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APPLICATION OF HOMOTOPY PERTURBATION TRANSFORM METHOD FOR SOLVING HEAT LIKE AND WAVE LIKE EQUATIONS WITH VARIABLE COEFFICIENTS

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ABSTRACT

In this paper, we apply homotopy perturbation transform method (HPTM) for solving various heat-like and wave-like equations. This method is the combined form of homotopy perturbation method and Laplace transform method. The nonlinear terms can be easily obtained by the use of He's polynomials. HPTM presents an accurate methodology to solve nonhomogeneous partial differential equations of variable coefficients. The aim of using the Laplace transform is to overcome the deficiency that is mainly caused by unsatisfied conditions in the other semi analytical methods such as Homotopy perturbation method (HPM), Variational iteration method (VIM) and Adomain decomposition method (ADM). The approximate solutions obtained by means of HPTM in a wide range of problem's domain were compared with those results obtained from the actual solution. The fact that proposed technique solves nonlinear problems can be considered as a clear advantage of this algorithm over the decomposition method.

Keywords: *Homotopy perturbation method, Laplace transform method, Parabolic-like equations, Hyperbolic-like equations, He's polynomials.*

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1. INTRODUCTION

The real world problems in scientific fields such as solid state physics, plasma physics, fluid mechanics, chemical kinetics and mathematical biology are nonlinear in general when formulate as partial differential equations and integral equations. In the last two decades, many powerful and simple methods have been proposed and applied successfully to solve various types of problems. Some various approximate methods have been developed such as the Adomain decomposition method [1-4], the Variational iteration method [5-12], the differential transform method [13-14], the Laplace decomposition method [15-16], the tanh-method [17-18] and the extended tanh-method [19-20].One of the analytical methods of recent vintage, namely the homotopy perturbation method (HPM), first proposed by He [21-28] by combining the standard homotopy and classical perturbation technique for solving various linear, nonlinear initial and boundary value problems [29-39] and has been modified later by some scientists to obtain more accurate results, rapid convergence and to reduce the amount of computation [40-43]. HPM, VIM and ADM methods can be used to solve the nonlinear partial differential equations with accurate approximations, but this approximation is acceptable only for a small range, because boundary conditions in one dimension are satisfied via these methods, consequently, this shows that most of the analytical techniques encounter the in-built deficiencies and involve huge computational work. The Adomain decomposition method is the most transparent method for solutions of the partial differential equations; however, this method is involved in the calculation of complicated Adomain's polynomials which narrow down its applications. The Laplace transform is totally incapable of handling the nonlinear equations because of the difficulties that are caused by the nonlinear terms. To overcome these deficiencies we combine the homotopy perturbation method with Laplace transform method to produce a highly effective technique to deal with these nonlinearities. Various ways have been proposed to recently to deal with nonlinearities as Adomain decomposition method [44]. Furthermore, the homotopy perturbation method is also combined with the Laplace transform method [45] and Variational iteration method [46] to produce a highly effective technique for solving many nonlinear problems.

The basic motivation of this paper is to propose a new modification of HPM to overcome the deficiency. The suggested HPTM provides the solution in a rapid convergent series which may leads the solution in closed form. The advantage of this method is its capability of combining two powerful methods for obtaining exact solution for nonlinear equations. The use of He's polynomials in nonlinear terms first proposed by Ghorbani [47-48].It is worth mentioning that the HPTM is applied without any discretization or restrictive assumptions or transformations and free from round-off errors. Also very accurate results are obtained in a wide range via one or two iteration steps. Unlike the method of separation of variables that require initial or boundary conditions, The HPTM provides an analytical solution by using

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the initial conditions only. The boundary conditions can be used only to justify the obtained results. The proposed method work efficiently and the results so far are very encouraging and reliable. We would like to emphasize that the HPTM may be considered as an important and significant refinement of the previously developed techniques and can be viewed as an alternative to the recently developed methods such as Adomain's decomposition method, Variational iteration method and Homotopy perturbation method. Several examples are given to verify the reliability and efficiency of the homotopy perturbation transform method. In this paper we have considered the effectiveness of the homotopy perturbation transform method (HPTM) for solving various heat-like and wave-like equations of variable coefficients.

