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SOME TYPES OF DOMINATION IN INTUITIONISTIC FUZZY GRAPHS

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ABSTRACT

In this paper, the concept of edge contraction, fusion of two vertices, pendent dominating set, complementary pendent dominating set, isolate dominating set, doubly isolate dominating set and perfect secure dominating set in IFG have been introduced. Also, some properties of the domination numbers d_p, d_{cp}, d_{is} and d_{dis} with known parameters of G are analysed.

Keywords: *Fusion, Edge contraction, pendent domination, isolated domination.*

AMS Classification: *05C72, 03E72, 03F55.*

1. INTRODUCTION

Graph theory has numerous applications in modern science and technology. The study of dominating sets in graphs was started by Ore and Berge [2, 8] and the domination number was introduced by Cockayne and Hedetniemi [4]. Isolated domination and pendent domination in graphs studied in [3, 7, 15]. Technology are featured with complex processes and phenomena for which complete and precise information is not always available. For such cases, mathematical models are developed to handle the types of systems containing elements of uncertainty.

The notion of fuzzy sets was introduced by Zadeh [16] as a method of representing uncertainty and vagueness. The first definition of fuzzy graphs was proposed by Kaufmann [6] from the fuzzy relations introduced by Zadeh. The concept of domination in fuzzy graphs introduced by A.Somasundaram and S.Somasundaram [14]. Intuitionistic fuzzy models give more precision, flexibility and compatibility to the system as compared to the classic and fuzzy models. In 1984, Atanassov [1] introduced intuitionistic fuzzy sets as a generalization of fuzzy sets added a new component which determines the degree of non-membership.

Intuitionistic fuzzy graph was introduced in [1, 5]. In [9], the concept of domination, total domination, connected domination have been introduced. In this way, the authors got motivated to work further on the theory of domination and to introduce new concepts such as pendent dominating set, complementary pendent dominating set, isolate dominating set, doubly isolate dominating set, perfect secure dominating set which have since been introduced. This paper is organized as follows: section 2 contains basic notations and definitions required for this work. In section 3, the definition of edge contraction, fusion of vertices, pendent dominating set, complementary pendent dominating set, isolate dominating set, doubly isolate dominating set and perfect secure dominating set of an IFG is given. Also, analyzed some properties of the domination numbers d_p, d_{cp}, d_{is} and d_{cis} with known parameters of G. Section 4 concludes the paper. For other notations and terminologies not mentioned in the paper, the readers may read [5, 9, 10, 11, 12].

2. PRELIMINARIES

In this section, some basic definitions relating to IFGs are given. Throughout this paper, simple and undirected IFGs are taken into consideration.

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Definition 2.1 [1]: Let a set E be fixed. An *intuitionistic fuzzy set* (IFS) A in E is an object of the form $A = \{\langle x, \mu_A(x),\ \nu_A(x)\rangle \mid x \in E\}$, where the function μ_A: $E \to [0, 1]$ and ν_A: $E \to [0, 1]$ determine the degree of membership and the degree of non-membership of the element $x \in E$ respectively, and for every
$x \in E, 0 \le \mu_A(x) + \nu_A(x) \le 1.$

Definition 2.2 [5]: An *intuitionistic fuzzy graph* (IFG) is of the form $G = (V, E)$ where

(i) $V = \{v_1, v_2, ..., v_n\}$, such that $\mu : V \to [0,1]$ and $\gamma : V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \le \mu_i + \gamma_i \le 1$ for every $v_i \in V$, $i = 1,2,\cdots,n$

(ii) $E \subset V \times V$ where $\mu : V \times V \to [0,1]$ and $\gamma : V \times V \to [0,1]$ are such that $\mu_{ij} \le \mu_i \oslash \mu_i$, $\gamma_{ij} \le \gamma_i \oslash \gamma_i$ and $0 \le \mu_{ij} + \gamma_{ij} \le 1$ where μ_{ij} and γ_{ij} are the membership and non- membership values of the edge (v_i, v_j); the values $\mu_i \oslash \mu_i$ and $\gamma_i \oslash \gamma_i$ can be determined by one of the six cartesian products $\times_t, t = 1,2,3,4,5,6$ for all i and j given in [11].

Note 1:
When $\mu_{ij} = \gamma_{ij} = 0$ for some i and j, there is no edge between v_i and v_j.
Otherwise, there exists an edge between v_i and v_j.

