Volume 9, No. 1, Jan. - 2018 (Special Issue)
International Journal of Mathematical Archive-9(1), 2018, 141-149
Available online through www.ijma.info ISSN 2229 – 5046

APPLICATION OF INTUITIONISTIC DIFFERENTIAL TRANSFORMATION METHOD TO SOLVE INTUITIONISTIC FUZZY VOLTERRA INTEGRO-DIFFERENTIAL EQUATION

SUVANKAR BISWAS[1,*] & TAPAN KUMAR ROY[2]

**[1,2]Department of Mathematics,
Indian Institute of Engineering Science and Technology,
Shibpur, Howrah-711103, West Bengal, India.**

E-mail: suvo180591@gmail.com

ABSTRACT

In this study, some properties of differentiability for intuitionistic fuzzy functions have been presented. Fuzzy differential transform method (FDTM) is applied to solve fuzzy Volterra integro-differential equations. In order to show the effectiveness of this method some illustrative examples are given.

Keywords: *Intuitionistic fuzzy derivative; Intuitionistic fuzzy Volterra intego-differential equation; Intuitionistic fuzzy differential transform method.*

1. INTRODUCTION

The concept of fuzzy set theory has been extended into intuitionistic fuzzy set (IFS) theory by Atanassov [3, 4, 15]. There are huge applications of IFS in different fields of science like medical diagnosis [5], microelectronic fault analysis [17], pattern recognition [11], decision-making problems [2 & 12], drug selection [9] and etc. The studies on improvement of IFS theory, together with intuitionistic fuzzy geometry, intuitionistic fuzzy logic, intuitionistic fuzzy topology, an intuitionistic fuzzy approach to artificial intelligence, and intuitionistic fuzzy generalized nets have been given in [18].

There are only few works have been done on intuitionistic fuzzy differential equation (IFDE) [1, 6, 8, 13, 14, 16] although like fuzzy differential equations there are a large area of possible applications of IFDE in civil engineering, biology, physics, medical science and so many other fields of science. V. Nirmala and S. C. Pandian have studied Intuitionistic fuzzy differential equation with initial condition by using Euler method [16]. The existence and uniqueness of a solution of the intuitionistic fuzzy differential equation using the method of successive approximation has been discussed by R. Ettoussi *et al.* [8]. The existence and uniqueness theorem of a solution to the nonlocal intuitionistic fuzzy differential equation using the concept of intuitionistic fuzzy semigroup and the contraction mapping principle has been given by S. Melliani [13]. A system of differential equation of first order with initial value as triangular intuitionistic fuzzy number has been solved in [14]. Numerical Solution of intuitionistic fuzzy differential equations by Runge-Kutta method has been given by S. Abbasbandy, T. Allahviranloo in [1]. Solution intuitionistic fuzzy differential equation with linear differential operator by Adomian decomposition method has been given by us in [6].

There are also only few works have been done on intuitionistic fuzzy integral equation [7, 10] but there are no works on numerical solution intuitionistic fuzzy integral or integro-differential equation. This is the first paper where a numerical method for the solution of solution intuitionistic fuzzy integral or integro-differential equation has been given. In this paper, intuitionistic fuzzy derivative and its properties have been given in Section 3. FDTM and its properties have been given in Section 4. Finally illustrative examples and a brief conclusion are provided in Section 5.

2. INTUITIONISTIC FUZZY DERIVATIVE

H-difference for Intuitionistic fuzzy number:

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

Let $\tilde{x}^i, \tilde{y}^i \in E^i$, if there exists $\tilde{z}^i \in E^i$ such that $\tilde{x}^i = \tilde{y}^i + \tilde{z}^i$ then $\tilde{z}^i$ is called H-difference of $\tilde{x}^i$ and $\tilde{y}^i$ and is denoted by $\tilde{x}^i \ominus \tilde{y}^i$ where E^i is the set of all intuitionistic fuzzy number.

Definition 1: (see [18]): Let $\tilde{f}^i : (a,b) \to E^i$ and $x_0 \in (a,b)$. we say that $\tilde{f}^i$ is differential at x_0 if there exist an element $\tilde{f}'^i(x_0) \in E^i$, such that

(1) for all h>0 sufficiently near to 0, there exist $\tilde{f}^i(x_0+h) \ominus \tilde{f}^i(x_0)$, $\tilde{f}^i(x_0) \ominus \tilde{f}^i(x_0\text{-h})$, and the limits

$$\lim_{h\to 0^+} \frac{\tilde{f}^i(x_0+h) \ominus \tilde{f}^i(x_0)}{h} = \lim_{h\to 0^+} \frac{\tilde{f}^i(x_0) \ominus \tilde{f}^i(x_0-\text{h})}{h} = \tilde{f}'^i(x_0) \text{ or}$$

(2) for all h<0 sufficiently near to 0, there exist $\tilde{f}^i(x_0+h) \ominus \tilde{f}^i(x_0)$, $\tilde{f}^i(x_0) \ominus \tilde{f}^i(x_0\text{-h})$, and the limits

$$\lim_{h\to 0^-} \frac{\tilde{f}^i(x_0+h) \ominus \tilde{f}^i(x_0)}{h} = \lim_{h\to 0^-} \frac{\tilde{f}^i(x_0) \ominus \tilde{f}^i(x_0-\text{h})}{h} = \tilde{f}'^i(x_0)$$

In the case when f is intuitionistic fuzzy valued function, we have the following theorem

Theorem 1: Let $\tilde{f}^i: R \to E^i$ be a intuitionistic fuzzy valued function with (α,β)-cut representation $\tilde{f}^i_{\alpha,\beta}(x) = \langle f_\alpha(x,\alpha), f_\beta(x,\beta)\rangle = \langle [f_{\alpha_l}(x,\alpha), f_{\alpha_r}(x,\alpha)][f_{\beta_l}(x,\beta), f_{\beta_r}(x,\beta)]\rangle$, for each $(\alpha,\beta) \in (0,1)$ then we have the following