2. HOMOTOPY PERTURBATION TRANSFORM METHOD

This method has been introduced by Y.Khan and Q.Wu [49] by combining the Homotopy perturbation method and Laplace transform method for solving various types of linear and nonlinear systems of partial differential equations. To illustrate the basic idea of HPTM, we consider a general nonlinear partial differential equation with the initial conditions of the form [49].

$$Du(x,t) + Ru(x,t) + Nu(x,t) = g(x,t), \tag{1}$$

$$u(x,0) = h(x), \qquad u_t(x,0) = f(x).$$

where D is the second order linear differential operator $D = \partial^2/\partial t^2$, R is the linear differential operator of less order than D; N represents the general nonlinear differential operator and $g(x,t)$ is the source term. Taking the Laplace transform (denoted in this paper by L) on both sides of Eq. (1):

$$L[Du(x,t)] + L[Ru(x,t)] + L[Nu(x,t)] = L[g(x,t)] \tag{2}$$

Using the differentiation property of the Laplace transform, we have

$$L[u(x,t)] = \frac{h(x)}{s} + \frac{f(x)}{s^2} - \frac{1}{s^2}L[Ru(x,t)] + \frac{1}{s^2}L[g(x,t)] - \frac{1}{s^2}L[Nu(x,t)] \tag{3}$$

Operating with the Laplace inverse on both sides of Eq. (3) gives

$$u(x,t) = G(x,t) - L^{-1}\left[\frac{1}{s^2}L[Ru(x,t) + Nu(x,t)]\right] \tag{4}$$

where $G(x,t)$ represents the term arising from the source term and the prescribed initial conditions. Now we apply the homotopy perturbation method

$$u(x,t) = \sum_{n=0}^{\infty} p^n u_n(x,t) \tag{5}$$

and the nonlinear term can be decomposed as

$$Nu(x,t) = \sum_{n=0}^{\infty} p^n H_n(u) \tag{6}$$

for some He's polynomials $H_n(u)$ (see [47-48]) that are given by

$$H_n(u_0, u_1, \ldots, u_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}\left[N\left(\sum_{i=0}^{\infty} p^i u_i\right)\right]_{p=0}, \; n = 0, 1, 2, 3\ldots \tag{7}$$

Substituting Eq. (7), Eq. (6) and Eq. (5) in Eq. (4) we get

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = G(x,t) - p\left(L^{-1}\left[\frac{1}{s^2}L\left[R\sum_{n=0}^{\infty} p^n u_n(x,t) + \sum_{n=0}^{\infty} p^n H_n(u)\right]\right]\right) \tag{8}$$

which is the coupling of the Laplace transform and the homotopy perturbation method using He's polynomials. Comparing the coefficient of like powers of p, the following approximations are obtained.

$$p^0 : u_0(x,t) = G(x,t)$$

$$p^1 : u_1(x,t) = -\frac{1}{s^2} L\left[R u_0(x,t) + H_0(u)\right],$$
$$p^2 : u_2(x,t) = -\frac{1}{s^2} L\left[R u_1(x,t) + H_1(u)\right], \tag{9}$$
$$p^3 : u_3(x,t) = -\frac{1}{s^2} L\left[R u_2(x,t) + H_2(u)\right],$$
$$\vdots$$

and so on

3. APPLICATIONS

In this section, we will present the exact solutions of the heat-like and wave-like equations with variable coefficients investigated by A.M.Wazwaz [50] and L.Jin [51] to assess the efficiency of the homotopy perturbation transform method.

Example: 3.1 Consider the one-dimensional parabolic-like equation with variable coefficients [50-51].

$$u_t(x,t) = \frac{x^2}{2} u_{xx}(x,t) \tag{10}$$

with the initial condition $u(x,0) = x^2$

taking Laplace Transform both of sides of Eq. (10) subject to the initial conditions, we have

$$L[u(x,t)] = \frac{x^2}{s} + \frac{x^2}{2s} L[u_{xx}(x,t)] \tag{11}$$

taking inverse Laplace transform, we get

$$u(x,t) = x^2 + L^{-1}\left[\frac{x^2}{2s} L[u_{xx}(x,t)]\right] \tag{12}$$

By homotopy perturbation method, we get

$$u(x,t) = \sum_{n=0}^{\infty} p^n u_n(x,t) \tag{13}$$

using eq. (13) in eq. (12), we get

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = x^2 + pL^{-1}\left[\frac{x^2}{2s} L\left(\sum_{n=0}^{\infty} p^n u_n(x,t)\right)_{xx}\right] \tag{14}$$

