Volume 9, No. 1, Jan. - 2018 (Special Issue)
International Journal of Mathematical Archive-9(1), 2018, 133-135
Available online through www.ijma.info ISSN 2229 – 5046

EXTENSION OF NASH EQUILIBRIUM THEOREMS USING SOFT SET

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ABSTRACT

John Nash describing Nash equilibrium using Brower's and Kakutani's fixed point theorems. Molodstov initiated the soft set theory which also deals with decision making under uncertainty. Here we extend some of the theorems on Nash equilibrium related to fixed point theory using soft set approach.

***Keywords**: Soft set, fixed point theory, game theory, Nash equilibrium.*

1. INTRODUCTION

Game theory is originally the mathematical study of competition and cooperation [9]. Strategic decision making is the main concept behind Game theory. In classical game theory the payoff functions are real valued functions and therefore the solution of games are obtained by using arithmetic operations. In recent years, many interesting applications of game theory have been expanded by using the ideas of fuzzy sets. The main concepts of game theory revolve around the fundamentals of Nash equilibrium [13].

Molodstov introduced a new concept soft set theory, which is the generalization of fuzzy set for dealing with uncertainty [17]. Applications of soft set theory in other disciplines and real life problems are now on fast track development in the decision making area. The traditional soft set is a mapping from parameter set to crisp subset of universe. Irfan Deli and Naim Cagman proposed a new concept soft game for dealing with uncertainty [7].

The knowledge of the existence of fixed point has relevant applications in decision making area. Mujahid elaborated the fixed point concept in fuzzy soft set theory [16] Among the different important applications of fixed point theorems such as Brower's, Kakutani's etc., Nash equilibrium's existence theorems [15].

The aim of this paper is to extend some of the theorems on Nash equilibrium related to fixed point theory using soft set approach.

2. PRELIMINARIES

In this portion, we present the basic definitions and results of game theory [9] soft set theory [5], [7], [17] and fixed point theory [15], [23].

Definition 2.1[9]: Nash equilibrium for two person game is a pair of strategies (σ_1^*, σ_2^*) such that

$$\pi_1 (\sigma_1^*, \sigma_2^*) \geq \pi_1 (\sigma_1, \sigma_2^*), \text{ for all } \sigma_1 \text{ € } \Sigma_1 \text{ and } \pi_2 (\sigma_1^*, \sigma_2^*) \geq \pi_2 (\sigma_1^*, \sigma_2), \text{ for all } \sigma_2 \text{ € } \Sigma_2.$$

Definition 2.2 [17]: Consider U be a universal set, E be the set of parameters and P (U) be the power set, $A \subseteq E$. A soft set F_A over U is set defined by function f_A representing a mapping f_A: E→ P (U) such that $f_A = \phi$ if x € A. A soft set over U can be represented by $F_A = \{(x, f_A(x)), x \epsilon E, f_A(x) \epsilon P(U)\}$.

Definition 2.3[15]: Consider a mapping f: X → X, where X ⊂ Rn. A point x ∈ X is called a fixed point of f if f(x) = x.

Theorem 2.1[10]: Assume that there exists a sequences $\{x_m\}$, $m \geq 0$ in V such that $x^m \lesssim x^{m+1} \epsilon f(x^m)$ for any $m \geq 0$ and $\{x \epsilon V: x^0 \lesssim x\}$ is finite. Then f has fixed point $x^* \epsilon f(x^*)$.

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International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

Definition 2.4[15]: Let F_A be the soft set over U, where U be the universal set. Consider a soft mapping $\Psi: F \rightarrow F$, if $\Psi(f_A^*) = f_A^*$, or $f_A^* \epsilon \Psi(f_A^*)$ then soft element $f_A^* \epsilon F$ is called fixed point of Ψ.

Some other definitions and results related to game theory, soft sets and fixed point theories are found in [3], [5], [6], [8], [9], [10], [13], [17] and [16].