(1) If $\tilde{f}^i$ is differentiable in the first form (1) in Definition 1 then $f_{\alpha_l}(x,\alpha), f_{\alpha_r}(x,\alpha)$ and $f_{\beta_l}(x,\beta), f_{\beta_r}(x,\beta)$ are differentiable functions and $\tilde{f}'^i_{\alpha,\beta}(x) = \langle [f'_{\alpha_l}(x,\alpha), f'_{\alpha_r}(x,\alpha)][f'_{\beta_l}(x,\beta), f'_{\beta_r}(x,\beta)]\rangle$

(2) If $\tilde{f}^i$ is differentiable in the second form (2) in Definition 1 then $f_{\alpha_l}(x,\alpha), f_{\alpha_r}(x,\alpha)$ and $f_{\beta_l}(x,\beta), f_{\beta_r}(x,\beta)$ are differentiable functions and $\tilde{f}'^i_{\alpha,\beta}(x) = \langle [f'_{\alpha_r}(x,\alpha), f'_{\alpha_l}(x,\alpha)][f'_{\beta_r}(x,\beta), f'_{\beta_l}(x,\beta)]\rangle$

Proof:

(1) If $h > 0$ and $\alpha,\beta \in [0,1]$ then we have

$$[\tilde{f}^i(x+h) \ominus \tilde{f}^i(x)]_{\alpha,\beta}$$
$$= \langle [f_{\alpha_l}(x+h,\alpha) - f_{\alpha_l}(x,\alpha), f_{\alpha_r}(x+h,\alpha) - f_{\alpha_r}(x,\alpha)]\ [f_{\beta_l}(x+h,\beta) - f_{\beta_l}(x,\beta), f_{\beta_r}(x+h,\beta) - f_{\beta_r}(x,\beta)]\rangle$$

and multiplying by $\frac{1}{h}$ we have

$$\frac{[\tilde{f}^i(x+h) \ominus \tilde{f}^i(x)]_{\alpha,\beta}}{h}$$
$$= \langle [\frac{f_{\alpha_l}(x+h,\alpha) - f_{\alpha_l}(x,\alpha)}{h}, \frac{f_{\alpha_r}(x+h,\alpha) - f_{\alpha_r}(x,\alpha)}{h}]\ [\frac{f_{\beta_l}(x+h,\beta) - f_{\beta_l}(x,\beta)}{h}, \frac{f_{\beta_r}(x+h,\beta) - f_{\beta_r}(x,\beta)}{h}]\rangle$$

Similarly we obtain

$$\frac{[\tilde{f}^i(x) \ominus \tilde{f}^i(x-h)]_{\alpha,\beta}}{h}$$
$$= \langle [\frac{f_{\alpha_l}(x,\alpha) - f_{\alpha_l}(x-h,\alpha)}{h}, \frac{f_{\alpha_r}(x,\alpha) - f_{\alpha_r}(x-h,\alpha)}{h}]\ [\frac{f_{\beta_l}(x,\beta) - f_{\beta_l}(x-h,\beta)}{h}, \frac{f_{\beta_r}(x,\beta) - f_{\beta_r}(x-h,\beta)}{h}]\rangle$$

Passing to the limit we have

$$\tilde{f}'^i_{\alpha,\beta}(x) = \langle [f'_{\alpha_l}(x,\alpha), f'_{\alpha_r}(x,\alpha)][f'_{\beta_l}(x,\beta), f'_{\beta_r}(x,\beta)]\rangle$$

(2) If $h < 0$ and $\alpha,\beta \in [0,1]$ then we have

$$[\tilde{f}^i(x+h) \ominus \tilde{f}^i(x)]_{\alpha,\beta}$$
$$= \langle [f_{\alpha_l}(x+h,\alpha) - f_{\alpha_l}(x,\alpha), f_{\alpha_r}(x+h,\alpha) - f_{\alpha_r}(x,\alpha)]\ [f_{\beta_l}(x+h,\beta) - f_{\beta_l}(x,\beta), f_{\beta_r}(x+h,\beta) - f_{\beta_r}(x,\beta)]\rangle$$

and multiplying by $\frac{1}{h}$ we have

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

$$\frac{[\tilde{f}^i(x+h) \ominus \tilde{f}^i(x)]_{\alpha,\beta}}{h}$$
$$= \langle [\frac{f_{\alpha_r}(x+h,\alpha) - f_{\alpha_r}(x,\alpha)}{h}, \frac{f_{\alpha_l}(x+h,\alpha) - f_{\alpha_l}(x,\alpha)}{h}] [\frac{f_{\beta_r}(x+h,\beta) - f_{\beta_r}(x,\beta)}{h}, \frac{f_{\beta_l}(x+h,\beta) - f_{\beta_l}(x,\beta)}{h}] \rangle$$

Similarly we obtain

$$\frac{[\tilde{f}^i(x) \ominus \tilde{f}^i(x-h)]_{\alpha,\beta}}{h}$$
$$= \langle [\frac{f_{\alpha_r}(x,\alpha) - f_{\alpha_r}(x-h,\alpha)}{h}, \frac{f_{\alpha_l}(x,\alpha) - f_{\alpha_l}(x-h,\alpha)}{h}] [\frac{f_{\beta_r}(x,\beta) - f_{\beta_r}(x-h,\beta)}{h}, \frac{f_{\beta_l}(x,\beta) - f_{\beta_l}(x-h,\beta)}{h}] \rangle$$

Passing to the limit we have

$$\tilde{f}'^i_{\alpha,\beta}(x) = \langle \left[f'_{\alpha_r}(x,\alpha), f'_{\alpha_l}(x,\alpha) \right] [f'_{\beta_r}(x,\beta), f'_{\beta_l}(x,\beta)] \rangle$$

This completes the proof of the theorem.