Comparing the coefficients of various powers of p , we get

$$p^0 : \quad u_0(x,t) = x^2$$
$$p^1 : \quad u_1(x,t) = \frac{x^2 t}{1!}$$
$$p^2 : \quad u_2(x,t) = \frac{x^2 t^2}{2!} \tag{15}$$
$$p^3 : \quad u_3(x,t) = \frac{x^2 t^3}{3!}$$
$$\vdots$$

and so on

Therefore the approximate solution is given by

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \ldots.$$

$$=x^2\left(1+t+\frac{t^2}{2!}+\frac{t^3}{3!}+\cdots\right)=x^2e^t \tag{16}$$

which is an exact solution. The results of the above example shows that our method is capable of reducing the huge computational work and generates the modification of homotopy perturbation method in the convergence rate and is same as obtained by the Adomain decomposition method [50] and Homotopy perturbation method [51].

Example: 3.2 Consider the two-dimensional parabolic-like equation with variable coefficients [50-51].

$$u_t(x,y,t)=\frac{y^2}{2}u_{xx}+\frac{x^2}{2}u_{yy} \tag{17}$$

with the initial condition $u(x,y,0)=y^2$

by applying aforesaid method, we get

$$\sum_{n=0}^{\infty}p^n u_n(x,y,t)=y^2+pL^{-1}\left[\frac{y^2}{2s}L\left(\sum_{n=0}^{\infty}p^n u_n(x,y,t)\right)_{xx}+\frac{x^2}{2s}L\left(\sum_{n=0}^{\infty}p^n u_n(x,y,t)\right)_{yy}\right] \tag{18}$$

Comparing the coefficients of various powers of p , we get

$$\begin{aligned}
p^0 &: \quad u_0(x,y,t)=y^2\\
p^1 &: \quad u_1(x,y,t)=x^2t\\
p^2 &: \quad u_2(x,y,t)=\frac{y^2t^2}{2!}\\
p^3 &: \quad u_3(x,y,t)=\frac{x^2t^3}{3!}
\end{aligned} \tag{19}$$

thus in general $u_{2n}(x,y,t)=\dfrac{y^2t^{2n}}{2n!}$ and $u_{2n+1}(x,y,t)=\dfrac{x^2t^{2n+1}}{(2n+1)!}$

Therefore the approximate solution is given by

$$\begin{aligned}
u(x,t)&=u_0(x,t)+u_1(x,t)+u_2(x,t)+u_3(x,t)+\ldots.\\
&=x^2\left(t+\frac{t^3}{3!}+\frac{t^5}{5!}\ldots\right)+y^2\left(1+\frac{t^2}{2!}+\frac{t^4}{4!}+\ldots\right)\\
&=x^2\sinh t+y^2\cosh t
\end{aligned} \tag{20}$$

Which is an exact solution and is same as obtained by the Adomain decomposition method [50] and Homotopy perturbation method [51].

Example: 3.3 Consider the three-dimensional parabolic-like equation with variable coefficients [50-51].

$$u_t-(xyz)^4-\frac{1}{36}\left(x^2u_{xx}+y^2u_{yy}+z^2u_{zz}\right)=0 \tag{21}$$

with the initial condition $u(x,y,z,0)=0$

by applying aforesaid method, we get

$$\sum_{n=0}^{\infty}p^n u_n(x,y,z,t)=(xyz)^4+pL^{-1}\left[\frac{1}{36s}L\left[x^2\left(\sum_{n=0}^{\infty}p^n u_n(x,y,z,t)\right)_{xx}+\right.\right.$$

$$+y^2\left(\sum_{n=0}^{\infty}p^n u_n(x,y,z,t)\right)_{yy}+z^2\left(\sum_{n=0}^{\infty}p^n u_n(x,y,z,t)\right)_{zz}\Bigg]\Bigg] \tag{22}$$

