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SOME PROPERTIES OF INTUITIONISTIC FUZZY BI-IDEALS OF NEAR RINGS

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ABSTRACT

In this present paper, we introduce the concept of intuitionistic fuzzy bi-ideals of near-rings. Also we investigate some algebraic nature of intuitionistic fuzzy bi-ideals of near-rings and some related properties of these fuzzy substructures.

Keywords: *Near-rings, Bi-ideals, Fuzzy bi-ideals, Intuitionistic fuzzy set, Intuitionistic fuzzy subring, Intuitionistic fuzzy ideal, Intuitionistic fuzzy bi-ideal.*

Mathematics Subject Classification: *16D25; 03E72; 16Y30; 03F55.*

1. INTRODUCTION

The notion of intuitionistic fuzzy set (IFS) was introduced by Atanassov [2] as a generalization of notion of fuzzy sets. The concept of near-rings was introduced by Pilz [9] and that of quasi-ideal in near ring was introduced by Yakabe [12]. The notion of bi-ideals was introduced by Chelvam and Ganesan [4].

In this paper we study the intuitionistic fuzzification of the notion of bi-ideals in near-rings. We show that every intuitionistic fuzzy bi-ideal of a near-ring is an intuitionistic fuzzy subnear-ring. We give characterizations of intuitionistic fuzzy bi-ideals in near-rings.

A near-ringisa non empty set N with two binary operations "+" and "."such that
- (i) (N,+) is a group not necessarily abelian
- (ii) (N, .) is a semi group
- (iii) $(x + y).z = x.z + y.z$, for all $x, y, z \in$ N.

Precisely speaking it is a right near-ring because it satisfies the right distributive law. If the condition (iii) is replace by $z(x + y) = z.x + z.y$ for all $x, y, z \in$ N, then it is called left near- ring .We denote *xy*instead of *x.y*. A near-ring N is called zerosymmetric if $x.0 = 0$ for all $x \in$N.
Given two subsets A and B of N, the product ABis defined as

$$AB = \{ab | a \in A,\ b \in B\}$$

A subgroup S of (N, +) is called left (right) N-subgroup of N if NS ⊆S(SN⊆S). A subgroup M of (N, +) is called subnear-ring of N if MM ⊆ M. A subnear-ring M is called invariant in N if MN ⊆ NM ⊆ M.

2. PRELIMINARIES

Throughout this paper *N* stands for a right zero symmetric near-ring.
Definition 2.1: An ideal of a near-ring N is a subset I of N such that
- (i) (I , +) is normal subgroup of (N , +)
- (ii) I N ⊆I
- (iii) $y(x + i) - yx \in$ I for all $x, y \in$ N and $i \in$ I

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Note that I is right ideal of N if I satisfies (i) and (ii), and I is left ideal of N if I satisfies (i) and (iii).

Definition 2.2: A subgroup Q of N is called a quasi-ideal of N if QN∩NQ∩N*Q⊆Q.

Proposition 2.3: Let Q be a quasi-ideal and M be a sub near-ring of a near-ring N then Q∩M is a quasi-ideal of N.

Proposition 2.4: Let N be a zero symmetric near-ring and Q be the subgroup of N. Then Q is a quasi-ideal of N if and only if QN ∩ NQ ⊆ Q.

Proof: Let $n \in$ N, $q \in$ Q be any element. As N is zero symmetric near-ring. Therefore $nq=n(0 + q) - n0 \in$ N* Q implies NQ⊆N*Q so that NQ∩N*Q =NQ.Hencewe have QN ∩ NQ ⊆ Q.

Definition 2.5: A subgroup B of N is called a bi-ideal of N if BNB∩(BN)*B⊆B.

Proposition 2.6: Let N be a zero symmetric near-ring and B be the subgroup of N. Then B is abi-ideal if andonly if BNB ⊆ B.

Proof: Since N is a zero symmetric, therefore NB⊆N*B. Hence BNB = BNB ∩ BNB ⊆ BNB ∩ (BN)*B ⊆ B.

Proposition 2.7: Intersection of a bi-ideal B and a sub near-ring S of a near-ring is a bi-ideal of S.

Lemma 2.8: Let N be a zero symmetric near-ring and Q be a quasi-ideal in N. Then every Q is bi-ideal.

Proof: Let Q be a quasi ideal in a zero symmetric near-ring N. Then (Q, +) is a subgroup of N and QN ∩ NQ ⊆ Q. Now, QNQ ⊆ QN and QNQ ⊆ NQ. Thus, QNQ ⊆ QN ∩ NQ ⊆ Q.Hence Q is bi-ideal of N.

