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A COMBINATION OF GREY RELATIONAL ANALYSIS AND MINIMIZATION OF REGRET METHOD IN INTERVAL-VALUED INTUITIONISTIC FUZZY SET: CASE STUDY IN SELECTION PROCESS OF SALES ON MANGO-BASED BEVERAGES

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ABSTRACT

The main scope of this research work is to propose a new formulation by combining the concept of Grey relational analysis and minimization of regret methods to do multiple attribute evaluations and studied for an Interval-valued Intuitionistic fuzzy set. In this research, a real-life problem is given to expose the practicality and effectiveness of the proposed method.

Keywords *- Intuitionistic fuzzy set, Interval-valued IFS, GRA, Minimization of Regret, mango-based beverages.*

Subject Classification Code: *03E72.*

1. INTRODUCTION

Atanassov(1986) introduced Intuitionistic fuzzy set (IFS) which is the generalization of the fuzzy set. IFS is characterized by a membership function, non-membership function, and a hesitancy function. It is a more efficient tool to deal with uncertainty and vagueness in real-life application than Fuzzy Set and it received more attention by the researchers since its appearance [Joshi, B. P., & Kharayat, P. S. (2015)]. Interval-valued Intuitionistic fuzzy set (IVIFS) is introduced by Atanassov, K., & Gargov, G. (1989). IVIFS is an extension of IFS, which is expressed by membership degree range and non-membership degree range rather than exact numbers [Wei, G., & Wang, X. (2007, December)]. Interval-valued Intuitionistic fuzzy set (IVIFS) is effective in dealing with fuzziness and uncertainty inherent in decision data and multi-attribute decision making (MADM). During the last few decades, especially in the business sector multiple attribute decision making (MADM) has been one of the fastest growing areas. MADM consists of finding the most satisfactory alternative from all the feasible alternatives [Hou, J. (2010)].

An approach to multiple attribute decision making (MADM) with an interval-valued Intuitionistic fuzzy set is developed based on the combined concept of GRA and minimum of regret methods. Grey relational analysis (GRA) was originally developed by Deng in 1989. GRA method has been successfully applied and it is the best method to make decisions in a business environment. The major advantages of the GRA method are, the results depend on the original data and the calculations are simple and straightforward [Wei, G. W. (2010)]. In order to derive the attribute weights in GRA method, an optimization model is made based on the concept of the score function and the principle of minimization of regret. First, obtain the weights using minimization of regret method and then rank and select the alternatives using Grey Relational Analysis [Ozturkoglu, Y., & Esendemir, E. (2014)].

To check the effectiveness of the proposed method a real-life problem is considered i.e. ordering the mango-based beverages regarding the sales result. In that problem, the ratings are derived from the domain expert in which all the information is presented as linguistic variables and it switches over into interval-valued Intuitionistic fuzzy numbers and the information about attribute weight is incompletely known in case of the time pressure, lack of knowledge or data, and the expert's limited expertise about the problem domain. [Hou, J. (2010)].

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2. BASIC DEFINITIONS

Definition 1: [George J.Klir/Bo Yaun (1995)] Let X be a Universal set. Let $A \subset R$. Define A: X→ [0, 1] is called membership grade of A. Any set A defined by its membership function $A(x)$ is called fuzzy set denoted by

$$\tilde{A} = \{(x, A(x)) : x \in X\} .$$

Definition 2: [Atanassov, K. T. (1986) & (1999)] An Intuitionistic fuzzy set (IFS) A in E is defined as an object of the following form

$$A=\{<x, \mu_A(x), \gamma_A(x)>\}: x \in E \}$$

where the functions:

$$\mu_A : E \to [0,1] \text{ and } \gamma_A : E \to [0,1]$$

define the degree of membership and the degree of non-membership of the element x∈E, respectively, and for every x∈E : $0 \le \mu_A + \gamma_A \le 1$.