Theorem 2: Let $\tilde{f}^i : R \to E^i$ be a intuitionistic fuzzy valued function with (α,β)-cut representation $\tilde{f}^i_{\alpha,\beta}(x) = \langle f_\alpha(x,\alpha), f_\beta(x,\beta) \rangle = \langle [f_{\alpha_l}(x,\alpha), f_{\alpha_r}(x,\alpha)][f_{\beta_l}(x,\beta), f_{\beta_r}(x,\beta)] \rangle$, for each $(\alpha,\beta) \in (0,1)$ then we have the following

(i) If $\tilde{f}^i$and $\widetilde{f}'^i$ are differentiable in the first form (1) or if $\tilde{f}^i$& $\widetilde{f}'^i$ are differentiable in the second form (2) in Definition 1 then $\tilde{f}''^i_{\alpha,\beta}(x) = \langle [f''_{\alpha_l}(x,\alpha), f''_{\alpha_r}(x,\alpha)][f''_{\beta_l}(x,\beta), f''_{\beta_r}(x,\beta)] \rangle$

(ii) If $\tilde{f}^i$ is differentiable in the first form (1) and $\widetilde{f}'^i$ is differentiable in the second form (2) or if $\tilde{f}^i$ is differentiable in the second form (2) and $\widetilde{f}'^i$ is differentiable in the first form (1) in Definition 1 then $\tilde{f}''^i_{\alpha,\beta}(x) = \langle [f''_{\alpha_r}(x,\alpha), f''_{\alpha_l}(x,\alpha)][f''_{\beta_r}(x,\beta), f''_{\beta_l}(x,\beta)] \rangle$

Proof: The results of this theorem are state forward repeated application of the Theorem 1.We present the details only for the first case of (i), the other cases are analogous. If $\tilde{f}^i$is differentiable in the first form (1), then from Theorem 1 we have

$$\tilde{f}'^i_{\alpha,\beta}(x) = \langle [f'_{\alpha_l}(x,\alpha), f'_{\alpha_r}(x,\alpha)][f'_{\beta_l}(x,\beta), f'_{\beta_r}(x,\beta)] \rangle$$

Now if $\widetilde{f}'^i$ is differentiable in the first form (1), then again applying Theorem 1on $\widetilde{f}'^i$ we have

$$\tilde{f}''^i_{\alpha,\beta}(x) = \langle [f''_{\alpha_l}(x,\alpha), f''_{\alpha_r}(x,\alpha)][f''_{\beta_l}(x,\beta), f''_{\beta_r}(x,\beta)] \rangle$$

This completes the proof of the first case of (i).

Similarly we can find any order derivative of $\tilde{f}^i$.

3. DIFFERENTIAL TRANSFORMATION METHOD

Definition 2: The differential transformation $\tilde{F}^i(k)$ of kth derivative of a function $\tilde{f}^i(x) : (a,b) \to E^i$ in one variable is as follows:

$$F_{\alpha_l}(k,\alpha) = \frac{1}{k!}[\frac{d^k f_{\alpha_l}(x,\alpha)}{dx^k}]_{x=x_0}, F_{\alpha_r}(k,\alpha) = \frac{1}{k!}[\frac{d^k f_{\alpha_r}(x,\alpha)}{dx^k}]_{x=x_0}, \forall\, k \in K = \{0,1,2,\dots.\}, \quad (1)$$
$$F_{\beta_l}(k,\beta) = \frac{1}{k!}[\frac{d^k f_{\beta_l}(x,\beta)}{dx^k}]_{x=x_0}, F_{\beta_r}(k,\beta) = \frac{1}{k!}[\frac{d^k f_{\beta_r}(x,\beta)}{dx^k}]_{x=x_0}, \forall\, k \in K = \{0,1,2,\dots.\}, \quad (2)$$

when $\tilde{f}^i(x)$ is differentiable in the form of (1) and

$$F_{\alpha_l}(k,\alpha) = \frac{1}{k!}[\frac{d^k f_{\alpha_r}(x,\alpha)}{dx^k}]_{x=x_0}, F_{\alpha_r}(k,\alpha) = \frac{1}{k!}[\frac{d^k f_{\alpha_l}(x,\alpha)}{dx^k}]_{x=x_0}, k \text{ is odd}, \quad (3)$$
$$F_{\alpha_l}(k,\alpha) = \frac{1}{k!}[\frac{d^k f_{\alpha_l}(x,\alpha)}{dx^k}]_{x=x_0}, F_{\alpha_r}(k,\alpha) = \frac{1}{k!}[\frac{d^k f_{\alpha_r}(x,\alpha)}{dx^k}]_{x=x_0}, k \text{ is even}, \quad (4)$$
$$F_{\beta_l}(k,\beta) = \frac{1}{k!}[\frac{d^k f_{\beta_r}(x,\beta)}{dx^k}]_{x=x_0}, F_{\beta_r}(k,\beta) = \frac{1}{k!}[\frac{d^k f_{\beta_l}(x,\beta)}{dx^k}]_{x=x_0}, k \text{ is odd}, \quad (5)$$
$$F_{\beta_l}(k,\beta) = \frac{1}{k!}[\frac{d^k f_{\beta_l}(x,\beta)}{dx^k}]_{x=x_0}, F_{\beta_r}(k,\beta) = \frac{1}{k!}[\frac{d^k f_{\beta_r}(x,\beta)}{dx^k}]_{x=x_0}, k \text{ is even}, \quad (6)$$

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

when $\tilde{f}^i(x)$ is differentiable in the form of (2).