Comparing the coefficients of various powers of p, we get

$$\begin{aligned} p^0 &: \quad u_0(x,y,z,t)=(xyz)^4 t \\ p^1 &: \quad u_1(x,y,z,t)=(xyz)^4\frac{t^2}{2!} \\ p^2 &: \quad u_2(x,y,z,t)=(xyz)^4\frac{t^3}{3!} \\ &\quad \vdots \end{aligned} \tag{23}$$

and so on

In general $u_n(x,y,z,t)=(xyz)^4\dfrac{t^{n+1}}{(n+1)!}$

Therefore the approximate solution is given by

$$\begin{aligned} u(x,t)&=u_0(x,t)+u_1(x,t)+u_2(x,t)+u_3(x,t)+\ldots. \\ &=(xyz)^4\left(e^t-1\right) \end{aligned} \tag{24}$$

Which is an exact solution and is same as obtained by the Adomain decomposition method [50] and Homotopy perturbation method [51].

Example: 3.4 Consider the one-dimensional hyperbolic-like equation with variable coefficients [50-51].

$$u_{tt}-\frac{x^2}{2}u_{xx}=0 \tag{25}$$

with the initial conditions $u(x,0)=x, \quad u_t(x,0)=x^2$

by applying aforesaid method, we get

$$\sum_{n=0}^{\infty}p^n u_n(x,t)=x+x^2t+p\frac{x^2}{2}L^{-1}\left[\frac{1}{s^2}L\left(\sum_{n=0}^{\infty}p^n u_n(x,t)\right)_{xx}\right] \tag{26}$$

Comparing the coefficients of various powers of p, we get

$$\begin{aligned} p^0 &: \quad u_0(x,t)=x+x^2t \\ p^1 &: \quad u_1(x,t)=x^2\frac{t^3}{3!} \\ p^2 &: \quad u_2(x,t)=x^2\frac{t^5}{5!} \\ p^3 &: \quad u_3(x,t)=x^2\frac{t^7}{7!} \\ &\quad \vdots \end{aligned} \tag{27}$$

and so on

Therefore the approximate solution is given by

$$\begin{aligned} u(x,t)&=u_0(x,t)+u_1(x,t)+u_2(x,t)+u_3(x,t)+\ldots. \\ &=x+x^2\sinh t \end{aligned} \tag{28}$$

Which is an exact solution and is same as obtained by the Adomain decomposition method [50] and Homotopy perturbation method [51].

Example: 3.5 Consider the two-dimensional hyperbolic-like equation with variable coefficients [50-51].

$$u_{tt}(x,y,t)=\frac{x^2}{12}u_{xx}(x,y,t)+\frac{y^2}{12}u_{yy}(x,y,t) \tag{29}$$

with the initial conditions $u(x,y,0)=x^4 \quad u_t(x,y,0)=y^4$

by applying aforesaid method, we get

$$\sum_{n=0}^{\infty}p^n u_n(x,y,t)=x^4+y^4t+pL^{-1}\left[\frac{1}{s^2}\left[\frac{x^2}{12}L\left(\sum_{n=0}^{\infty}p^n u_n(x,y,t)\right)_{xx}+\frac{y^2}{12}L\left(\sum_{n=0}^{\infty}p^n u_n(x,y,t)\right)_{yy}\right]\right] \tag{30}$$

Comparing the coefficients of various powers of p , we get

$$\begin{aligned}
p^0 &: \quad u_0(x,y,t)=x^4+y^4t\\
p^1 &: \quad u_1(x,y,t)=x^4\frac{t^2}{2!}+y^4\frac{t^3}{3!}\\
p^2 &: \quad u_2(x,y,t)=x^4\frac{t^4}{4!}+y^4\frac{t^5}{5!}\\
&\quad\vdots
\end{aligned} \tag{31}$$

and so on

Therefore the approximate solution is given by

$$\begin{aligned}
u(x,t)&=u_0(x,t)+u_1(x,t)+u_2(x,t)+....\\
&=x^4\cosh t+y^4\sinh t
\end{aligned} \tag{32}$$

Which is an exact solution and is same as obtained by the Adomain decomposition method [50] and Homotopy perturbation method [51].

