

INTUITIONISTIC L - FUZZY ALMOST IDEALS

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ABSTRACT

L- Fuzzy Almost Ideal is a generalisation of L-fuzzy ideal and L-Fuzzy Almost Ideal are defined even when the lattice L is not necessarily distributive. In our previous papers, the concept of L-Fuzzy Almost Ideal was introduced and concept of primality in LFAI was studied. The aim of this current paper is to define the intuitionistic version of LFAI and investigate some of its properties.

Keywords: L-Fuzzy Sets, L-Fuzzy Ideals, L-Fuzzy Almost Ideals, Non-Distributive lattice, Intuitionistic L-fuzzy Almost Ideal, Primality of Intuitionistic L-fuzzy Almost Ideal, Almost bi- level set and Primary Intuitionistic L-fuzzy Almost Ideal.

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I. INTRODUCTION

The concept of Intuitionistic Fuzzy Sets (IFS) was proposed by K. T. Atanassov [3] and this initiated the investigation of various algebraic structures over IFS. In [2], the concept of Intuitionistic L- Fuzzy Sets was introduced by K. Atanassov, S. Stoeva and later extended in [4]. In [10], Intuitionistic L-fuzzy rings and Intuitionistic L-fuzzy ideals have been studied by K. Meena. In [12], M. Palanivelrajan and S. Nandhakumar defined Intuitionistic Fuzzy Semi - Primary Ideals over Rings and discussed some of its operators. F. J. Lobillo, O. Cortadellas and G. Navarro [8] have reviewed and collected the various definitions of prime fuzzy ideal and semiprime fuzzy ideal and extended the concepts to non-commutative rings.

In our previous papers [16] and [17], the concept of L-Fuzzy Almost Ideal (LFAI) was introduced and concept of primality in LFAI was studied. L- Fuzzy Almost Ideal is a generalisation of L-fuzzy ideal and L-Fuzzy Almost Ideal are defined even when the lattice L is not necessarily distributive.

The aim of this current paper is to define the intuitionistic version of LFAI and investigate some of its properties.

II. PRELIMINARIES

In this section, we list some basic concepts and well known results of fuzzy set and intuitionistic fuzzy set theory. Throughout this paper, R will be a ring with unity. Let X be a nonempty subset and (L, \vee, \wedge, \leq) be a lattice, which has least and greatest elements, say 0 and 1 respectively.

In this section L is assumed to be distributive.

Definition 2.1: Let X be a nonempty set. A mapping $\mu: X \rightarrow [0,1]$ is called a fuzzy subset of X.

Definition 2.2: Let X be a nonempty set. A mapping $\mu: X \rightarrow L$ is called a L-fuzzy subset of X.

Definition 2.3: A fuzzy subset μ of a ring R is called a fuzzy ideal of R if,

- (i) $\mu(x-y) \geq \mu(x) \wedge \mu(y)$, for all $x, y \in R$
- (ii) $\mu(xy) \geq \mu(x) \vee \mu(y)$, for all $x, y \in R$.

Definition 2.4: let μ be any fuzzy subset of a set X , $t \in [0, 1]$. Then the set

$\mu_t = \{x \in X \mid \mu(x) \geq t\}$ is called a level set of μ . More generally if μ is L- fuzzy set defined by $\mu: X \rightarrow L$ then the set $\mu_t = \{x \in X \mid \mu(x) \geq t\}$ is called a level set of μ .

Definition 2.5: [3] An Intuitionistic Fuzzy Set (IFS) $A \in X$ is defined as an object of the form

$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid \forall x \in X \}$, where the function $\mu_A: X \rightarrow [0,1]$ and

$\nu_A: X \rightarrow [0,1]$ denote the degree of membership and the degree of non-membership function of A respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, for every x in X .