Definition 3: [Atanassov, K., & Gargov, G. (1989).] An interval-valued fuzzy set A (over a basic set E) is specified by a function M_A : E → INT (X[0,1]), where INT (X[0,1]) is the set of all intervals within [0,1], i.e. for all x∈E, $M_A(x)$ is an interval [a,b], $0 \le a \le b \le 1$.

Definition 4: [Atanassov, K., & Gargov, G. (1989).] An interval-valued Intuitionistic fuzzy set A over E is defined as an object of the form

$$A= \{< x, M_A(x), N_A(x)> / x \in E \}$$

where $M_A(x)$ C [0,1] and $N_A(x)$ C [0,1] are intervals, and for all x∈E,

$$\sup M_A(x) + \sup N_A(x) \le 1.$$

Definition 5: [Li, D. F. (2014).] The value of

$$\pi_A (x) = 1- \mu_A(x) - \gamma_A(x)$$

is called the degree of non-determinacy (or uncertainty) of the element x∈E to the intuitionistic fuzzy set A.

Definition 6: [Li, D. F. (2014).] Let A= <μ, γ> be an Intuitionistic fuzzy set. A score function M of an Intuitionistic fuzzy set is defined as

$$S(A) = \mu - \gamma$$

where S(A) ∈ [-1,1].

Definition 7: [Li, D. F. (2014).] The score of an interval-valued Intuitionistic fuzzy set A = $([\mu_{AL}, \mu_{AU}],[\gamma_{AL}, \gamma_{AU}]$ is defined as

$$S(A) = \frac{\mu_{AL} + \mu_{AU} - \gamma_{AL} - \gamma_{AU}}{2}$$

where S(A) ∈ [-1,1].

Definition 8: [Wei, G. W. (2011).] Let $\tilde{a}_1 = (\mu_1, \gamma_1)$ and $\tilde{a}_2 = (\mu_2, \gamma_2)$ be two intuitionistic fuzzy numbers, then the normalized Hamming distance between $\tilde{a}_1 = (\mu_1, \gamma_1)$ and $\tilde{a}_2 = (\mu_2, \gamma_2)$ is defined as

$$d(\tilde{a}_1, \tilde{a}_2) = \frac{1}{2} |\mu_1 - \mu_2| + |\gamma_1 - \gamma_2| \ .$$

Definition 9: [Wei, G. W. (2011).] Let $\tilde{a}_1 = ([a_1, b_1], [c_1, d_1])$ and $\tilde{a}_2 = ([a_2, b_2], [c_2, d_2])$ be two interval-valued intuitionistic fuzzy values, then the normalized Hamming distance between $\tilde{a}_1 = ([a_1, b_1],[c_1, d_1])$ and $\tilde{a}_2 = ([a_2, b_2],[c_2, d_2])$ is defined as

$$d(\tilde{a}_1, \tilde{a}_2) = \frac{1}{4} |a_1 - a_2| + |b_1 - b_2| + |c_1 - c_2| + |d_1 - d_2|.$$

3. MADM Problems with Interval-Valued Intuitionistic Fuzzy Information.

Let A = $\{A_1, A_2, \ldots, A_m\}$ be a finite set of alternative and O = $\{O_1, O_2, \ldots, O_n\}$ be the finite set of attributes in the form of IVIFN, where F = $(< [\mu_{ijL}, \mu_{ijU}], [\gamma_{ijL}, \gamma_{ijU}] >)$where $[\mu_{ijL}, \mu_{ijU}]$ represent the satisfaction degree of alternative with respect to the attribute and $[\gamma_{ijL}, \gamma_{ijU}]$ represent the dissatisfaction degree of alternative with respect to the attribute respectively. Let $w = (w_1, w_2, \ldots, w_n)$ be the weight vector of the attribute O_i(i = 1,2,…,n), where $w_i \ge 0$ and $\sum_{i=1}^{n} w_i = 1$. In the real situations, it is very often that the information about attribute weights is incompletely known.For convenience, let H be a set of the known weight information, which can be constructed by the following forms:

1) A weak ranking: $w_i \geq w$;
2) A strict ranking: $w_i – w_j \geq \delta_i$, $\delta_i > 0$;
3) A ranking of differences: $w_i – w_j \geq w_k – w_i$, for $j \neq k \neq l$;
4) A ranking with multiples: $w_i \geq \delta_i w_j$, $0 \leq \alpha_i \leq 1$;
5) An interval form: $0 \leq \delta_i \leq w_i \leq \delta_i + \varepsilon_i \leq 1$ [Luo, Y., & Wei, G. (2009, May).]