Example: 3.6 Consider the three-dimensional hyperbolic-like equation with variable coefficients [50-51].

$$u_{tt}=\left(x^2+y^2+z^2\right)+\frac{1}{2}\left(x^2u_{xx}+y^2u_{yy}+z^2u_{zz}\right) \tag{33}$$

with the initial conditions $u(x,y,z,0)=0 \quad u_t(x,y,z,0)=x^2+y^2-z^2$

by applying aforesaid method, we get

$$\sum_{n=0}^{\infty}p^n u_n(x,y,z,t)=\left(x^2+y^2-z^2\right)t+\left(x^2+y^2+z^2\right)\frac{t^2}{2!}+pL^{-1}\left[\frac{1}{2s^2}.\right.$$

$$\left.\left\{x^2L\left(\sum_{n=0}^{\infty}p^n u_n(x,y,z,t)\right)_{xx}+y^2L\left(\sum_{n=0}^{\infty}p^n u_n(x,y,z,t)\right)_{yy}+z^2L\left(\sum_{n=0}^{\infty}p^n u_n(x,y,z,t)\right)_{zzz}\right\}\right] \tag{34}$$

Comparing the coefficients of various powers of p , we get

$$\begin{aligned}
p^0 &: \quad u_0(x,y,z,t)=\left(x^2+y^2-z^2\right)t+\left(x^2+y^2+z^2\right)\frac{t^2}{2!}\\
p^1 &: \quad u_1(x,y,z,t)=\left(x^2+y^2\right)\left(\frac{t^3}{3!}+\frac{t^4}{4!}\right)+z^2\left(-\frac{t^3}{3!}+\frac{t^4}{4!}\right)\\
&\quad\vdots
\end{aligned} \tag{35}$$

and so on

Therefore the approximate solution is given by

$$u(x,t)=u_0(x,t)+u_1(x,t)+u_2(x,t)+....$$
$$=\left(x^2+y^2\right)e^t+z^2e^{-t}-\left(x^2+y^2+z^2\right) \tag{36}$$

Which is an exact solution and is same as obtained by the Adomain decomposition method [50] and Homotopy perturbation method [51].

4. CONCLUSIONS

The main concern of this article is to construct an analytic solution for heat-like and wave-like partial differential equations of variable coefficients. We have achieved this goal by applying homotopy perturbation transform method (HPTM). The main advantage of this algorithm is the fact that it provides its user with an analytical approximation, in many cases an exact solution, in a rapidly convergent sequence with elegantly computed terms. Analytical solutions enable researchers to study the effect of different variables under study easily. Its small size of computation in comparison with the computational size required in other numerical methods and its rapid convergence show that the method is reliable and introduces a significant improvement in solving partial differential equations over existing methods. The solution procedure by using He's polynomials is simple, but the calculation of Adomain's polynomials is complex. The fact that the HPTM solves nonlinear problems without using the Adomain's polynomials can be considered as a clear advantage of this algorithm over the decomposition method. Also the proposed scheme exploits full advantage of Variational iteration method (VIM), Adomain's decomposition method (ADM) and Variational iteration decomposition method (VIDM). Finally, we conclude that HPTM can be considered as a nice refinement in existing numerical technique and might find wide applications in different fields of Sciences.

REFERENCES

[1] G. Adomain, Solving Frontier problem of Physics: The Decomposition Method, Kluwer Acad. Publ., Boston, 1994.

[2] A. M. Wazwaz, A new algorithm for calculating Adomain polynomials for nonlinear operators, Applied Mathematics and Computation 111 (2000) 53-59.

[3] A. M. Wazwaz, Constructing of solitary wave solutions and rational solutions for the KdV equation by A domain Decomposition Method, Chaos Solitons Fractals 12 (2001) 2283-2293.

[4] A. Sadighi, D. D. Ganji, Y. Sabzehmeidani, A Decomposition method for Volume Flux and Average Velocity of thin film flow of a third grade film down an inclined plane, Advance in Theoretical and Applied Mechanics 1 (2008) 45-49.

[5] He, J. H., Variational iteration method- a kind of nonlinear analytical technique: some examples, International journal of Non linear Mechanics, 34 (1999) 699-708.