Definition 2.9: Let X be a non-empty set. A mapping μ: X→ [0, 1] is a fuzzy set in X.The complement of μ, denoted by μ^c, is the fuzzy set in X given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in$X. For anyI ⊆ X, $\chi_I$denote the characteristicfunction of I.

Definition 2.10: For any fuzzy set μ in X and $r\in$ [0, 1], we define two sets, U $(\mu, r) = \{x \in X \mid \mu(x) \geq r\}$ and L$(\mu, r) = \{x \in X \mid \mu(x) \leq r\}$, which are called an upper andlower r-level cut of μ respectively and can be used to the characterization of μ.

Definition 2.11: A fuzzy set μ in N is a fuzzy subnear-ring of N if for all $x, y \in$N,

(i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$ and
(ii) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$.

Definition 2.12: A fuzzy set μ in N is a fuzzy bi-ideal of N if for all $x, y \in$N,

(i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$ and
(ii) $\mu(xyz) \geq \min\{\mu(x), \mu(z)\}$ for all $x, y, z \in$N.

3. INTUITIONISTIC FUZZY SETS AND BI-IDEALS

Definition 3.1: An intuitionistic fuzzy set A in a non-empty set X is an object having the form A= $\{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$, where the functions μ_A: X → [0, 1]andν_A:X → [0, 1]denote the degree of membership and the degree of non-membership of each element $x \in$Xto the setA, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$for all $x \in$X.

For the sake of simplicity, we shall use the symbol A = (μ_A, ν_A) for the intuitionistic fuzzy set A = $\{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$.

Definition 3.2: An intuitionistic fuzzy set A = (μ_A, ν_A) of a group (G, +) is said to be an intuitionistic fuzzy subgroup of G if for all $x, y \in$ G

(i) $\mu_A(x + y) \geq min\{\mu_A(x), \mu_A(y)\}$
(ii) $\mu_A(-x) = \mu_A(x)$
(iii) $\nu_A(x + y) \leq max\{\nu_A(x), \nu_A(y)\}$
(iv) $\nu_A(-x) = \nu_A(x)$

Equivalently, $\mu_A(x - y) \geq min\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(x - y) \leq max\{\nu_A(x), \nu_A(y)\}$for all $x, y \in$G.

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Let A = (μ_A, ν_A) and B = (μ_B, ν_B) be two intuitionistic fuzzy subsetof a near-ring N. We define the product of A and B as AB = (μ_{AB} , ν_{AB}). If S ⊆ N, then, we define the characteristic functionχ_s on N is defined as

$$\chi_s(x) = \begin{cases} (1,0) \text{ if } x \in S \\ (0,1) \text{ if } x \in N\backslash S \end{cases}$$

The characteristic function on N is χ_N and $\chi_N(x)$ = (1, 0) for all$x$ ∈N

Definition 3.3: An intuitionistic fuzzy set A = (μ_A, ν_A) in N is an intuitionistic fuzzy subnear-ring of N if for all x, y∈ N,

(i) $\mu_A(x - y) \geq min\{\mu_A(x), \mu_A(y)\}$
(ii) $\mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\}$
(iii) $\nu_A(x - y) \leq max\{\nu_A(x), \nu_A(y)\}$
(iv) $\nu_A(xy) \leq max\{\nu_A(x), \nu_A(y)\}$.

Definition 3.4: An intuitionistic fuzzy set A = (μ_A, ν_A) in N is an intuitionistic fuzzy bi-ideal of N if for all x, y, z ∈N,

(i) $\mu_A(x - y) \geq min\{\mu_A(x), \mu_A(y)\}$
(ii) $\mu_A(xyz) \geq min\{\mu_A(x), \mu_A(z)\}$
(iii) $\nu_A(x - y) \leq max\{\nu_A(x), \nu_A(y)\}$
(iv) $\nu_A(xyz) \leq max\{\nu_A(x), \nu_A(z)\}$.

Lemma 3.5: Let A = (μ_A, ν_A) be an intuitionistic fuzzy setinN. Then A is an intuitionistic fuzzy bi-ideal of N if and only if the fuzzy sets μ_A *and* $\nu^c_A$are fuzzy bi-ideals of N.

Proof: If A= (μ_A, ν_A) is an intuitionistic fuzzy bi-ideal of N, then clearly μ_A is a fuzzy bi-ideal of N.

For all x, y∈N,

$$\begin{aligned} \nu^c_A(x - y) &= 1 - \nu_A(x-y) \\ &\geq 1 - \max\{\nu_A(x), \nu_A(y)\} \\ &= \min\{1 - \nu_A(x), 1 - \nu_A(y)\} \\ &= \min\{\nu^c_A(x), \nu^c_A(y)\}. \end{aligned}$$

For all x, y, z∈N,

$$\begin{aligned} \nu^c_A(xyz) &= 1 - \nu_A(xyz) \\ &\geq 1 - \max\{\nu_A(x), \nu_A(z)\} \\ &= \min\{1 - \nu_A(x), 1 - \nu_A(z)\} \\ &= \min\{\nu^c_A(x), \nu^c_A(z)\}. \end{aligned}$$

Thus ν^c_A is a fuzzy bi-ideal of N.