Definition 2.3 [9]: The *cardinality* of a subset S of V is defined as $|S| = \sum_{v_i \in S}\left(\frac{1+\mu_i+\gamma_i}{2}\right)$ for all $v_i \epsilon S$.

Definition 2.4 [9]: The *order* of an IFG $G = (V, E)$, and denoted by $o(G)$, and is defined as $o(G) = \sum_{v_i \in V}\left(\frac{1+\mu_i+\gamma_i}{2}\right)$ for all $v_i \epsilon V$.

Definition 2.5 [9]: An edge (v_i, v_j) is said to be a strong edge of an IFG G, if $\mu_{ij} \ge {\mu_{ij}}^{\infty}$ and $\gamma_{ij} \ge {\gamma_{ij}}^{\infty}$

Definition 2.6: Let $G = (V, E)$ be an IFG, then the vertex $v_i \in V$ is said to be pendent vertex if $N(v_i) = \{v_j\},\ v_j \in V$.

3.TYPES OF DOMINATION INTUITIONISTIC FUZZY GRAPHS

Definition 3.1: Let $G = (V, E)$ be an IFG. For any pair of vertices $v_i, v_j \in V$, the *fusion of the vertices* v_i, v_j in G is denoted by $G \bowtie \{v_i, v_j\} = (V', E', \langle\mu_r, \gamma_r\rangle, \langle\mu_{rs}, \gamma_{rs}\rangle)$, where

(i) $V' = V \cup \{v_k\}, v_k = v_i v_j$

(ii) E'=(v_r, v_s), such that either one of the following is true

- $v_s = v_i$ or $v_s = v_j$
- $v_r = v_i$ or $v_s = v_j$
- $v_r \ne v_i$ or $v_s \ne v_j$

(iii) $\langle\mu_r, \gamma_r\rangle$ denote the degrees of membership and non-membership of vertices of G, and is given by

$$\langle\mu_r, \gamma_r\rangle = \begin{cases} \langle\mu_r, \gamma_r\rangle \ \ if\ v_r \ne v_k \\ \max(\mu_i, \mu_j), \min(\gamma_i, \gamma_j)\ \ if\ v_r = v_k \end{cases}$$

(iv) $\langle\mu_{rs}, \gamma_{rs}\rangle$ denote the degrees of membership and non-membership of edges of G, and is given by

$$\langle\mu_{rs}, \gamma_{rs}\rangle = \begin{cases} \max(u_r, \mu_i), \min(\gamma_r, \gamma_i) & if\ v_s = v_i, (v_r, v_i) \in E \\ \max(u_r, \mu_j), \min(\gamma_r, \gamma_j) & if\ v_s = v_j, (v_r, v_j) \in E \\ \langle\mu_{rs}, \gamma_{rs}\rangle & if\ v_s \ne v_j, v_r \ne v_i, (v_r, v_s) \in E \\ \max(u_i, \mu_j), \min(\gamma_i, \gamma_j) & if\ v_s = v_j, v_r = v_i, (v_i, v_j) \in E \end{cases}$$

Definition 3.2: Let $G = (V, E)$ be an IFG. Let $e = (v_i, v_j)$, be an edge in G, then *edge contraction* e of G is denoted by $G \ominus e = (V', E', \langle\mu_r, \gamma_r\rangle, \langle\mu_{rs}, \gamma_{rs}\rangle)$, where

(v) $V' = V \cup \{v_k\},\ \ v_k = v_i v_j$

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(vi) $E'=(v_r, v_s)$, such that either one of the following is true
- $v_s = v_i$ or $v_s = v_j$
- $v_r = v_i$ or $v_s = v_j$

(vii) $\langle \mu_r, \gamma_r \rangle$ denote the degrees of membership and non-membership of vertices of G, and is given by

$$\langle \mu_r, \gamma_r \rangle = \begin{cases} \langle \mu_r, \gamma_r \rangle \ if\ v_r \neq v_k \\ \max(\mu_i, \mu_j), \min(\gamma_i, \gamma_j)\ if\ v_r = v_k \end{cases}$$