[6] He, J. H, Wu, X. H., Variational iteration method: new development and applications, Computer and Mathematics with Applications 54 (2007) 881-894.

[7] He, J. H, Wu, G. C., Austin, F., The variational iteration method which should be followed, Nonlinear Science Letter A 1 (2009) 1-30.

[8] Soltani, L. A., Shirzadi, A., A new modification of variational iteration method, Computer and Mathematics with Applications 59 (2010) 2528-2535.

[9] Faraz, N., Khan, Y., Yildirim, A., Analytical approach to two dimensional viscous flows with a shrinking sheet via variational iteration algorithm-II, Journal of King Saud University (2010), doi: 10.1016/ j.jksus. 2010. 06.010.

[10] Wu, G. C., Lee, E. W. M., Fractional variational iteration method and its applications, Physics Letter A. (2010), doi: 10.1016/ j.physleta. 2010. 04.034.
[11] Hesameddini.E, Latifizadeh, H., Reconstruction of variational iteration algorithms using the Laplace transform, International Journal of Nonlinear Sciences and Numerical Simulation 10 (2009) 1377-1382.

[12] Chun, C., Fourier series based variational iteration method for a reliable treatment of heat equations with variable coefficients, International Journal of Nonlinear Sciences and Numerical Simulation 10 (2009) 1383-1388.

[13] Y. Keskin, G. Oturanc, Reduced Differential Transform Method for Partial differential equations, International Journal of Nonlinear Sciences and Numerical Simulation ,10 (6) (2009),741-749.

[14] Y. Keskin, G.Oturanc, Reduced Differential Transform Method for Fractional Partial Differential equations, Nonlinear Sciences Letters A, 1 (2) (2010), 61-72.

[15] E.Yusufoglu, Numerical solution of Duffing equation by the Laplace decomposition method, Applied Mathematics and Computation, 177 (2006), 572-580.

[16] Y. Khan, An efficient modification of the Laplace decomposition method for nonlinear equations, International Journal of Nonlinear Sciences and Numerical Simulation, 10 (2009), 1373-1376.

[17] D. J. Evans, K. R. Raslan, The Tanh-Function method for solving some important nonlinear partial differential equation, International Journal of Computer Mathematics, 82 (2005), 897-905.

[18] E. J. Parkes, B. R. Duffy, An Automated Tanh-Function method for finding Solitary Wave Solutions to Nonlinear Evolutions Equations, Computational Physics Communication, 98 (1998), 288-300.

[19] E. Fan Tanh-Function Method and its Applications to nonlinear equations. Physics Letters A, 277 (2000), 212-218.

[20] E. Fan, Travelling Wave Solutions for Generalized Hirota-Satsuma Coupled KdV Systems. Z Naturforschung A 56 (2001), 312-318.

[21 J. H. He, Homotopy perturbation techniques, Computer Methods in Applied Mechanics and Engineering 178 (1999) 257-262.

[22] J. H. He, Homotopy perturbation method: a new nonlinear analytical technique, Applied Mathematics and Computation 135 (2003) 73-79.

[23] J. H. He, Comparison of Homotopy perturbation method and Homotopy analysis method, Applied Mathematics and Computation 156 (2004) 527-539.

[24] J. H. He, The homotopy perturbation method for nonlinear oscillator with discontinuities, Applied Mathematics and Computation 151 (2004) 287-292.
.

[25] J. H. He, Some asymptotic methods for strongly nonlinear equation, International Journal of Modern Physics B 20 (2006) 1144-1199.

[26] J. H. He, Homotopy perturbation method for solving Boundary Value Problems, Physics Letters A 350 (2006), 87-88.

[27] J. H. He, A coupling of homotopy technique and perturbation technique for nonlinear problems, International Journal of Nonlinear Mechanics 35(1) (2000) 37-43.

[28] J. H. He, A note on the Homotopy perturbation method, Thermal Science 14 (2) (2010) 565-568.

[29] G. Domairry, N. Nadim, Assessment of homotopy analysis method and homotopy perturbation method in nonlinear heat transfer equation, International Communication in Heat and Mass Transfer 35 (2008) 93-102.