Conversely, suppose that μ_A and ν^c_A are fuzzy bi-ideals of N, then clearly the conditions (i) and (ii) of Definition 3.4 are valid.

Now for all x, y∈N,

$$\begin{aligned} 1 - \nu_A(x-y) &= \nu^c_A(x-y) \\ &\geq \min\{\nu^c_A(x), \nu^c_A(y)\} \\ &= 1 - \max\{\nu_A(x), \nu_A(y)\}. \end{aligned}$$

Therefore $\nu_A(x-y) \leq max\{\nu_A(x), \nu_A(y)\}$.

For all x, y, z∈N,

$$\begin{aligned} 1 - \nu_A(xyz) &= \nu^c_A(xyz) \\ &\geq \min\{\nu^c_A(x), \nu^c_A(z)\} \\ &= 1 - \max\{\nu_A(x), \nu_A(z)\}. \end{aligned}$$

Therefore $\nu_A(xyz) \leq \max\{\nu_A(x), \nu_A(z)\}$.

Thus A = (μ_A, ν_A) is an intuitionistic fuzzy bi-ideal of N.

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Theorem 3.6: Let A = (μ_A, ν_A) be an intuitionistic fuzzy set in N. Then A is an intuitionistic fuzzy bi-ideal of N if and only if A = (μ_A, $\mu^c{}_A$) and A = ($\nu^c{}_A$, ν_A) are intuitionistic fuzzy bi-ideals of N.

Proof: If A= (μ_A, ν_A) is an intuitionistic fuzzy bi-ideal of *N*, then μ_A= $(\mu^c{}_A)^c$ and $\nu^c{}_A$ are fuzzy bi-ideals of *N*, from Lemma 3.5. Therefore A = (μ_A, $\mu^c{}_A$) and A= ($\nu^c{}_A$, ν_A) are intuitionistic fuzzy bi-ideals of N.

Conversely, if A and A are intuitionistic fuzzy bi-ideals of N, then the fuzzy sets μ_A and $\nu^c{}_A$ are fuzzy bi-ideals of N. Therefore A = (μ_A, ν_A) is an intuitionistic fuzzy bi-ideal of N.

Theorem 3.7: Let A = (μ_A, ν_A) be an intuitionistic fuzzy setinN. Then Ais a fuzzybi-ideal of N if and only if all the non-empty sets U(μ_A, *r*) *and* L(ν_A, *t*)are bi-ideals of N for all $r \in Im(\mu_A)$ *and* $t \in Im(\nu_A)$ respectively.

Proof: Suppose that A = (μ_A, ν_A) is an intuitionistic fuzzy bi-ideal ofN. For *x, y*∈U(μ_A, *r*), we have $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\} \geq r$. Hence, $x-y$∈U(μ_A, *r*). Let *x, z* ∈ U(μ_A, *r*) and *y* ∈ N. Then $\mu_A(xyz) \geq \min\{\mu_A(x), \mu_A(z)\} \geq r$ and so *xyz* ∈ U(μ_A, *r*).

Hence U(μ_A, *r*) is a bi-ideal of N for all *r*∈Im(μ_A). Similarly we can show that L(ν_A, *t*) is also a bi-ideal of N for all *t*∈Im(ν_A).

Conversely suppose that U(μ_A, *r*) and L(ν_A, *t*) are bi-ideals of *N* for all *r* ∈ Im(μ_A) and *t* ∈ Im(ν_A) respectively. Suppose that *x, y* ∈ N and $\mu_A(x-y) < \min\{\mu_A(x), \mu_A(y)\}$. Choose *r* such that $\mu_A(x-y) < r < \min\{\mu_A(x), \mu_A(y)\}$. Then we get *x, y*∈U($\mu_A$, *r*) but $x-y$∉U(μ_A, *r*), a contradiction.

Hence $\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\}$. A similar argument shows that$\mu_A(xyz) \geq \min\{\mu_A(x), \mu_A(z)\}$ for all *x, y, z*∈N. Likewise we can show that $\nu_A(x-y) \leq \max\{\nu_A(x), \nu_A(y)\}$ and $\nu_A(xyz) \leq \max\{\nu_A(x), \nu_A(z)\}$. Hence A = ($\mu_A$, ν_A) is an intuitionistic fuzzy bi-ideal of *N*.

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