(viii) $\langle \mu_{rs}, \gamma_{rs} \rangle$ denote the degrees of membership and non-membership of edges of G, and is given by

$$\langle \mu_{rs}, \gamma_{rs} \rangle = \begin{cases} \max(u_r, \mu_i), \min(\gamma_r, \gamma_i) & if\ v_s = v_i, (v_r, v_i) \in E \\ \max(u_r, \mu_j), \min(\gamma_r, \gamma_j) & if\ v_s = v_j, (v_r, v_j) \in E \\ \langle \mu_{rs}, \gamma_{rs} \rangle & if\ v_s \neq v_j, v_r \neq v_i, (v_r, v_s) \in E \\ \langle 0,0 \rangle & if\ v_s = v_j, v_r = v_i, (v_i, v_j) \in E \end{cases}$$

Definition 3.3: Let G be an IFG. A subset S of V is called *pendent dominating set* in G if,
(i) For every $v_j \in V - S$, there exist $v_i \in S$ such that $\mu_{ij} \geq \mu_{ij}^{\infty}$ and $\gamma_{ij} \geq \gamma_{ij}^{\infty}$
(ii) The sugraph $H = (V', E')$ induced by S contains atleast one pendent vertex.

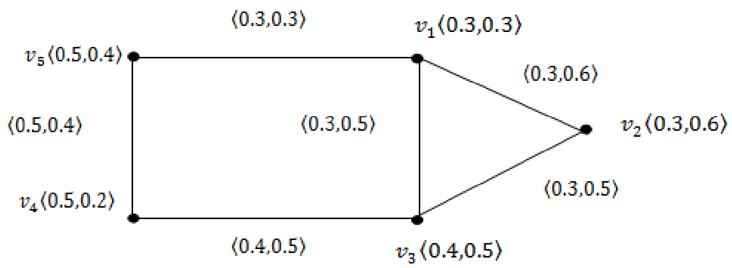
Defintion 3.4: The minimum cardinality of a pendent dominating set is called the lower pendent domination number of G, denoted by $d_p(G)$. The maximum cardinality of a pendent dominating set is called the upper pendent domination number of G, denoted by $d_P(G)$.

Defintion 3.5: Let G be an IFG. A subset S of V is called *complementary pendent dominating set* in G if,
(i) For every $v_j \in V - S$, there exist $v_i \in S$ such that $\mu_{ij} \geq \mu_{ij}^{\infty}$ and $\gamma_{ij} \geq \gamma_{ij}^{\infty}$
(ii) The sugraph $H = (V', E')$ induced by $V - S$ contains atleast one pendent vertex.

Defintion3.6: The minimum cardinality of a complementary pendent dominating set is called the lower complementary pendent domination number of G, denoted by $d_{cp}(G)$.

The maximum cardinality of a complementary pendent dominating set is called the upper complementary pendent domination number of G, denoted by $d_{CP}(G)$.

Example 3.1: Consider an IFG G with $V = \{v_1, v_2, v_3, v_4, v_5\}$,
$E = \{(v_1, v_2), (v_2, v_3), (v_5, v_4), (v_5, v_1), (v_4, v_3), (v_1, v_3)\}$.



Here, $d_p(G) = 0.9$ and $d_{cpd}(G) = 0.925$.

Definition 3.7: Let G be an IFG. A subset S of V is called *isolated dominating set* in G if,
i. For every $v_j \in V - S$, there exist $v_i \in S$ such that $\mu_{ij} \geq \mu_{ij}^{\infty}$ and $\gamma_{ij} \geq \gamma_{ij}^{\infty}$
ii. The sugraph $H = (V', E')$ induced by S has atelast one vertex v_i suchthat $\mu_{ij} = 0$ and $\gamma_{ij} = 0$.

Defintion 3.8: The minimum cardinality of an isolated dominating set is called the lower isolated domination number of G, denoted by $d_{is}(G)$.

The maximum cardinality of an isolated dominating set is called the upper isolated domination number of G, denoted by $d_{IS}(G)$.