4. PROPOSED METHOD

A new procedure is formed by utilizing the method of grey relation analysis and minimization of regret [Hou, J. (2010) & Luo, Y., & Wei, G. (2009, May)]:

Step-1: Convert the decision matrix into a score matrix by the equation

$$S(A) = \frac{\mu_{AL} + \mu_{AU} - \gamma_{AL} - \gamma_{AU}}{2} \quad (1)$$

where $S(A) \in [-1,1]$.

Step-2: According to the conception of regret, the regret matrix, $R = (r_{ij})_{mxn}$ can be utilized from the score function.

Step-3: Determine the weight vector of the attributes using the single objective programming method.

$$\text{i.e.} \begin{cases} \min f(w) = \sum_{i=1}^{m} f_i(w) = \sum_{i=1}^{m} \sum_{i=1}^{n} w_j \, r_{ij} \\ \text{subject to} : w = (w_1, w_2, \ldots\ldots, w_n) \in H \end{cases}$$

Step-4: Determine the positive ideal solution based on interval-valued Intutionistic fuzzy numbers

$$\tilde{r}^+ = ([a_1^+, b_1^+], [c_1^+, d_1^+], [a_2^+, b_2^+], [c_2^+, d_2^+], \ldots\ldots, [a_n^+, b_n^+][c_n^+, d_n^+])$$

where

$$\tilde{r}_j^+ = ([a_j^+, b_j^+], [c_j^+, d_j^+])$$
$$= ([\, \max_i a_{ij} , \max_i b_{ij}], [\, \min_i c_{ij} , \min_i d_{ij}])\ j \in 1,2, \ldots, n.$$

Step-5: Calculate the grey relational coefficient of each alternative from positive ideal solution using the following equation

$$\xi_{ij}^+ = \frac{\min_{1 \leq i \leq m} \min_{1 \leq i \leq n} d(\tilde{r}_{ij}, r_j^+) + \rho \max_{1 \leq i \leq m} \max_{1 \leq i \leq n} d(\tilde{r}_{ij}, r_j^+)}{d(\tilde{r}_{ij}, r_j^+) + \rho \max_{1 \leq i \leq m} \max_{1 \leq i \leq n} d(\tilde{r}_{ij}, r_j^+)} \quad (2)$$

$i = 1, 2, \ldots, m$ and $j \in 1, 2, \ldots, n$. where the identification coefficient $\rho = 0.5$.

Step-6: Evaluate the degree of grey relational coefficient of each alternative by the equation

$$\xi_{ij}^+ = \sum_{j=1}^{n} \omega_i . \xi_{ij}^+ \qquad i = 1,2, \ldots, m \quad (3)$$

Step-7: Rank all the alternative A_i ($i = 1,2,\ldots..m$) and select the best one in accordance with ξ_i^+($i = 1,2,\ldots..m$). If an alternative has the highest ξ_i^+Value, then it is the most important alternative.

5. APPLICATION OF THE PROPOSED METHOD

To illustrate the proposed method, a selection process of sales on mango-based beverages problem is examined.