3. SOFT SET APPROACH TO NASH EQUILIBRIUM USING FIXED POINT CONCEPT

In this section, fist we have defined Nash equilibrium of strategies using soft set and fixed point theories. Then we have introduced some results of Nash equilibrium through soft set approach and fixed point theory.

3.1 Nash equilibrium of pure strategies using soft set approach

Consider finite strategic structure of a game $G = \{N,\{S_i\},\{f_i\},i \epsilon N\}$,where $N=\{1,2,3\ldots n\}$ be set of players. A strategic set $S_i = \{f_{A1}^*, f_{A2}^* \ldots f_{An}^*\}$ is a pure strategy Nash equilibrium iff $f_A^* \epsilon (\Psi(f_A^*)_{-i})$ for all $i \epsilon N$, where $f_i((f_A^*)_{-i})$ represents the best responses of i^{th} player. That is $\Psi((f_A^*)_{-i}) = \{(f_A)_i) \epsilon S_i, f_i(f_{Ai}, (f_A)_{-i})\}$.We proceed to define the composite correspondence $\Psi: S \rightarrow S$ as $\Psi(f_{A1}, f_{A2} \ldots f_{An}) = \Psi_1((f_A)_{-1} \times (f_A)_{-2} \times \ldots (f_A)_{-n})$.Now $(f_{A1}^*, f_{A2}^* \ldots f_{An}^*)$ is Nash equilibrium is equivalent to $f_A^*{}_i \epsilon (\Psi(f_A^*)_{-i})$, $\forall i \epsilon N$. This means that $(f_{A1}^*, f_{A2}^*, \ldots f_{An}^*) \epsilon \Psi_1((f_A^*)_{-1}) \times \Psi_2((f_A^*)_{-2}) \times \ldots \Psi_n((f_A^*)_{-n})$ which turns that $(f_{A1}^*, f_{A2}^* .. f_{An}^*) \epsilon \Psi(f_A^*{}_{-1}, f_A^*{}_{-2} .. f_A^*{}_{-n})$. This makes us to conclude that f_A^* is a fixed point of the best correspondence Ψ.

3.2 Nash equilibrium of mixed strategies using soft set theory

Let $G = \{N, \{S_i\}, \{g_i\}\ i \epsilon N\}$ be a finite strategic structure of game. Consider a mixed strategic profile $\{g_{B1}^*, g_{B2}^*, .. g_{Bn}^*\}$ is a Nash equilibrium iff $g_B^* \epsilon (\Psi(g_B^*)_{-i})$ for all $i \epsilon N$ where $g_i((g_B^*)_{-i})$ represents the set of best responses of i^{th} player.

Theorem 3.1: Every finite game has mixed strategy Nash equilibrium.

Proof: We have a mixed strategic set $\{g_{B1}^*, g_{B2}^* \ldots g_{Bn}^*\}$ is a Nash equilibrium iff $g_B^* \epsilon (\Psi_i(g_B^*)_{-i})$ for all $i \epsilon N$, where Ψ_i is the best response of i^{th} player, defined by $\Psi_i((g_B^*)_{-i}) = \{(g_B)_i) \epsilon S_i, g_i(g_{Bi}, (g_B)_{-i})\}$.

Let the composite correspondence $\Psi: \Delta(S_1)\times\Delta(S_2)\ldots\times \Delta(S_n) \rightarrow \Delta(S_1)\times\Delta(S_2) \times\ldots\times\Delta(S_n)$ as follows $\Psi(g_{B1}, g_{B2}, \ldots g_{Bn}) = \Psi((g_B)_{-1} \times (g_B)_{-2} \times \ldots (g_B)_{-n})$.Clearly a mixed strategy profile $g_B^* = (g_B)_1^*, (g_B)_2^* \ldots (g_B)_n^*$ is a Nash equilibrium iff g_B^* is a fixed point of the best response of Ψ. Similar argument shows that Ψ is a best response to g_B^*. Since $g_B^*$and Ψ are mutual best response, we have Nash equilibrium.