Definition 3.9: Let G be an IFG. A subset S of V is called *doubly isolated dominating set* in G if,

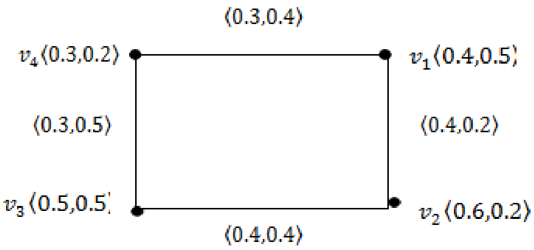
(i) For every $v_j \in V - S$, there exist $v_i \in S$ such that $\mu_{ij} \geq {\mu_{ij}}^{\infty}$ and $\gamma_{ij} \geq {\gamma_{ij}}^{\infty}$

(ii) The sugraph $H = (V', E')$ induced by $(V - S)$ has atelast one vertex v_i suchthat $\mu_{ij} = 0$ and $\gamma_{ij} = 0$.

Defintion 3.10: The minimum cardinality of a doubly isolated dominating set is called the lower doubly isolated domination number of G, denoted by $d_{dis}(G)$.

The maximum cardinality of a doubly isolated dominating set is called the upper doubly isolated domination number of G, denoted by $d_{DIS}(G)$.

Example 3.2: Consider an IFG G with $V = \{v_1, v_2, v_3, v_4\}, E = \{(v_1, v_2), (v_2, v_3), (v_1, v_4), (v_4, v_3)\}$



Here, $\{v_3, v_1\}$ is a minimal isolated dominating set and $\{v_4, v_2\}$ is a minimal doubly dominating set. $d_{is}(G) = 0.8$, $d_{dis}(G) = 1.25$.

Defintion 3.11: Let G be an IFG. A subset S of V is called *perfect equitable dominating set* in G if,

(i) For every $v_j \in V - S$, there exist exactly one $v_i \in S$ such that $\mu_{ij} \geq {\mu_{ij}}^{\infty}$ and $\gamma_{ij} \geq {\gamma_{ij}}^{\infty}$

(ii) For every $v_j \in V - S$, there exist $v_i \in S$ such that $(v_i, v_j) \in E$, $|deg_{\mu_i} - deg_{\mu_i}| \leq 1, \mu_{ij} \geq {\mu_{ij}}^{\infty}$ and $|deg_{\gamma_i} - deg_{\gamma_i}| \leq 1, \gamma_{ij} \geq {\gamma_{ij}}^{\infty}$.

Defintion 3.12: The minimum cardinality of perfect equitable dominating set is called the lower perfect equitable domination number of G, denoted by $d_{pe}(G)$.

The maximum cardinality of an perfect equitable dominating set is called the upper perfect equitable domination number of G, denoted by $d_{PE}(G)$.

4. SOME PROPERTIES OF TYPES OF DOMINATION IN IFG

Theorem 4.1: For any complete IFG, G $d_{cp}(G) = (o(G) - \sum_{i=1}^{2}(\frac{1+\mu_i+\gamma_i}{2}))$

Proof: Let G be a complete IFG. Every induced subgraph of an complete IFG is complete. For any two vertices v_i, v_j in G such that $\mu_{ij} = \min(\mu_i, \mu_j)$ and $\gamma_{ij} = \max(\gamma_i, \gamma_j)$. The set $D = \{V(G) - \{v_i, v_j\}\}$ be a complementary pendent dominating set of G. Hence, $d_{cd}(G) = (o(G) - \sum_{i=1}^{2}(\frac{1+\mu_i+\gamma_i}{2}))$

Theorem 4.2: Let K_{V_1,V_2} be a complete bipartite IFG. Then $d_{cp}(K_{V_1,V_2}) \leq (o(V_1) + o(V_2) - \sum_{v_i \in P} (\frac{1+\mu_i+\gamma_i}{2}))$, if $|V_1| \geq 2$, $|V_2| \geq 2$ and P is any arbitary path of length 2 in K_{V_1,V_2}

Proof: Let V_1, V_2 are two disjoint non empty sets in K_{V_1,V_2}. Choose an arbitary path $P = v_1 v_2 v_3$ with length $n = 2$ in K_{V_1,V_2}. Then the set $S = V - \{v_1, v_2, v_3\}$, $V = V_1 \cup V_2$ be the complementary pendent dominating set of K_{V_1,V_2}. Hence, $d_{cp}(K_{V_1,V_2}) \leq (o(V_1) + o(V_2) - \sum_{v_i \in P} (\frac{1+\mu_i+\gamma_i}{2})$. Suppose, consider a path $P = v_1 v_2 v_3 v_4$ with length $n = 3$ in K_{V_1,V_2}. Any subset S'$= V - \{v_1, v_2, v_3 v_4\}$ is not an complementary pendent dominating. The subgraph induced by $V - S'$ has no pendent vertices. Thus, $d_{cp}(K_{V_1,V_2}) > (o(V_1) + o(V_2) - \sum_{v_i \in P} (\frac{1+\mu_i+\gamma_i}{2})$.