So, if $\tilde{f}^i(x)$ is differentiable in the form of (1) then $\tilde{f}^i(x)$ can be presented as follows:

$$f_{\alpha_l}(x,\alpha) = \sum_{k=0}^{\infty}(x-x_0)^k F_{\alpha_l}(k,\alpha), \tag{7}$$
$$f_{\alpha_r}(x,\alpha) = \sum_{k=0}^{\infty}(x-x_0)^k F_{\alpha_r}(k,\alpha), \tag{8}$$
$$f_{\beta_l}(x,\beta) = \sum_{k=0}^{\infty}(x-x_0)^k F_{\beta_l}(k,\beta), \tag{9}$$
$$f_{\beta_r}(x,\beta) = \sum_{k=0}^{\infty}(x-x_0)^k F_{\beta_r}(k,\beta), \tag{10}$$

and if $\tilde{f}^i(x)$ is differentiable in the form of (2) then $\tilde{f}^i(x)$ can be presented as follows:

$$f_{\alpha_l}(x,\alpha) = \sum_{k=0,even}^{\infty}(x-x_0)^k F_{\alpha_l}(k,\alpha) + \sum_{k=1,odd}^{\infty}(x-x_0)^k F_{\alpha_r}(k,\alpha), \tag{11}$$
$$f_{\alpha_r}(x,\alpha) = \sum_{k=0,even}^{\infty}(x-x_0)^k F_{\alpha_r}(k,\alpha) + \sum_{k=1,odd}^{\infty}(x-x_0)^k F_{\alpha_l}(k,\alpha). \tag{12}$$
$$f_{\beta_l}(x,\beta) = \sum_{k=0,even}^{\infty}(x-x_0)^k F_{\beta_l}(k,\beta) + \sum_{k=1,odd}^{\infty}(x-x_0)^k F_{\beta_r}(k,\beta), \tag{13}$$
$$f_{\beta_r}(x,\beta) = \sum_{k=0,even}^{\infty}(x-x_0)^k F_{\beta_r}(k,\beta) + \sum_{k=1,odd}^{\infty}(x-x_0)^k F_{\beta_l}(k,\beta). \tag{14}$$

Theorem 3: Let us consider $\tilde{f}^i(x)$, $\tilde{u}^i(x)$ and $\tilde{v}^i(x)$ are intuitionistic fuzzy valued functions and their intuitionistic fuzzy differential transformations (IFDT) denoted by $\tilde{F}^i(k)$, $\tilde{U}^i(k)$ and $\tilde{V}^i(k)$, respectively. If $\tilde{f}^i(x)$, $\tilde{u}^i(x)$ and $\tilde{v}^i(x)$ are all differentiable in the form (1) or all differentiable in the form of (2). Then

(1) if $\tilde{f}^i(x) = \tilde{u}^i(x) + \tilde{v}^i(x)$, then $\tilde{F}^i(k) = \tilde{U}^i(k) + \tilde{V}^i(k)$, $k \in K = \{0,1,2,....\}$

(2) if $\tilde{f}^i(x) = \tilde{u}^i(x) - \tilde{v}^i(x)$, then $\tilde{F}^i(k) = \tilde{U}^i(k) - \tilde{V}^i(k)$, $k \in K$

(3) if $\tilde{f}^i(x) = \tilde{u}^i(x) \ominus \tilde{v}^i(x)$, then $\tilde{F}^i(k) = \tilde{U}^i(k) \ominus \tilde{V}^i(k)$, $k \in K$

Proof: The results are obvious from Definition 2**.**

Theorem 4: Let us consider $\tilde{f}^i(x)$, $\tilde{u}^i(x)$ and $\tilde{v}^i(x)$ are intuitionistic fuzzy valued functions and their intuitionistic fuzzy differential transformations denoted by $\tilde{F}^i(k)$, $\tilde{U}^i(k)$ and $\tilde{V}^i(k)$, respectively. If $\tilde{f}^i(x)$, $\tilde{u}^i(x)$ are differentiable in the form (1) and $\tilde{v}^i(x)$ is differentiable in the form (2) or $\tilde{f}^i(x)$, $\tilde{u}^i(x)$ are differentiable in the form (2) and $\tilde{v}^i(x)$ is differentiable in the form (1) then

(1) if $\tilde{f}^i(x) = \tilde{u}^i(x) + \tilde{v}^i(x)$, then $\tilde{F}^i(k) - \tilde{V}^i(k) = \tilde{U}^i(k)$, $k \in K = \{0,1,2,....\}$

(2) if $\tilde{f}^i(x) = \tilde{u}^i(x) - \tilde{v}^i(x)$, then $\tilde{F}^i(k) = \tilde{U}^i(k) \ominus \tilde{V}^i(k)$, $k \in K$

(3) if $\tilde{f}^i(x) = \tilde{u}^i(x) \ominus \tilde{v}^i(x)$, then $\tilde{F}^i(k) = \tilde{U}^i(k) - \tilde{V}^i(k)$, $k \in K$

Proof: The results are obvious from Definition 2**.**

Theorem 5: Let us consider the intuitionistic fuzzy-valued functions $\tilde{g}^i(x) \in$ E where $\tilde{g}^i(x)$ is differentiable in the form of (1) and $\tilde{f}^i(x) = \int_{x_0}^{x} \tilde{g}^i(t)\,dt$, then $\tilde{F}^i(k) = \frac{\tilde{G}^i(k-1)}{k}$, $k \geq 1$ where $\tilde{F}^i(k)$ and $\tilde{G}^i(k)$ are the intuitionistic fuzzy differential transformations of $\tilde{f}^i$ and $\tilde{g}^i$ respectively.