Definition 2.6: [2] Let (L, \leq) be any lattice with an involutive order reversing operation $N: L \rightarrow L$. Let X be any non-empty set. An Intuitionistic L-Fuzzy Set (ILFS) A in X is defined as an object of the form

$A = \{ \langle x, \mu(x), \nu(x) \rangle \mid \forall x \in X \}$, where the functions $\mu: X \rightarrow L$ and $\nu: X \rightarrow L$ define the degree of membership and the degree of non-membership respectively and for every x in X satisfy $\mu(x) \leq N(\nu(x))$.

Definition 2.7: [9] An Intuitionistic L-fuzzy subset $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid \forall x \in X \}$ of a ring R is called an Intuitionistic L-fuzzy subring of R if,

- (i) $\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y)$, for all $x, y \in R$
- (ii) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$, for all $x, y \in R$
- (iii) $\nu_A(x - y) \leq \nu_A(x) \vee \nu_A(y)$, for all $x, y \in R$
- (iv) $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$, for all $x, y \in R$

Definition 2.8: [9] Let $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid \forall x \in X \}$ be an Intuitionistic L-fuzzy set of R . It is called an Intuitionistic L-fuzzy ideal (ILFI) of R if,

- (i) $\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y)$, for all $x, y \in R$
- (ii) $\mu_A(xy) \geq \mu_A(x) \vee \mu_A(y)$, for all $x, y \in R$
- (iii) $\nu_A(x - y) \leq \nu_A(x) \vee \nu_A(y)$, for all $x, y \in R$
- (iv) $\nu_A(xy) \leq \nu_A(x) \wedge \nu_A(y)$, for all $x, y \in R$

Definition 2.9: If A, B are L-fuzzy sets their product is defined as follows;

$$A \circ B(x) = \bigvee_{\{(y,z) \mid x=yz\}} A(y) \wedge B(z)$$

F.J.Lobillo, O.Cortadellas and G.Navarro have collected and listed the various definitions for primality of fuzzy ideals as follows:

Definition 2.10: (D₀- prime) A non-constant fuzzy ideal $P: R \rightarrow [0,1]$ is said to be prime if, whenever $x_t y_s \leq P$ for any singletons x_t and y_s , then $x_t \leq P$ or $y_s \leq P$.

Definition 2.11: (D₁- prime) A non-constant fuzzy ideal $P: R \rightarrow [0, 1]$ is said to be prime if, whenever $I \circ J \leq P$ for some fuzzy ideal I and J , $I \leq P$ or $J \leq P$.

Definition 2.12: (D₂- prime) A non-constant fuzzy ideal $P: R \rightarrow [0, 1]$ is said to be prime if the level ideal P_α is prime for any $P(0) \geq \alpha > P(1)$.

Definition 2.13: (D₃- prime) A non-constant fuzzy ideal $P: R \rightarrow [0, 1]$ is said to be prime if, for any $x, y \in R$, whenever $P(xy) = P(0)$, then $P(x) = P(0)$ or $P(y) = P(0)$.

Definition 2.14: (D₄- prime) A non-constant fuzzy ideal $P: R \rightarrow [0, 1]$ is said to be prime if, for any $x, y \in R$, whenever $P(xy) = P(x)$ or $P(xy) = P(y)$.

III. L-FUZZY ALMOST IDEAL

Notation: Consider $\mu: X \rightarrow L$. If L is totally ordered then for all $x, y \in R$, $\mu(x)$ and $\mu(y)$ are comparable. That is either $\mu(x) > \mu(y)$ or $\mu(x) = \mu(y)$ or $\mu(x) < \mu(y)$. But if L is not totally ordered then are four possibilities $\mu(x) > \mu(y)$ or $\mu(x) = \mu(y)$ or $\mu(x) < \mu(y)$ or $\mu(x)$ and $\mu(y)$ are not comparable. We use the notation $\mu(x) \not\leq \mu(y)$ to mean, " $\mu(x) > \mu(y)$ or $\mu(x) = \mu(y)$ or $\mu(x)$ and $\mu(y)$ are not comparable".