Theorem 4.3: Let G be an IFG, then the following results hold:

(i) If $G = (V, E)$ is an intuitionistic fuzzy cycle or path with $|V| \geq 4$, then $d_{cp}(G) \leq (\frac{o(G)}{3})$

(ii) $d(G) \leq d_{cp}(G)$

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Proof

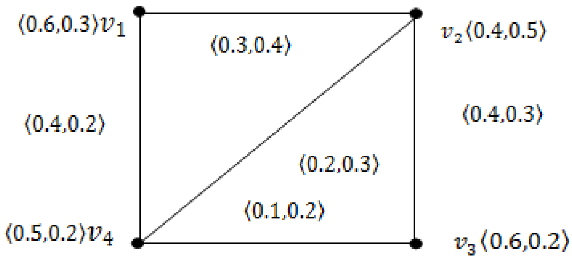
(i) Let $G = (V, E)$ be an intuitionistic fuzzy cycle. Then $S = \{v_1, v_3, v_5 \cdots, v_{2i-1}\}$ be an dominating set of G. The subgraph induced by $(V - S)$ contains a pendent vertex and S itself a complementary pendent dominating set of G. Then, $d_{cp}(G) \leq (\frac{o(G)}{3})$

(ii) Since every complementary pendent dominating set is also a dominating set of G, then $d(G) \leq d_{cp}(G)$

Theorem 4.4: Let $G = (V, E)$ be an IFG and $|V| = n$. Then $d(G) + d_{pd}(G) \leq o(G)$.

Proof: Let $S \subseteq V$ be a dominating set of G. Then S is a dominating set and the subgraph $H = (V', E')$ induced by S containing a pendent vertex. Then, $d_{pd}(G) \leq |S|$. Since S is dominating set $V - S$ is also a dominating set
$d(G) \leq |V - S|$. Hence, $d(G) + d_{pd}(G) \leq o(G)$.

Example 4.1: Consider an IFG, G with $V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1, v_2), (v_2, v_3), (v_1, v_4), (v_2, v_4), (v_4, v_3)\}$.



Here, $\{v_1, v_2\}$ is dominating set. $d(G) = 1.1$, $d_{pd}(G) = 2.2$, o(G)=2.45, then $d(G) + d_{pd}(G) \leq o(G)$.

Theorem 4.5: Let $G = (V, E)$ be an IFG with $supp_G(V) \geq 2$. Then any vertex v_k in G is doubly dominating set if and only if there exist two vertices v_i, v_j in G suchthat $deg_\mu(v_i) = \delta_\mu(G)$ and $deg_\mu(v_j) = \Delta_\mu(G)$.

Proof: Let $S = \{v_j\}$ be a doubly dominating set. Since S is a dominating set and then $deg_\mu(v_j) = \Delta_\mu(G)$. Also, the subgraph $H = (V', E')$ induced by $(V - S)$ contain an isolated vertex v_i such that $\mu_{ij} = 0, \gamma_{ij} = 0$. Thus, the only neighbour of v_i in G is v_j. Hence, $deg_\mu(v_i) = \delta_\mu(G)$

Conversely, there exist a vertex v_j with degree $\Delta_\mu(G)$, $\{v_j\}$ is dominating set of G. Since $deg_\mu(v_i) = \delta_\mu(G)$, v_i is an isolated vertex in $(V(G) - \{v_i\})$. Thus, $S = \{v_j\}$.

5. CONCLUSION

Domination in graph theory has many interesting applications in real world applications such as locating radar stations, nuclear power plants, communication networks and voting situations. Finding the minimal dominating set can be used to optimize time and distance while traveling, to optimize the performance of computer communication networks. The concept of edge contraction, fusion of two vertices, pendent dominating set, complementary pendent dominating set, isolate dominating set, doubly isolate dominating set, perfect secure dominating set in IFG have been introduced. Further, the domination numbers d_{pe}, d_{cp} and d_{dis} for complete, path and cycle of an IFG are investigated.

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