Suppose a marketing agency want to interrogation and determine the sales processing of mango-based beverages in Trichy city. So, they consider the following six mango-based beverages(alternative) X1: Tropicana, X2: Maaza, X3: Slice, X4: Frooti, X5: Minute Maid and X6: Real. To evaluate the mango-based beverages, selected attributes are G1: Movie Theater, G2: Restaurant, G3: Malls and G4: School/College Canteen. The ratings of the alternative with respect to attribute are collected from the expert using linguistic terms are represented in table 2 using the linguistic variable a decision matrix is formed under IVIFN (table 3). In this problem, the weight vector is incompletely known, and the partial weight vector is shown below which is given by the expert

$H= \{0.30 \leq w_1 \leq 0.35,\ 0.10 \leq w_2 \leq 0.15,\ 0.20 \leq w_3 \leq 0.26,\ 0.33 \leq w_4 \leq 0.38,\ w_1 + w_2 + w_3 + w_4 = 1,\ 0 \leq w_j \leq 1\ (j = 1, 2, 3, 4)\}$

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Table-1: Linguistic variable for rating the alternative

Linguistic term	**IVIFN**
Extremely high	< [0.7,0.8],[0.1,0.2] >
Very high	< [0.6,0.7],[0.1,0.3] >
High	< [0.5,0.6],[0.2,0.3] >
Medium	< [0.4,0.6],[0.2,0.4] >
Low	< [0.3,0.4],[0.4,0.6] >
Very low	< [0.2,0.5],[0.3,0.4] >
Extremely low	< [0.1,0.3],[0.5,0.6] >

Table-2: Decision Matrix in terms of Linguistic variables

	G1	G2	G3	G4
X1	EL	EL	H	EL
X2	VH	L	M	H
X3	EH	L	H	VH
X4	L	EL	L	H
X5	VL	EL	L	EL
X6	EL	EL	L	EL

Table-3: Interval -valued Intuitionistic Fuzzy Decision Matrix

	G1	G2	G3	G4
X1	<[0.1,0.3],[0.5,0.6]>	<[0.1,0.3],[0.5,0.6]>	<[0.5,0.6],[0.2,0.3]>	<[0.1,0.3],[0.5,0.6]>
X2	<[0.6,0.7],[0.1,0.3]>	<[0.3,0.4],[0.4,0.6]>	<[0.4,0.6],[0.2,0.4]>	<[0.5,0.6],[0.2,0.3]>
X3	<[0.7,0.8],[0.1,0.2]>	<[0.3,0.4],[0.4,0.6]>	<[0.4,0.6],[0.2,0.4]>	<[0.6,0.7],[0.1,0.3]>
X4	<[0.3,0.4],[0.4,0.6]>	<[0.1,0.3],[0.5,0.6]>	<[0.3,0.4],[0.4,0.6]>	<[0.5,0.6],[0.2,0.3]>
X5	<[0.2,0.5],[0.3,0.4]>	<[0.1,0.3],[0.5,0.6]>	<[0.3,0.4],[0.4,0.6]>	<[0.1,0.3],[0.5,0.6]>
X6	<[0.1,0.3],[0.5,0.6]>	<[0.1,0.3],[0.5,0.6]>	<[0.3,0.4],[0.4,0.6]>	<[0.1,0.3],[0.5,0.6]>

Step-1: Convert the decision matrix into a Score Matrix by (1)

$$S = \left(s_{ij}\right)_{6\times 4} = \begin{pmatrix} -0.35 & -0.35 & 0.3 & -0.35 \\ 0.45 & -0.15 & 0.2 & 0.3 \\ 0.6 & -0.15 & 0.2 & 0.45 \\ -0.15 & -0.35 & -0.15 & 0.3 \\ 0 & -0.35 & -0.15 & -0.15 \\ -0.35 & -0.35 & -0.15 & -0.35 \end{pmatrix}$$

Step-2: According to the concept of regret, the regret matrix R= (r_{ij}) can be obtained

$$R = \left(r_{ij}\right)_{6\times 4} = \begin{pmatrix} 0.625 & 0.625 & 0 & 0.625 \\ 0 & 0.6 & 0.25 & 0.15 \\ 0 & 0.75 & 0.4 & 0.15 \\ 0.45 & 0.65 & 0.45 & 0 \\ 0 & 0.35 & 0.15 & 0.15 \\ 0.05 & 0.05 & 0 & 0.05 \end{pmatrix}$$