In our previous papers we had given the following definitions

Definition 3.1: Let R be a ring with unity. Let L be a lattice (L, \vee, \wedge, \leq) not necessarily distributive with least and greatest element 0 and 1 respectively. $\mu : R \rightarrow L$ with $\mu(0)=1$ and $\mu(1) = 0$ is said to be L-fuzzy almost ideal if ,

- (i) $\mu(x-y) \preceq \mu(x) \wedge \mu(y)$, for all $x, y \in R$.
- (ii) $\mu(xy) \preceq \mu(x)$ and $\mu(xy) \preceq \mu(y)$, for all $x, y \in R$.

Note: If μ is a L-fuzzy almost ideal, then μ_1 is an ideal. But the other level sets need not be ideals.

Definition 3.2: A non-constant L-fuzzy almost Ideal $\mu: R \rightarrow L$ is said to be D_0 -Prime L-Fuzzy Almost Ideal if, whenever $x_t y_s \leq \mu$ for any singletons x_t and y_s , then $x_t \leq \mu$ or $y_s \leq \mu$.

Definition 3.3: A non-constant L-fuzzy almost Ideal $\mu: R \rightarrow L$ is said to be D_1 -Prime L-Fuzzy Almost Ideal if, whenever $I \circ J \leq \mu$ for some L-fuzzy almost Ideal I and J, $I \leq \mu$ or $J \leq \mu$.

Definition 3.4: A non-constant L-fuzzy almost Ideal $\mu: R \rightarrow L$ is said to be D_2 -Prime L-Fuzzy Almost Ideal if, the level set μ_α is prime ideal for any $\mu(0) \geq \alpha > \mu(1)$.

Definition 3.5: A L-fuzzy almost Ideal $\mu: R \rightarrow L$ is said to be D_3 -Prime L-Fuzzy Almost Ideal if it is non-constant and for all $x, y \in R$ whenever $\mu(xy) = \mu(0)$ then $\mu(x) = \mu(0)$ or $\mu(y) = \mu(0)$.

Definition 3.6: Let $\mu: R \rightarrow L$ be a L-fuzzy almost ideal. It is said to be D_4 - Prime L -fuzzy almost ideal if for all $x, y \in R$, $\mu(xy) = \mu(x)$ or $\mu(y)$.

Definition 3.7: A L-fuzzy almost ideal $\mu: R \rightarrow L$ is said to be semiprime L-fuzzy almost ideal if for all $x \in R$, $\mu(x^n) \succeq \mu(x)$ for all $n \in \mathbb{Z}_+$.

Definition 3.8: A L-fuzzy almost ideal defined by $\mu: R \rightarrow L$ is a Primary L-fuzzy almost ideal if for all $x, y \in R$ either $\mu(xy) = \mu(x)$ or else $\mu(xy) \not\succeq \mu(y^m)$ for some $m \in \mathbb{Z}_+$.

IV. INTUITIONISTIC L-FUZZY ALMOST IDEAL

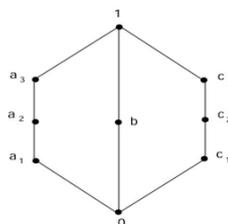
Definition 4.1: Let R be a ring with unity. Let L be a lattice (L, \vee, \wedge, \leq) not necessarily distributive with least and greatest element 0 and 1 respectively, with an involutive order reversing operation $N : L \rightarrow L$. Any ILFS defined by (μ, ν) where $\mu(0) = 1, \mu(1) = 0$ said to be Intuitionistic L-fuzzy Almost Ideal (ILFAI) if

- (i) $\mu(x - y) \preceq \mu(x) \wedge \mu(y)$, for all $x, y \in R$.
- (ii) $\mu(xy) \preceq \mu(x)$ and $\mu(xy) \preceq \mu(y)$, for all $x, y \in R$.
- (iii) $\nu(x - y) \succeq \nu(x) \vee \nu(y)$, for all $x, y \in R$.
- (iv) $\nu(xy) \succeq \nu(x)$ and $\nu(xy) \succeq \nu(y)$, for all $x, y \in R$.