Proof: Using the definition of IFDT under differentiability in the form of (1), we get for $0 \leq \alpha \leq 1$

$$\tilde{f}^i_{\alpha,\beta}(x) = \int_{x_0}^{x} \tilde{g}^i_{\alpha,\beta}(t)\,dt = \left\langle [\int_{x_0}^{x} g_{\alpha_l}(t,\alpha)\,dt, \int_{x_0}^{x} g_{\alpha_r}(t,\alpha)\,dt][\int_{x_0}^{x} g_{\beta_l}(t,\beta)\,dt, \int_{x_0}^{x} g_{\beta_r}(t,\beta)\,dt] \right\rangle$$
$$= \langle [\int_{x_0}^{x} \sum_{k=0}^{\infty}(t-x_0)^k G_{\alpha_l}(k,\alpha)\,dt,\ \int_{x_0}^{x} \sum_{k=0}^{\infty}(t-x_0)^k G_{\alpha_r}(k,\alpha)\,dt]$$
$$[\int_{x_0}^{x} \sum_{k=0}^{\infty}(t-x_0)^k G_{\beta_l}(k,\beta)\,dt,\ \int_{x_0}^{x} \sum_{k=0}^{\infty}(t-x_0)^k G_{\beta_r}(k,\beta)\,dt]\rangle$$
$$= \langle [\sum_{k=0}^{\infty} \frac{(x-x_0)^{k+1}}{k+1} G_{\alpha_l}(k,\alpha), \sum_{k=0}^{\infty} \frac{(x-x_0)^{k+1}}{k+1} G_{\alpha_r}(k,\alpha)][\sum_{k=0}^{\infty} \frac{(x-x_0)^{k+1}}{k+1} G_{\beta_l}(k,\beta),\ \sum_{k=0}^{\infty} \frac{(x-x_0)^{k+1}}{k+1} G_{\beta_r}(k,\beta)] \rangle$$

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

Now changing the index we have

$$= \langle [\sum_{k=1}^{\infty} \frac{(x-x_0)^k}{k} G_{\alpha_l}(k-1,\alpha),\ \sum_{k=1}^{\infty} \frac{(x-x_0)^k}{k} G_{\alpha_r}(k-1,\alpha)]$$
$$[\sum_{k=1}^{\infty} \frac{(x-x_0)^k}{k} G_{\beta_l}(k-1,\beta),\ \sum_{k=1}^{\infty} \frac{(x-x_0)^k}{k} G_{\beta_r}(k-1,\beta)] \rangle$$

Now by using Definition 2 we have

$$\tilde{F}^i(k) = \frac{\tilde{G}^i(k-1)}{k},\ k \geq 1.$$

Theorem 6: Suppose that $\widetilde{U}^i(k)$ and $G(k)$ are the differential transformation of the functions $\tilde{u}(x)$ (is an intuitionistic fuzzy valued function) and $g(x)$ (is a positive real-valued function), respectively. If $\tilde{f}^i(x) = \int_{x_0}^{x} g(t)\tilde{u}^i(t)\,dt$, then under differentiability of $\tilde{f}^i$ in the form of (1) we have for $0 \leq \alpha \leq 1$ and $k \geq 1$

$$F_{\alpha_l}(k,\alpha) = \sum_{l=0}^{k-1} G(l) \frac{U_{\alpha_l}(k-l-1,\alpha)}{k},$$
$$F_{\alpha_r}(k,\alpha) = \sum_{l=0}^{k-1} G(l) \frac{U_{\alpha_r}(k-l-1,\alpha)}{k},$$
$$F_{\beta_l}(k,\beta) = \sum_{l=0}^{k-1} G(l) \frac{U_{\beta_l}(k-l-1,\beta)}{k},$$
$$F_{\beta_r}(k,\beta) = \sum_{l=0}^{k-1} G(l) \frac{U_{\beta_r}(k-l-1,\beta)}{k},$$

and under differentiability of $\tilde{f}^i$ in the form of (2) we have for $0 \leq \alpha \leq 1$ and $k \geq 1$

$$F_{\alpha_l}(k,\alpha) = \sum_{l=0,even}^{k-1} G(l) \frac{U_{\alpha_l}(k-l-1,\alpha)}{k} + \sum_{l=0,odd}^{k-1} G(l) \frac{U_{\alpha_r}(k-l-1,\alpha)}{k},$$
$$F_{\alpha_r}(k,\alpha) = \sum_{l=0,even}^{k-1} G(l) \frac{U_{\alpha_r}(k-l-1,\alpha)}{k} + \sum_{l=0,odd}^{k-1} G(l) \frac{U_{\alpha_l}(k-l-1,\alpha)}{k}$$
$$F_{\beta_l}(k,\beta) = \sum_{l=0,even}^{k-1} G(l) \frac{U_{\beta_l}(k-l-1,\beta)}{k} + \sum_{l=0,odd}^{k-1} G(l) \frac{U_{\beta_r}(k-l-1,\beta)}{k},$$
$$F_{\beta_r}(k,\beta) = \sum_{l=0,even}^{k-1} G(l) \frac{U_{\beta_r}(k-l-1,\beta)}{k} + \sum_{l=0,odd}^{k-1} G(l) \frac{U_{\beta_l}(k-l-1,\beta)}{k}$$

Proof: Let $\tilde{f}^i$ be differentiable in the form of (1). Using the definition of IFDT under differentiability in the form of (1), we get for $0 \leq \alpha \leq 1$

$$\tilde{F}^i(0,\alpha) = \left[\int_{x_0}^{x} g(t)\tilde{u}^i(t)\,dt\right]_{x=x_0} = 0$$

$$\tilde{F}^i(1,\alpha) = \left[\frac{d}{dx}\left(\int_{x_0}^{x} g(t)\tilde{u}^i(t)\,dt\right)\right]_{x=x_0}$$
$$= \left\langle \left[\left[\frac{d}{dx}\int_{x_0}^{x} g(t)u_{\alpha_l}(t,\alpha)\,dt\right]_{x_0}, \left[\frac{d}{dx}\int_{x_0}^{x} g(t)u_{\alpha_r}(t,\alpha)\,dt\right]_{x=x_0}\right] \right.$$
$$\left. \left[\left[\frac{d}{dx}\int_{x_0}^{x} g(t)u_{\beta_l}(t,\beta)\,dt\right]_{x_0}, \left[\frac{d}{dx}\int_{x_0}^{x} g(t)u_{\beta_r}(t,\beta)\,dt\right]_{x=x_0}\right] \right\rangle$$
$$= \left\langle [g(x_0)u_{\alpha_l}(x_0,\alpha), g(x_0)u_{\alpha_l}(x_0,\alpha)][g(x_0)u_{\beta_l}(x_0,\beta), g(x_0)u_{\beta_l}(x_0,\beta)] \right\rangle = \tilde{G}^i(0)\tilde{U}^i(0)$$