[30] A. Rajabi, Homotopy perturbation method for fin efficiency of convective straight fins with temperature dependent thermal conductivity, Physics Letters A 364 (2007) 33-37.

[31] M. Akbarzade, J. Langari, Application of Homotopy perturbation method and Variational iteration method to three dimensional diffusion problem, International Journal of Mathematical Analysis 5 (18) (2011), 871-880.

[32] B.Raftari, A.Yildirim, The application of homotopy perturbation method for MHD flows of UCM fluids above porous stretching sheet, Computers and Mathematics with Applications 59 (2010) 3328-3337.

[33] V. G. Gupta and S. Gupta, A Reliable Algorithm for solving nonlinear Kawahara equation and its generalization, International Journal of Computational Science and Mathematics 2 (2010) 407-416.

[34] L. Xu, He's homotopy perturbation method for boundary layer equation in unbounded domain, Computers and Mathematics with Applications 54 (2007), 1067-1070.

[35] A.Yildirim, Application of He's homotopy perturbation method for solving the Cauchy reaction-diffusion equations, Computers and Mathematics with Applications 57 (4) (2009) 612-618.

[36] D. D.Ganji, The application of He's homotopy perturbation method for nonlinear equation arising in heat transfer, Physics Letters A 355 (2006) 337-341.

[37] J. Biazar, Z. Ayati, H. Ebrahimi, Homotopy perturbation method for general form of porous medium equation, Journal of Porous Media 12 (11) (2009) 1121-1127.

[38] A. M Siddiqui, R. Mahmood, Q. K.Ghori, Thin film flow of a third grade fluid on a moving belt by He's homotopy perturbation method, International Journal of Nonlinear Sciences and Numerical Simulation 7(1) (2006) 7-12.

[39] J. Biazar., K. Hosseini, P.Gholamin, Homotopy perturbation method Fokker- Planck equation, International Mathematical Forum 19 (3) (2008) 945-954.

[40] Z. Odibat, S.Momani, Modified homotopy perturbation method: Application to Riccati differential equation of fractional order, Chaos Solitons Fractals 36 (2008) 167-174.

[41] Z. Odibat, A new modification of the homotopy perturbation method for linear and nonlinear operators, Applied Mathematics and Computation 189 (1) (2007) 746- 753.

[42] J. Nadjafi, M.Tamamgar, Modified Homotopy perturbation method for solving integral equations, International Journal of Modern Physics B 24 (2) (2010) 4741-4746.

[43] M. A. Jafari and A Aminataei, Improvement of the homotopy perturbation method for solving diffusion equations Physica Scripta 82(1) (2010).

[44] J. Biazar, M. Gholami, P.Ghanbari, Extracting a general iterative method from an a domain decomposition method and compare it to the variational iteration method, Computers and Mathematics with Applications 59 (2010) 622-628.

[45] M. Madani, M.Fatizadeh, Homotopy perturbation algorithm using Laplace transformation, Nonlinear Science Letters A 1 (2010) 263-267.

[46] M. A Noor, S. T. Mohyud-Din, Variational homotopy perturbation method for solving higher dimensional initial boundary value problem, Mathematical Problems in Engineering (2008) Article ID 696734 doi: 10.1155 / 2008 / 696734.

[47] A. Ghorbani, J.Saberi-Nadjafi, He's homotopy perturbation method for calculating A domain's polynomials, International Journal of Nonlinear Sciences and Numerical Simulation 8 (2007) 229-232.

[48] A. Ghorbani, Beyond Adomain's polynomials: He's polynomials, Chaos Solitons Fractals 39 (2009) 1486-1492.

[49] Y. Khan, Q.Wu, Homotopy perturbation transform method for solving nonlinear equations using He's polynomials, Computers and Mathematics with Applications (Article in Press) doi: 10.1016 / j.camwa 2010.08.022.

[50] A. M.Wazwaz, Exact Solutions for Heat-Like and Wave-Like Equations withVariable coefficients, Applied Mathematics and Computation, 149 (2004), 15-29.

[51] L. Jin, Homotopy perturbation method for Partial differential equations with variable coefficients, international Journal of Contempory Mathematical Sciences, 28 (3), (2008), 1395-1407.

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