Note: For any ILFS $\mu(0)=1$ implies that $\nu(0) = 0$.

In [10] and [12] several results regarding L-fuzzy almost ideals have been proved. The intuitionistic version for many of these results can be similarly proved. Hence we just list these theorems omitting the proofs.

Figure - I



Example 4.2: Let L be the lattice given by Hasse diagram in figure-I. Note that it is not distributive. $N: L \rightarrow L$ is defined by $N(0) = 1, N(a_i) = a_{3-i}, N(b) = b, N(c_i) = c_{3-i}, N(1) = 0$.

Let A be an ILFS defined by (μ, ν) where $\mu(x) = 0, \nu(x) = 1$ if $x \in Z - \langle 3 \rangle - \langle 5 \rangle$

$$\begin{aligned} \mu(x) = a_1, v(x) = a_1 & \text{ if } x \in \langle 3 \rangle - \langle 3^2 \rangle - \langle 5 \rangle \\ \mu(x) = a_2, v(x) = a_1 & \text{ if } x \in \langle 3^2 \rangle - \langle 3^3 \rangle - \langle 5 \rangle \\ \mu(x) = a_3, v(x) = a_1 & \text{ if } x \in \langle 3^3 \rangle - \langle 3^4 \rangle - \langle 5 \rangle \\ \mu(x) = b = v(x) & \text{ if } x \in \langle 15 \rangle \\ \mu(x) = c_1, v(x) = c_1 & \text{ if } x \in \langle 5 \rangle - \langle 5^2 \rangle - \langle 3 \rangle \\ \mu(x) = c_2, v(x) = c_1 & \text{ if } x \in \langle 5^2 \rangle - \langle 5^3 \rangle - \langle 3 \rangle \\ \mu(x) = c_3, v(x) = c_1 & \text{ if } x \in \langle 5^3 \rangle - \langle 5^4 \rangle - \langle 3 \rangle \\ \mu(x) = 1, v(x) = 0 & \text{ if } x = 0 \end{aligned}$$

By routine calculation we can verify that A is Intuitionistic L-fuzzy almost ideal.

Definition 4.3: An Intuitionistic L-fuzzy subset $A = \{ \langle x, \mu(x), v(x) \rangle \forall x \in X \}$ of a ring R is called an Intuitionistic L-fuzzy almost subring of R if,

- (i) $\mu(x - y) \leq \mu(x) \wedge \mu(y)$, for all $x, y \in R$.
- (ii) $\mu(xy) \leq \mu(x) \wedge \mu(y)$, for all $x, y \in R$.
- (iii) $v(x - y) \geq v(x) \vee v(y)$, for all $x, y \in R$.
- (iv) $v(xy) \geq v(x) \vee v(y)$, for all $x, y \in R$.

Theorem 4.4: If $A(\mu, v)$ is an Intuitionistic L-fuzzy almost ideal of R then it is Intuitionistic L-fuzzy almost subring of R.

Proof: It is a straightforward exercise using the definitions.

V. ALMOST BI- LEVEL SET

Definition 5.1: If $A(\mu, v)$ is a Intuitionistic L-fuzzy set of R, bi-level set $A_{t,s}$ is defined as

$$A_{t,s} = \{ x \in R \mid \mu(x) \geq t \text{ and } v(x) \leq s \}.$$

Note: In an ILFAI the bi-level sets in general need not be ideals. However we have:

Theorem 5.2: Let $A(\mu, v)$ be an Intuitionistic L-fuzzy ideal of R then

$$A_{1,0} = \{ x \in R \mid \mu(x) = 1 \text{ and } v(x) = 0 \} \text{ is an ideal.}$$

Definition 5.3: If $A(\mu, v)$ is an Intuitionistic L-fuzzy set, we define Almost bi-level set $\approx A_{t,s}$ by $\approx A_{t,s} = \{ x \in R \mid \mu(x) \leq t, v(x) \geq s \}$.

