma_B(x)), \frac{1}{2} (\mu_A(x) + \mu_B(x)) \rangle}{x \in E} \]

\[ = \left\{ \begin{array}{l}
\frac{\mu_A(x)}{2} + \frac{\mu_B(x)}{2} \\
\frac{\gamma_A(x)}{2} + \frac{\gamma_B(x)}{2}
\end{array} \right\}_{x \in E} \]

By definition of \( \bar{A} \)

\[ = A @ B \quad \text{By definition of } @ \]

\[ = \text{R.H.S} \]

**CONCLUSION**

We have defined the different operations of intuitionistic fuzzy sets. Using this, we have proved the different algebraic relation between this operators in the intuitionistic fuzzy sets.

**REFERENCES**

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