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

$$\tilde{F}^i(2,\alpha)=\frac{1}{2}\left[\frac{d^2}{dx^2}\left(\int_{x_0}^{x} g(t)\tilde{u}^i(t)\,dt\right)\right]_{x=x_0}$$

$$=\frac{1}{2}\Big\langle[G_{\alpha_l}(1,\alpha)U_{\alpha_l}(0,\alpha)+G_{\alpha_l}(0,\alpha)U_{\alpha_l}(1,\alpha),G_{\alpha_r}(1,\alpha)U_{\alpha_r}(0,\alpha)+G_{\alpha_r}(0,\alpha)U_{\alpha_r}(1,\alpha)]$$

$$[G_{\beta_l}(1,\beta)U_{\alpha_l}(0,\beta)+G_{\beta_l}(0,\beta)U_{\beta_l}(1,\beta),G_{\beta_r}(1,\beta)U_{\alpha_r}(0,\alpha)+G_{\alpha_r}(0,\alpha)U_{\alpha_r}(1,\alpha)]\Big\rangle$$

In general, we have

$$F_{\alpha_l}(k,\alpha)=\sum_{l=0}^{k-1}G(l)\frac{U_{\alpha_l}(k-l-1,\alpha)}{k},$$

$$F_{\alpha_r}(k,\alpha)=\sum_{l=0}^{k-1}G(l)\frac{U_{\alpha_r}(k-l-1,\alpha)}{k},$$

$$F_{\beta_l}(k,\beta)=\sum_{l=0}^{k-1}G(l)\frac{U_{\beta_l}(k-l-1,\beta)}{k},$$

$$F_{\beta_r}(k,\beta)=\sum_{l=0}^{k-1}G(l)\frac{U_{\beta_r}(k-l-1,\beta)}{k},$$

Similarly we can prove the results for $\tilde{f}^i$ differentiable in the form of (2).

Theorem 7: Suppose that $\tilde{U}^i(k)$ and $G(k)$ are the differential transformation of the functions $\tilde{u}^i(x)$ (is an intuitionistic fuzzy valued function) and $g(x)$ (is a positive real-valued function), respectively. If $\tilde{f}^i(x)=g(t)\int_a^x \tilde{u}^i(t)\,dt$, then under differentiability of $\tilde{f}^i$ in the form of (1) we have for $0\le\alpha\le1$ and $k\ge1$

$$F_{\alpha_l}(k,\alpha)=\sum_{l=0}^{k-1}G(l)\frac{U_{\alpha_l}(k-l-1,\alpha)}{k-1},$$

$$F_{\alpha_r}(k,\alpha)=\sum_{l=0}^{k-1}G(l)\frac{U_{\alpha_r}(k-l-1,\alpha)}{k-1},$$

$$F_{\beta_l}(k,\beta)=\sum_{l=0}^{k-1}G(l)\frac{U_{\beta_l}(k-l-1,\beta)}{k-1},$$

$$F_{\beta_r}(k,\beta)=\sum_{l=0}^{k-1}G(l)\frac{U_{\beta_r}(k-l-1,\beta)}{k-1},$$

and under differentiability of $\tilde{f}^i$ in the form of (2) we have for $0\le\alpha\le1$ and $k\ge1$

$$F_{\alpha_l}(k,\alpha)=\sum_{l=0,even}^{k-1}G(l)\frac{U_{\alpha_l}(k-l-1,\alpha)}{k-1}+\sum_{l=0,odd}^{k-1}G(l)\frac{U_{\alpha_r}(k-l-1,\alpha)}{k-1},$$

$$F_{\alpha_l}(k,\alpha)=\sum_{l=0,even}^{k-1}G(l)\frac{U_{\alpha_l}(k-l-1,\alpha)}{k-1}+\sum_{l=0,odd}^{k-1}G(l)\frac{U_{\alpha_r}(k-l-1,\alpha)}{k-1},$$

$$F_{\beta_r}(k,\beta)=\sum_{l=0,even}^{k-1}G(l)\frac{U_{\beta_r}(k-l-1,\beta)}{k-1}+\sum_{l=0,odd}^{k-1}G(l)\frac{U_{\beta_l}(k-l-1,\beta)}{k-1}.$$

$$F_{\beta_r}(k,\beta)=\sum_{l=0,even}^{k-1}G(l)\frac{U_{\beta_r}(k-l-1,\beta)}{k-1}+\sum_{l=0,odd}^{k-1}G(l)\frac{U_{\beta_l}(k-l-1,\beta)}{k-1}.$$

Proof: It is completely similar to the previous one, so the proof is omitted.

Theorem 8: Suppose that $\tilde{U}^i(k)$ and $G(k)$ are the differential transformation of the functions $\tilde{u}^i(x)$ (is an intuitionistic fuzzy valued function) and $g(x)$ (is a positive real-valued function), respectively. If $\tilde{f}^i(x) = g(x)\tilde{u}^i(x)$, then under differentiability of $\tilde{f}^i$ in the form of (1) we have

$$\tilde{F}^i(k,\alpha) = \sum_{l=0}^{k} G(l)\tilde{U}^i(k-l,\alpha), 0 \le \alpha \le 1.$$