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Step-3: Determine the weight vector of the attribute using the single objective programming model.

min $f(w) = 1.125\, w_1 + 3.025\, w_2 + 1.25\, w_3 + 1.125\, w_4$

Subject to: $0.30 \le w_1 \le 0.35$,

$$\begin{cases} 0.10 \le w_2 \le 0.15 \\ 0.20 \le w_3 \le 0.26 \\ 0.33 \le w_4 \le 0.38 \\ w_1 + w_2 + w_3 + w_4 = l, \\ 0 \le w_j \le 1 (j = 1,2,3,4) \end{cases}$$

Solving this single objective programming model, the attained weight vector of the attribute is

$w_1 = 0.35,\ w_2 = 0.10,\ w_3 = 0.20,\ w_4 = 0.35$

Step-4: Determine the positive ideal solution from IVIFDM

$\tilde{r}^+$ = [([0.5,0.6],[0.2,0.3]), ([0.6,0.7],[0.1,0.3]), ([0.7,0.8],[0.1,0.2]), ([0.5,0.6],[0.2.0.3]), ([0.3, 0.5], [0.3,0.4]), ([0.3, 0.4], [0.4, 0.6])]

Step-5: Calculate the grey relational coefficient of each alternative for PIS using (2)

$$\xi_{ij}^{+} = \begin{pmatrix} 0.3658 & 0.3658 & 1.0000 & 0.3658 \\ 1.0000 & 0.3846 & 0.6 & 0.7142 \\ 1.0000 & 0.3333 & 0.4838 & 0.7142 \\ 0.4545 & 0.3658 & 0.4545 & 1.0000 \\ 0.8823 & 0.4838 & 0.6521 & 0.4838 \\ 0.6521 & 0.6521 & 1.0000 & 0.6521 \end{pmatrix}$$

Step-6: Evaluate the degree of grey relational coefficient by (3)

$\xi_1^+ = 0.49264,\ \xi_2^+ = 0.75843,\ \xi_3^+ = 0.73006,\ \xi_4^+ = 0.636555,\ \xi_5^+ = 0.656935,\ \xi_6^+ = .72165.$

Step-7: Depending on the degree of grey relational coefficient the alternatives are arranged in an order is: *X2 > X3 > X6 > X5 > X4 > X1* and the most desirable alternative that is the best sale among the one is ***X2* (Maaza).**

6. DISCUSSION

To correlate the proposed method (especially in finding attribute weight), a few GRA methods are adapted and the decisions are compiled in table 4

Table-4: Comparison Results

Method	Order of the alternatives	Best alternative
Proposed method	*X2>X3>X6>X5>X4>X1 (0.75843>0.73006>0.72165>0.656935>0.636555>0.49264)*	*X2 (0.75843)*
Hou, J. (2010)	*X2>X6>X3>X5>X4>X1 (0.756146>0.728635>0.725452>0.660301>0.625645>0.505324)*	*X2 (0.756146)*
Wei, G. W. (2011)	*X2>X6>X3>X5>X4>X1 (0.694146>0.676033>0.665776>0.603144>0.59383>0.467034)*	*X2 (0.694146)*

While seeing the comparison table it doesn't have any major changes in ordering the sales of mango-based beverages. So, the proposed method is reliable compared with another method.

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7. CONCLUSION

The objective of the study is to combine the concept of GRA and Minimum of regret. For that, a proposed method is accomplished and studied for a real-life problem. The proposed method is used in this paper to order and select the best sale regarding mango-based beverages. The values are gathered in the form of linguistic variables and after that converted into a IVIFS. In this paper, the attribute weights are incompletely known and latterly it was obtained by the proposed method. It is found that Maaza has retained a maximum number of consumer/customer whereas other brands have retained a lesser number of consumers/customer in some areas in Trichy city, Tamil Nadu.

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