3.3 Nash equilibrium of pure strategies of sequentially bi matrix game using soft set approach

Consider the following

- Let $X = (x_{ij})$ be soft pay off matrix of P_1, where $x_{ij} = (f_A)_i$, $Y = (y_{ij})$ be pay off matrix of P_2 ,where $y_{ij} = (f_B)_j$
- Let $S_1 = \{f_{A1}, f_{A2} \ldots f_{An1}\}$ and $S_2 = \{f_{B1}, f_{B2} \ldots f_{Bn2}\}$ be pure strategies of Player 1 & 2 respectively.
- For any $(f_B)_j \epsilon S_2$, $\Psi((f_A)_{-i}) = \{ (f_A)_i^* \epsilon S_1 , x_i^*{}_j = \max_{f_A \epsilon S_1} x_{ij} \}$ is the best response of Player P_1.
- For any $(f_A)_i \epsilon S_1$, $\Psi((f_B)_{-j}) = \{ (f_B)_j^* \epsilon S_2 , y_{ij}^* = \max_{f_B \epsilon S_2} y_{ij} \}$ is the best response of Player P_2.
- The set of best response of $((f_A)_i, (f_B)_j) \epsilon S_1 \times S_2$ denotes $f(f_A, f_B) = (\Psi((f_A)_{-i}) \times \Psi((f_B)_{-j})$. A pair $((f_A)_i^*, (f_B)_j^*)$ is a Nash equilibrium of pure strategies if $((f_A)_i^*, (f_B)_j^*) \epsilon \Psi((f_A)_{-i}^*, (f_B)_{-j}^*)$.

Definition 3.3.1: A soft payoff matrix X is said to be sequentially monotone if their exist a sequence of best responses $(f_A)_i^k \epsilon \Psi((f_A)_{-i}^k)$ such that $f_A^k \subseteq f_A^{k+1}$ for any $k =\{1,2.3\ldots.n_1-1\}$. Another soft payoff matrix Y is said to be sequentially monotone if their exist a sequence of best responses $(f_B)_j^k \epsilon \Psi((f_B)_{-j}^k)$ such that $f_B^k \subseteq f_B^{k+1}$,
for any $k =\{1,2.3\ldots.n_2-1\}$.

Theorem 3.3.1: Any sequentially monotone bi matrix game has Nash equilibrium of pure strategies.

Proof: Consider the set S be product of S_1 and S_2, so it has minimum element. Assume that $x^k = (f_A, f_B) \epsilon S$. Then by sequential monotonicity, their exist $(f_A)_i^k \epsilon \Psi((f_A)_{-i}^k)$ and $(f_B)_j^k \epsilon \Psi((f_B)_{-j}^k)$,such that $(f_A^k, f_B^k) \subseteq (f_A^{k+1}, f_B^{k+1})$ which implies $(f_A, f_B) \subseteq (f_A^k, f_B^k)$ Define $x^{k+1} = (f_A^k, f_B^k)$,then $x^{k+1} \epsilon \Psi((f_A)_{-i}, (f_B)_{-j}) = f((f_A, f_B)) = f(x^k)$ and $x^k \lesssim x^{k+1}$.Then by theorem 2.1,the bi matrix game has a Nash equilibrium of pure strategies.

CONCLUSION

Soft set theory is one of the methods for solving problems of uncertainty. Recently, many authors have already studies the notion of fixed point on soft set, because fixed point theory is an important area of mathematics with many

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applications in various fields of computer science and engineering science etc. In this paper, first we have introduce Nash equilibrium of pure strategies and mixed strategies using soft set approach, then we have to extend this idea to Nash equilibrium of bi matrix game used fixed point and soft set theories.

ACKNOWLEDGEMENT

I express my sincere gratitude to my research guide Dr. M. S. Samuel, Director, Dept. of Computer Applications, MACFAST (Research guide ,M.G university) for his continuous support and guidance that helped me a lot in this paper work.

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Source of support: Proceedings of UGC Funded International Conference on Intuitionistic Fuzzy Sets and Systems (ICIFSS-2018), Organized by: Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.