Proposition 5.4: If $t_1 \leq t_2$ and $s_1 \geq s_2$, $\approx A_{t_1, s_1} \subseteq \approx A_{t_2, s_2}$.

Theorem 5.5: Let R be a ring with unity and (L, \vee, \wedge, \leq) a lattice. Let $A(\mu, v)$ be a Intuitionistic L-fuzzy set. If for all $(t, s) \in \text{Im}(\mu, v)$, $\approx A_{t,s}$ is an ideal then A is an Intuitionistic L-fuzzy almost ideal.

Remark 5.6: Converse of the above theorem is not true. In example 4.2 $\approx A_{t,s}$ for $t = a_1, s = a_3$ is not an ideal.

Theorem 5.7: Let L be a completely distributive lattice. If $A(\mu, v)$ is a Intuitionistic L-fuzzy ideal with $\mu(0) = 1, \mu(1) = 0$, then $A(\mu, v)$ is an Intuitionistic L-fuzzy almost ideal.

Remark 5.8: Theorem 5.7 shows that the concept of Intuitionistic L-fuzzy almost ideal is indeed a generalization of the Intuitionistic L-fuzzy ideal.

Remark 5.9: Note that in example 4.2 (μ, v) is a Intuitionistic L-fuzzy almost ideal but it is not an Intuitionistic L-fuzzy ideal.

Theorem 5.10: Let L be a distributive lattice. Let $A(\mu, v)$ be any Intuitionistic L-fuzzy ideal. If $\text{Im}(\mu)$ and $\text{Im}(v)$ are chains then $A(\mu, v)$ is an Intuitionistic L-fuzzy ideal.

Proof: Let L be a distributive lattice.

Let $A(\mu, \nu)$ be any Intuitionistic L-fuzzy almost ideal. So

- (i) $\mu(x - y) \triangleleft \mu(x) \wedge \mu(y)$, for all $x, y \in R$.
- (ii) $\mu(xy) \triangleleft \mu(x)$ and $\mu(xy) \triangleleft \mu(y)$, for all $x, y \in R$.
- (iii) $\nu(x - y) \triangleright \nu(x) \vee \nu(y)$, for all $x, y \in R$.
- (iv) $\nu(xy) \triangleright \nu(x)$ and $\nu(xy) \triangleright \nu(y)$, for all $x, y \in R$.

Since $\text{Im}(\mu)$ is a chain all elements in $\text{Im}(\mu)$ are comparable. By axiom (i)

$$\mu(x-y) \triangleleft \mu(x) \wedge \mu(y).$$

But $\mu(x-y)$ and $\mu(x) \wedge \mu(y)$ are comparable. So, $\mu(x-y) \geq \mu(x) \wedge \mu(y)$.

Similarly, by axiom (ii) $\mu(xy) \triangleleft \mu(x)$ and $\mu(xy) \triangleleft \mu(y)$ gives us the result $\mu(xy) \geq \mu(x)$ and $\mu(xy) \geq \mu(y)$.

Therefore $\mu(xy) \geq \mu(x) \vee \mu(y)$.

Similarly using the fact $\text{Im}(\nu)$ is a chain we can prove

$$\begin{aligned} \nu_A(x - y) &\leq \nu_A(x) \vee \nu_A(y), \\ \nu_A(xy) &\leq \nu_A(x) \wedge \nu_A(y). \end{aligned}$$

VI. PRIMALITY OF INTUITIONISTIC L-FUZZY ALMOST IDEAL

Definition 6.1: A non-constant Intuitionistic L-fuzzy almost Ideal A defined by (μ, ν) is said to be D_2 – Prime Intuitionistic L-Fuzzy Almost Ideal if the level set $\approx A_{t,s}$ is prime ideal for any $(t, s) \in \text{Im}(\mu, \nu)$.