Proof:

$$\tilde{f}^i(x) \approx (\sum_{k=0}^{n}(t-x_0)^k \tilde{G}^i(k))(\sum_{k=0}^{n}(t-x_0)^k \tilde{U}^i(k))$$
$$= (\tilde{G}^i(0) + (x-x_0)\tilde{G}^i(1) + (x-x_0)^2\tilde{G}^i(2) + ... + (x-x_0)^n\tilde{G}^i(n))\ (\tilde{U}^i(0\ +)(x-x_0)\tilde{U}^i(1)$$
$$+(x-x_0)^2\tilde{U}^i(2) + ... + (x-x_0)^n\tilde{U}^i(n))$$
$$= \{\tilde{G}^i(0)\tilde{U}^i(0)\} + \{\tilde{G}^i(0)\tilde{U}^i(1) + \tilde{G}^i(1)\tilde{U}^i(0)\}(x-x_0) + \{\tilde{G}^i(0)\tilde{U}^i(2) + \tilde{G}^i(1)\tilde{U}^i(1) + \tilde{G}^i(2)\tilde{U}^i(0)\}$$
$$(x-x_0)^2 + ... + \{\tilde{G}^i(0)\tilde{U}^i(n) + \tilde{G}^i(1)\tilde{U}^i(n-1) + ... + \tilde{G}^i(n-1)\tilde{U}^i(1) + \tilde{G}^i(n)\tilde{U}^i(0)\}(x-x_0)^n$$

So we have

$$\tilde{f}^i(x) \approx \sum_{k=0}^{n}\sum_{l=0}^{k}\tilde{G}^i(l)\tilde{U}^i(k-l)(x-x_0)^k$$

Therefore from the definition of IFDT, we have

$$\tilde{F}^i(k,\alpha) = \sum_{l=0}^{k} G(l)\tilde{U}^i(k-l,\alpha), 0 \le \alpha \le 1$$

.

4. APPLICATIONS AND NUMERICAL RESULTS

In this section, we apply the present method on some test problems. In all problems, our approximation space is based on differentiable in the form of (1). The figures are drawn by using MATLAB R2010a.

Example 1: Let us consider the following nonlinear fuzzy intuitionistic Volterra integro-differential equation

$\tilde{u}^{i}{}''(x) = \tilde{f}^i(x) + \frac{1}{2}\int_0^x \tilde{u}^{i2}(t)dt$, where $\tilde{f}^i_{\alpha,\beta}(x) = (1+x+x^2)\langle[\alpha, 2-\alpha][1-\beta, 1+2\beta]\rangle$

with initial conditions

$\tilde{u}^i_{\alpha,\beta}(0) = \langle[\alpha, 2-\alpha][1-\beta, 1+2\beta]\rangle$, $\tilde{u}^{i}{}'_{\alpha,\beta}(0) = \langle[1+\alpha, 3-\alpha][2-\beta, 2+2\beta]\rangle$.

Now using the properties of FDTM we have the result for $0 \le \alpha \le 1,\ k = 0$

$$U_{\alpha_l}(k+2,\alpha) = \frac{1}{(k+1)(k+2)}[\{\delta(k) + \delta(k-1) + \delta(k-2)\}\alpha],$$

$$U_{\alpha_r}(k+2,\alpha) = \frac{1}{(k+1)(k+2)}[\{\delta(k) + \delta(k-1) + \delta(k-2)\}(2-\alpha)],$$

$$U_{\beta_l}(k+2,\beta) = \frac{1}{(k+1)(k+2)}[\{\delta(k) + \delta(k-1) + \delta(k-2)\}(1-\beta)],$$

$$U_{\beta_r}(k+2,\beta) = \frac{1}{(k+1)(k+2)}[\{\delta(k) + \delta(k-1) + \delta(k-2)\}(1+2\beta)],$$

and for $k > 0$

$$U_{\alpha_l}(k+2,\alpha) = \frac{1}{(k+1)(k+2)}[\{\delta(k) + \delta(k-1) + \delta(k-2)\}\alpha + \frac{1}{2k}\sum_{l=0}^{k-1}U_{\alpha_l}(l,\alpha)U_{\alpha_l}(k-l-1,\alpha)],$$

$$U_{\alpha_r}(k+2,\alpha)=\frac{1}{(k+1)(k+2)}[\{\delta(k)+\delta(k-1)+\delta(k-2)\}(2-\alpha)+\frac{1}{2k}\sum_{l=0}^{k-1}U_{\alpha_r}(l,\alpha)U_{\alpha_r}(k-l-1,\alpha)]$$
$$U_{\beta_l}(k+2,\beta)=\frac{1}{(k+1)(k+2)}[\{\delta(k)+\delta(k-1)+\delta(k-2)\}(1-\beta)+\frac{1}{2k}\sum_{l=0}^{k-1}U_{\beta_l}(l,\beta)U_{\beta_l}(k-l-1,\beta)],$$
$$U_{\beta_r}(k+2,\beta)=\frac{1}{(k+1)(k+2)}[\{\delta(k)+\delta(k-1)+\delta(k-2)\}(1+2\beta)+\frac{1}{2k}\sum_{l=0}^{k-1}U_{\beta_r}(l,\beta)U_{\beta_r}(k-l-1,\beta)].$$

Then after considering FDT up to $k=6$ we can get an approximate Taylor series solution of order 6 as follows

$$u_{\alpha_l}(x,\alpha)=\alpha+(1+\alpha)x+\frac{\alpha}{2}x^2+\frac{\alpha^2+2\alpha}{12}x^3+\frac{\alpha^2+3\alpha}{24}x^4+\frac{2\alpha^2+2\alpha+1}{120}x^5+\frac{7\alpha^3+2\alpha^2+6\alpha}{1440}x^6+O(7)$$
$$u_{\alpha_r}(x,\alpha)=(2-\alpha)+(3-\alpha)x+\frac{2-\alpha}{2}x^2+\frac{\alpha^2-6\alpha+8}{12}x^3+\frac{\alpha^2-7\alpha+10}{24}x^4+\frac{2\alpha^2-10\alpha+13}{120}x^5$$
$$+\frac{-\alpha^3+14\alpha^2-50\alpha+52}{1440}x^6+O(7)$$
$$u_{\beta_l}(x,\beta)=(1-\beta)+(2-\beta)x+\frac{1-\beta}{2}x^2+\frac{3\beta^2-7\beta+4}{6}x^3+\frac{6\beta^2-19\beta+13}{12}x^4+\frac{2\beta^2-6\beta+5}{6}x^5$$
$$+\frac{-3\beta^3+13\beta^2-20\beta+10}{24}x^6+O(7)$$
$$u_{\beta_r}(x,\beta)=(1+2\beta)+(2+2\beta)x+\frac{1+2\beta}{2}x^2+\frac{12\beta^2+14\beta+4}{6}x^3+\frac{24\beta^2+38\beta+13}{12}x^4$$
$$+\frac{8\beta^2+12\beta+5}{6}x^5+\frac{24\beta^3+52\beta^2+40\beta+10}{24}x^6+O(7)$$

