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A COMPARATIVE STUDY OF VARIOUS DISTANCE MEASURES IN INTUITIONISTIC FUZZY SETS AND THEIR EXTENSIONS

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ABSTRACT

In this paper, we define the Hamming distance measure and normalized Hamming distance measure of Intuitionistic Fuzzy Sets of Second Type and Intuitionistic Fuzzy Sets of Third Type. Also the comparison is made between the measures.

Keywords: *Intuitionistic Fuzzy Set (IFS), Intuitionistic Fuzzy Set of Second Type (IFSST), Intuitionistic Fuzzy Sets of Third Type (IFSTT), Hamming distance, normalized Hamming distance.*

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1. INTRODUCTION

After inspired from K.T. Atanassov Intuitionistic Fuzzy Set[1], the present authors defined IFSTT[11] which is an extension of IFS. Many researchers studied the application of IFS and their extension in real life situations, like pattern recognition, medical diagnosis, career determination, electoral system and market prediction by using distance measures [3,4,5,6]. In this research article, we define the Hamming distance and normalized Hamming distance measures of IFSST and IFSTT, and also the comparison is made with the existing measure on IFS. The rest of the paper is designed as follows: Insection2, we give the definitions of IFS and their extensions. In section3, we define the distance measures of IFSST and IFSTT with suitable example. The paper concludes in section4.

2. PRELIMINARIES

In this section, we give some definitions of IFS and their extensions.

Definition 2.1: [1] Let X be a non-empty set. An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

where $\mu_A(x): X \to [0,1]$ and $\nu_A(x): X \to [0,1]$ denote the degree of membership and non-membership functions of A respectively, and

$$0 \le \mu_A(x) + \nu_A(x) \le 1, \text{ for each} x \in X.$$

Definition 2.2: [1] The degree of non-determinacy (uncertainty) of an element $x \in X$ in the IFS A is defined by

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) .$$

Definition 2.3: [7] Let$X$ be a non-empty set. An Intuitionistic Fuzzy Sets of Second Type (IFSST) A in X is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

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where $\mu_A(x): X \to [0,1]$ and $\nu_A(x): X \to [0,1]$ denote the degree of membership and non-membership functions of A respectively, and

$$0 \leq \mu_A^2(x) + \nu_A^2(x) \leq 1, \text{ for each } x \in X.$$

Definition 2.4: [7] The degree of non-determinacy (uncertainty) of an element $x \in X$ in the IFSST A is defined by

$$\pi_A(x) = \sqrt{1 - \mu_A^2(x) - \nu_A^2(x)}\,.$$

Definition 2.5: [8−14] Let X be the non-empty set. An Intuitionistic Fuzzy Sets of Third Type (IFSTT) A in X is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

where $\mu_A(x): X \to [0,1]$ and $\nu_A(x): X \to [0,1]$ denote the membership and non- membership functions of A, respectively, and

$$0 \leq \mu_A^3(x) + \nu_A^3(x) \leq 1, \text{ for each } x \in X.$$

Definition 2.6: [11] The degree of non-determinacy (uncertainty) of an element $x \in X$ in the IFSTT A is defined by $\pi_A(x) = \sqrt[3]{1 - \mu_A^3(x) - \nu_A^3(x)}\,.$

Definition 2.7: [15] Let A, B, C be the IFSs in X, then the distance measure d between A and B is a mapping $d: X \times X \to [0,1]$ satisfying the following axioms:

(a) $0 \leq d(A,B) \leq 1$ (boundedness)
(b) $d(A,B) = d(B,A)$(symmetric)
(c) $d(A,B) = 0$ if and only if $A = B$
(d) $d(A,C) + d(C,B) \geq d(A,B)$ (triangle inequality)
(e) if $A \subseteq B \subseteq C$, then $d(A,C) \geq d(A,B)$ and $d(A,C) \geq d(B,C)$.

Definition 2.8: [15] Let

$$A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle : x_i \in X\}, \forall i = 1,2,\ldots.,n,$$

and

$$B = \{\langle x_i, \mu_B(x_i), \nu_B(x_i)\rangle : x_i \in X\}, \forall i = 1,2,\ldots.,n,$$

be two IFSs in X. Then the distance measures between A and B is defined by:

The Hamming Distance:

$$d_H(A,B) = \frac{1}{2}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|)$$

The normalized Hamming Distance:

$$d_{n-H}(A,B) = \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|)$$

Example: Let $X = \{1,2,3,4,5,6,7\}$. Let $A = \{\langle 1,0.7,0.3\rangle, \langle 2,0.2,0.8\rangle, \langle 4,0.6,0.4\rangle, \langle 5,0.5,0.5\rangle, \langle 6,1,0\rangle\}$, and $B = \{\langle 1,0.2,0.8\rangle, \langle 4,0.6,0.4\rangle, \langle 5,0.8,0.2\rangle, \langle 7,1,0\rangle\}$ be two IFSs in X.

Using the definition 2.8, we get the hamming distance between A and B is

$$d_H(A,B) = 3$$

and the normalized hamming distance is

$$d_{n-H}(A,B) = 0.428571$$

3. DISTANCEMEASUREOFIFSSTANDIFSTT

In this section, we define the Hamming and normalized Hamming distance measures of IFSST and IFSTT with suitable example. Also, the comparison is made between the measures.