Definition 6.2: A non-constant Intuitionistic L-fuzzy almost Ideal A defined by (μ, ν) is said to be D_3 – Prime Intuitionistic L-Fuzzy Almost Ideal if, for all $x, y \in R$ whenever

$$\mu(xy) = \mu(0) \text{ then } \mu(x) = \mu(0) = 1 \text{ and } \nu(x) = 0 \text{ or } \mu(y) = \mu(0) = 1 \text{ and } \nu(y) = 0$$

Theorem 6.3: Any D_3 –Prime Intuitionistic L-Fuzzy Ideal is an D_3 –Prime Intuitionistic L-Fuzzy Almost Ideal.

Theorem 6.4: Let $A(\mu, \nu)$ be any Intuitionistic L-fuzzy ideal of R . Then $A(\mu, \nu)$ is D_3 –Prime if and only if

$$A_{1,0} = \{x \in R \mid \mu(x) = \mu(0) = 1, \nu(x) = \nu(0) = 0\} \text{ is a prime ideal of } R.$$

Proof: Assume $A(\mu, \nu)$ is a D_3 –Prime Intuitionistic L-fuzzy almost ideal. Suppose $x, y \in (\mu_1, \nu_1)$ which implies that $\mu(xy) = 1$ and $\nu(xy) = 0$. Since $\mu(x) = 1$ or $\mu(y) = 1$ and $\nu(x) = 0$ or $\nu(y) = 0 \Rightarrow x \in (\mu_1, \nu_1)$ or $y \in (\mu_1, \nu_1)$. Thus (μ_1, ν_1) is a prime ideal.

Conversely, Suppose $A_{1,0}$ is a prime ideal. Take $\mu(xy) = \mu(0) = 1$ and $\nu(xy) = \nu(0) = 0$ which implies $x, y \in A_{1,0}$. Since $A_{1,0}$ is a prime ideal this implies $x \in A_{1,0}$ or $y \in A_{1,0}$. Therefore $\mu(x) = 1$ and $\nu(x) = 0$ or $\mu(y) = 1$ and $\nu(y) = 0$. Hence (μ, ν) is D_3 –Prime Intuitionistic L-Fuzzy Almost Ideal.

Definition 6.5: A non-constant Intuitionistic L-fuzzy almost Ideal A defined by (μ, ν) is said to be D_4 - Prime Intuitionistic L-fuzzy almost ideal if for all $x, y \in R$, $\mu(xy) = \mu(x)$ and $\nu(xy) = \nu(x)$ or $\mu(xy) = \mu(y)$ and $\nu(xy) = \nu(y)$.

Theorem 6.6: Let R be a commutative ring with unity. If the ILFAI A defined by (μ, ν) is a D_4 - Prime Intuitionistic L-fuzzy almost ideal then it is D_3 –Prime Intuitionistic L-fuzzy almost ideal.

Theorem 6.7: Let R be a commutative ring with unity. If $\mu: R \rightarrow L$ and $\nu: R \rightarrow L$ is a D_2 - Prime Intuitionistic L-fuzzy almost ideal then it is D_3 –Prime Intuitionistic L-fuzzy almost ideal.

VII. SEMIPRIME INTUITIONISTIC L-FUZZY ALMOST IDEAL

Definition 7.1: A Intuitionistic L-fuzzy almost ideal $\mu: R \rightarrow L$ and $\nu: R \rightarrow L$ is said to be semiprime Intuitionistic L – fuzzy almost ideal if for all $x \in R$ and $n \in \mathbb{Z}_+$

- (i) $\mu(x^n) \triangleright \mu(x)$
- (ii) $\nu(x^n) \triangleleft \nu(x)$

Theorem 7.2: Any semiprime Intuitionistic L-fuzzy Ideal μ with $\mu(0) = 1, \mu(1) = 0$ is a semiprime Intuitionistic L-fuzzy Almost Ideal.

Theorem 7.3: Any D_4 -Prime Intuitionistic L-fuzzy Almost Ideal with $\mu(0) = 1, \mu(1) = 0$ is a semiprime Intuitionistic L-fuzzy Almost Ideal.