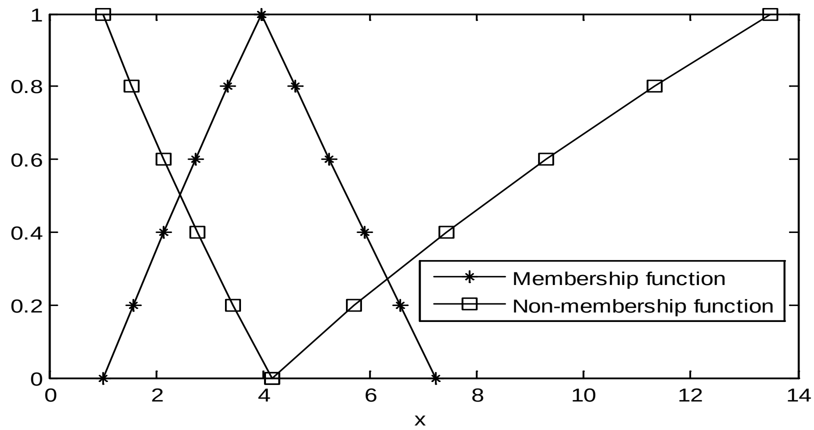

Figure-3: IFDTM solution for the membership function at $x=1$ for Example 3.

4. CONCLUSION

IFDT and its properties for various cases have been given. IFDTM for the solution of intuitionistic fuzzy Volterra integral equation and intuitionistic fuzzy Volterra integro-differential equation has been nicely presented. Three examples are given to illustrate the method. A nonlinear intuitionistic fuzzy Volterra integro-differentiable equation is illustrated.

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

REFERENCES

1. Abbasbandy S., Allahviranloo T., Numerical Solution of Fuzzy Differential Equations by Runge-Kutta and the Intuitionistic Treatment, in Journal of Notes on Intuitionistic Fuzzy Sets, 8(3), 2002, 43-53.
2. Atanassov K. T., Intuitionistic fuzzy sets. VII ITKR's session, Sofia (deposited in Central Science and Technical Library of the Bulgarian Academy of Sciences 1697/84), 1983.
3. Atanassov K. T., Intuitionistic fuzzy sets. Fuzzy Sets Syst, 20, 1986, 87–96.
4. Atanassov K. T., Operators over interval valued intuitionistic fuzzy sets. Fuzzy Sets System, 64(2), 1994, 159–174.
5. Biswas S., Roy A.R., and De S. K., An application of intuitionistic fuzzy sets in medical diagnosis. Fuzzy Sets Syst., 117(2), 2001, 209–213.
6. Biswas S., Banerjee S., Roy T. K., Solving intuitionistic fuzzy differential equations with linear differential operator by Adomian decomposition method, Notes on Intuitionistic Fuzzy Sets, 22(4), 2016, 25-41.
7. Deshpande B., and Handa A., Application of coupled fixed Point technique in solving integral equations on modified intuitionistic fuzzy metric spaces, Advances in Fuzzy Systems, 2014, Article ID 348069.
8. Ettoussi R., Melliani S., Elomari M., and Chadli L. S., Solution of intuitionistic fuzzy differential equations by successive approximations method, Notes on IFS, 21(2), 2015, 51–62.
9. Kharal K., Homeopathic drug selection using intuitionistic fuzzy sets. Homeopathy, 98(1), 2009, 35–39.
10. Lei Q., Xu., Bustince H., Burusco A., Definite Integrals of Atanassov's Intuitionistic Fuzzy Information, IEEE Transactions on Fuzzy Systems, 23(5), 2015, 1519 – 1533.
11. Li D. F., Cheng C. T., New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions. Pattern Recognit. Lett, 23, 2002, 221–225.
12. Li D. F., Multiattribute decision making models and methods using intuitionistic fuzzy sets. J. Comp. Syst. Sci, 70, 2005, 73–85.
13. Melliani S., Elomari M., Atraoui M., and Chadli L. S., Intuitionistic fuzzy differential equation with nonlocal condition, Notes on Intuitionistic Fuzzy Sets, 21(4), 2015, 58–68.
14. Mondal S. P., and Roy T. K., System of Differential Equation with Initial Value as Triangular Intuitionistic Fuzzy Number and its Application, Int. J. Appl. Comput. Math., 1, 2015, 449– 474.
15. Nikolova M., Nikolov N., Cornelis C., Deschrijver G., Survey of the research on intuitionistic fuzzy sets. Adv. Stud. Contempor. Math., 4(2), 2002, 127–157.
16. Nirmala V., and Pandian S. C., Numerical Approach for Solving Intuitionistic Fuzzy Differential Equation under Generalised Differentiability Concept, Applied Mathematical Sciences, 9(67), 2015, 3337 – 3346.
17. Shu M. H., Cheng C. H., Chang J. R., Using intuitionistic fuzzy sets for fault-tree analysis on printed circuit board assembly. Microelectron. Reliab., 46(12), 2006, 2139–2148.
18. Ye J., Multicriteria fuzzy decision-making method based on a novel accuracy function under interval valued intuitionistic fuzzy environment. Expert Syst. Applications, 36, 2009, 6899–6902.

Source of support: Proceedings of UGC Funded International Conference on Intuitionistic Fuzzy Sets and Systems (ICIFSS-2018), Organized by: Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