Definition 3.1: Let A, B, C be the IFSSTs in X, then the distance measure d between A and B is a mapping $d: X \times X \to [0,1]$ satisfying the following axioms:

(a) $0 \leq d(A,B) \leq 1$ (boundedness)

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(b) $d(A,B) = d(B,A)$(symmetric)
(c) $d(A,B) = 0$ if and only if $A = B$
(d) $d(A,C) + d(C,B) \geq d(A,B)$ (triangle inequality)
(e) if $A \subseteq B \subseteq C$, then $d(A,C) \geq d(A,B)$ and $d(A,C) \geq d(B,C)$.

Definition 3.2: Let

$$A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle : x_i \in X\}, \forall i = 1,2,\ldots.,n,$$

and

$$B = \{\langle x_i, \mu_B(x_i), \nu_B(x_i)\rangle : x_i \in X\}, \forall i = 1,2,\ldots.,n,$$

be two IFSSTs in X. Then the distance measures between A and B is defined by:

The Hamming Distance:

$$d_H(A,B) = \frac{1}{2}\sqrt{\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)|^2 + |\nu_A(x_i) - \nu_B(x_i)|^2 + |\pi_A(x_i) - \pi_B(x_i)|^2)}$$

The normalized Hamming Distance:

$$d_{n-H}(A,B) = \frac{1}{2n}\sqrt{\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)|^2 + |\nu_A(x_i) - \nu_B(x_i)|^2 + |\pi_A(x_i) - \pi_B(x_i)|^2)}$$

Example: Let $X = \{1,2,3,4,5,6,7\}$. Let $A = \{\langle 1,0.7,0.3\rangle, \langle 2,0.2,0.8\rangle, \langle 4,0.6,0.4\rangle, \langle 5,0.5,0.5\rangle, \langle 6,1,0\rangle\}$, and $B = \{\langle 1,0.2,0.8\rangle, \langle 4,0.6,0.4\rangle, \langle 5,0.8,0.2\rangle, \langle 7,1,0\rangle\}$ be two IFSSTs in X.

Using the definition 3.2, we get the hamming distance between A and B is

$$d_H(A,B) = 1.12991$$

and the normalized hamming distance is

$$d_{n-H}(A,B) = 0.161416$$

Definition 3.3: Let A, B, C be the IFSTTs in X, then the distance measure d between A and B is a mapping $d: X \times X \to [0,1]$ satisfying the following axioms:

(a) $0 \leq d(A,B) \leq 1$ (boundedness)
(b) $d(A,B) = d(B,A)$(symmetric)
(c) $d(A,B) = 0$ if and only if $A = B$
(d) $d(A,C) + d(C,B) \geq d(A,B)$ (triangle inequality)
(e) if $A \subseteq B \subseteq C$, then $d(A,C) \geq d(A,B)$ and $d(A,C) \geq d(B,C)$.

Definition 3.4: Let

$$A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle : x_i \in X\}, \forall i = 1,2,\ldots.,n,$$

and

$$B = \{\langle x_i, \mu_B(x_i), \nu_B(x_i)\rangle : x_i \in X\}, \forall i = 1,2,\ldots.,n,$$

be two IFSTTs in X. Then the distance measures between A and B is defined by:

The Hamming Distance:

$$d_H(A,B) = \frac{1}{2}\sqrt{\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)|^3 + |\nu_A(x_i) - \nu_B(x_i)|^3 + |\pi_A(x_i) - \pi_B(x_i)|^3)}$$

The normalized Hamming Distance:

$$d_{n-H}(A,B) = \frac{1}{2n}\sqrt{\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)|^3 + |\nu_A(x_i) - \nu_B(x_i)|^3 + |\pi_A(x_i) - \pi_B(x_i)|^3)}$$

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Example: Let $A = \{\langle 1, 0.7, 0.3\rangle,\ \langle 2, 0.2, 0.8\rangle,\ \langle 4, 0.6, 0.4\rangle,\ \langle 5, 0.5, 0.5\rangle,\ \langle 6, 1, 0\rangle\}$, and $B = \{\langle 1, 0.2, 0.8\rangle,\ \langle 4, 0.6, 0.4\rangle,$ $5, 0.8, 0.2, 7, 1, 0$ be two IFSTTs in X.

Using the definition 3.4, we get the hamming distance between A and B is

$$d_H(A, B) = 0.843573$$

and the normalized hamming distance is

$$d_{n-H}(A, B) = 0.12051$$

The following table gives the comparison between the distance measures among IFS and their extensions.

Measures	IFS	IFSST	IFSTT
Hamming Distance	3	1.12991	0.843573
Normalized Hamming Distance	0.428571	0.161416	0.12051

4. CONCLUSION

In this paper, we have defined the Hamming and normalized Hamming distance measures of IFSST and IFSTT and the comparison is made with the suitable example. It is concluded from the table that the normalized hamming distance measure of IFSTT gives the shortest value among all the measures. It is open to check the application of these measures in real life situations.

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