VIII. PRIMARY INTUITIONISTIC L-FUZZY ALMOST IDEAL

Definition 8.1: A Intuitionistic L-fuzzy almost ideal defined by $\mu: R \rightarrow L$ is a Primary Intuitionistic L-fuzzy almost ideal if, for all $x, y \in R$ either $\mu(xy) = \mu(x)$ and $\nu(xy) = \nu(x)$ or else $\mu(xy) \not\geq \mu(y^m)$ and $\nu(xy) \not\leq \nu(y^m)$, for some $m \in \mathbb{Z}_+$.

Proposition 8.2: If (L, \vee, \wedge, \leq) is a lattice with an involutive order reversing function $N: L \rightarrow L$, then

- (i) $N(a \vee b) = N(a) \wedge N(b)$
- (ii) $N(a \wedge b) = N(a) \vee N(b)$

Proof:

(i) For any two elements a and b in L

$$a \vee b \geq a$$

$$N(a \vee b) \leq N(a)$$

$$\text{Also } N(a \vee b) \leq N(b)$$

$$\text{So } N(a \vee b) \leq N(a) \wedge N(b)$$

(1)

Similarly for any two elements c and d in L

$$N(c \wedge d) \geq N(c) \vee N(d)$$

$$N(N(c \wedge d)) \leq N(N(c) \vee N(d))$$

$$c \wedge d \leq N(N(c) \vee N(d))$$

Putting $c=N(a)$ and $d=N(b)$

$$N(a) \wedge N(b) \leq N(a \vee b)$$

(2)

By combining (1) and (2)

$$N(a) \wedge N(b) = N(a \vee b)$$

(ii) Similar to (i).

Theorem 8.3: Let (L, \vee, \wedge, \leq) be a lattice with least and greatest element 0 and 1 respectively and an involutive order reversing function $N: L \rightarrow L$. Let R be a ring with unity. Suppose $\mu: R \rightarrow L$ defines a L-fuzzy Almost Ideal. Define $\nu: R \rightarrow L$ by $\nu(x) = N(\mu(x))$. Then (μ, ν) defines a Intuitionistic L-fuzzy Almost Ideal. Further

- (i) If μ defines a D_3 -prime L-fuzzy Almost Ideal then (μ, ν) defines a D_3 -prime Intuitionistic L-fuzzy Almost Ideal.
- (ii) If μ defines a semiprime L-fuzzy Almost Ideal then (μ, ν) defines a semiprime Intuitionistic L-fuzzy Almost Ideal.
- (iii) If μ defines a primary L-fuzzy Almost Ideal then (μ, ν) defines a primary Intuitionistic L-fuzzy Almost Ideal.

Proof: Since $\mu: R \rightarrow L$ defines a L-fuzzy Almost Ideal we have for all $x, y \in R$

$$\mu(x - y) \not\leq \mu(x) \wedge \mu(y) \text{ and } \mu(xy) \not\leq \mu(x) \text{ and } \mu(xy) \not\leq \mu(y).$$

If $\nu(x - y) > \nu(x) \vee \nu(y)$ then using proposition 8.2

$$N(\nu(x - y)) < N(\nu(x) \vee \nu(y)) = N(\nu(x)) \wedge N(\nu(y)).$$

So $\mu(x - y) < \mu(x) \wedge \mu(y)$ which is a contradiction.

So, $\nu(x - y) \not\geq \nu(x) \vee \nu(y)$, for all $x, y \in R$.

Similarly one can prove $\nu(xy) \not\geq \nu(x)$ and $\nu(xy) \not\geq \nu(y)$, for all $x, y \in R$. Hence (μ, ν) defines a Intuitionistic L-fuzzy Almost Ideal.

Statements (i), (ii), (iii) follow directly from definition.

IX. CONCLUSION

In this paper the concepts of Intuitionistic L-fuzzy almost ideal and Intuitionistic L-fuzzy almost subring have been introduced and illustrative example has been given. Some properties have been studied. In this paper we have developed the concept of primality or primeness of Intuitionistic L-fuzzy almost ideal. Some interesting results regarding this concept have been derived.

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