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SOME FIXED POINT THEOREMS ON GENERALIZED INTUITIONISTIC FUZZY METRIC SPACES

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ABSTRACT

In this paper, we introduce the concepts of convergent sequence, Cauchy sequence in generalized intuitionistic fuzzy metric space and some common fixed point theorems for some generalized contraction mappings are established.

Keywords: *Intuitionistic fuzzy metric spaces, D*- Fuzzy metric spaces, Contraction mappings.*

AMS Subject Classification: *47H10, 54H25.*

1. INTRODUCTION

The theory of fuzzy sets was introduced simultaneously by Zadeh's [15] in 1965. It gives the foundation of fuzzy mathematics. Later, several researchers have applied this theory to the well-known results in the classical set theory. The concept of an intuitionistic fuzzy set was first introduced by Atanassov in [2] and many works by the same author in [3]. Sedghi *et al.* [11] modified the definition of D-metric space and introduced an idea of D*- metric space and established some fixed point theorems in such space. Veerapandi and Pillai [14] established some fixed point theorems of contractive mappings on D*- metric spaces. In this context, it is worth mentioning the work of Deng [6] and Erceg [8]. On the other hand, different authors generalized the idea of fuzzy metric space in different directions. Sedghi *et al.* [12] introduced the concept of M- fuzzy metric space which is a generalization of fuzzy metric space due to George and Veeramani [9]. Recently Bag [5] modified the definition of M- fuzzy metric space introduced by Sedghi *et al.*[12] and call it D*- fuzzy metric space. It has been possible to achieve two decomposition theorems of D*- fuzzy metric into a family of D*- metrics.

Park introduced and discussed in [13] a notion of intuitionistic fuzzy metric space which is based both on the idea of intuitionistic fuzzy set due to Atanassov [3], and the concept of a fuzzy metric space given George andVeeramani [9]. In this paper, we introduce the concepts of convergent sequence, Cauchy sequence in generalized intuitionistic fuzzy metric space and some common fixed point theorems for some generalized contraction mappings are established.

2. PRELIMINARIES

Definition 2.1: A 5 – tuple $(X, D^*, F^*, *, \Diamond)$ is called a generalized intuitionstic fuzzy metric space, if X is an arbitrary (non-empty) set and D*, F* are fuzzy set on $X^3 \times [0, \infty)$, satisfying the following conditions: for each x, y, z a ∈ X and t, s ∈ [0, ∞),

(i) $D^*(x, y, z, t) + F^*(x, y, z, t) \le 1$,
(ii) $D^*(x, y, z, 0) = 0$,
(iii) $D^*(x, y, z, t) = 1$ iff $x = y = z$, for all $t > 0$,
(iv) $D^*(x, y, z, t) = D^*(p\{x, y, z\}, t)$, where p is a permutation function,
$D^*(x, y, a, t) * D^*(a, z, z, s) \le D^*(x, y, z, t + s)$,
(v) $\lim_{t\to\infty} D^*(x, y, z, t) = 1$,
(vi) $F^*(x, y, z, 0) = 1$,
(vii) $F^*(x, y, z, t) = 0$ iff $x = y = z$, for all $t > 0$,
(viii) $F^*(x, y, z, t) = F^*(p\{x, y, z\}, t)$, where pis a permutation function,
(ix) $F^*(x, y, a, t) \Diamond F^*(a, z, z, s) \ge F^*(x, y, z, t+s)$,
(x) $\lim_{t\to\infty} F^*(x, y, z, t) = 0$.

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Example 2.2: Let (X, D) is a generalized intuitionistic fuzzy metric space.

Define D* : X×X×X×[0, ∞) byD* (x, y, z, t) = $\begin{cases} 1 & \text{if } t > D(x,y,z) \\ \frac{1}{2} & \text{If } 0 < t \le D(x,y,z) \\ 0 & \text{if } t \le 0 \end{cases}$

and Define F* : X×X×X×[0, ∞) by F* (x, y, z, t) = $\begin{cases} 0 & \text{if } t < F(x,y,z) \\ \frac{1}{2} & \text{if } 0 > t \ge F(x,y,z) \\ 1 & \text{if } t \ge 1. \end{cases}$

Then (X, D*, F*, *, ◊) be a generalized intuitionistic fuzzy metric space.

Where X is a non empty set and for all x, y, z ∈ X, (X, D*, F*) is a D* and F* metric spaces, for all t ∈ [0, ∞) and D* and F* are functions defined on X above.

Then we shall prove that (X, D*, F*) is a generalized intuitionistic fuzzy metric space.

Proof:

(i) D*(x, y, z, 0) = 0, for all x, y, z ∈ X.
(ii) D*(x, y, z, t) = D*(p(x, y, z), t) for all t∈ [0, ∞), for all x, y, z ∈ X .
(iii) D*(x, y, z, t) = 1 for all t > 0 ⇔t > D(x, y, z), for all t > 0.
(iv) For all x, y, z, a ∈ X, s, t ∈[0, ∞) ⇔ D(x, y, z) = 0 ⇔ x= y = z and
(v) F*(x, y, z, 0) = 1 for all x, y, z ∈ X.
(vi) F*(x, y, z, t) = F*(p(x, y, z), t), for allt∈[0, ∞), for all x, y, z ∈ X .
(vii) F*(x, y, z, t) = 0 , for all t > 0 ⇔t <F(x, y, z) for all t > 0.
(viii) For all x, y, z, a∈ X, s, t ∈[0, ∞) ⇔ F(x, y, z) = 1 ⇔ x=y = z.

We consider the following cases:

Case-I: Suppose D*(x, y, z, t) =1 and D*(a, z, z, s) = 1.

Then t >D*(x, y, a) and s >D*(a, z, z,). Now, D*(x, y, z) ≤ D*(x, y, a) + D*(a, z, z) ≤ t+s.

Thus t + s> D*(x, y, z).

So, D*(x, y, z, t+s) = 1 = 1 * 1 = D*(x, y, a, t) * D*(a, z, z, s) and F*(x, y, z, t) = 0 and F*(a, z, z, s) = 0.

Then t < F*(x, y, a) and s<F*(a, z, z,).Now, F*(x, y, z) ≥ F*(x, y, a) + F*(a, z, z) ≥ t+s.

Thus t + s< F*(x, y, z). So, F*(x, y, z, t+s) = 0 = 0◊ 0 = F*(x, y, a, t) ◊ F*(a, z, z, s).

Case-II: Suppose, D*(x, y, a, t) =½ , D*(a, z, z, s) =1 and F*(x, y, a, t) =½ , F*(a, z, z, s) = 0.

Then 0 < t ≤ D*(x, y, a) and s> D*(a, z, z) ≥ 0 and 0 > t ≥ F*(x, y, a) and s< F*(a, z, z) ≤1. Thus s + t > 0.

So, D*(x, y, z, t+s) ≥ ½ = ½ * 1 = D*(x, y, z, s) * D*(a, z, z, t) and
F*(x, y, z, t + s) ≤ ½ = ½ ◊ 0 = F*(x, y, a, s) ◊ F(a, z, z, t).

Case-III: Suppose D*(x, y, a, t) = 0; D*(a, z, z, s) = 1 and F*(x, y, a, t) = 1;
F*(a, z, z, s) = 0. Then D*(x, y, z, t+s) ≥ 0 = 0 * 1 = D*(x, y, z, t) * D*(a, z, z, t) and
F*(x, y, z, t + s) ≤ 1 = 1 ◊ 0 = F*(x, y, a, t) ◊ F*(a, z, z, s).

Case-IV: Similarly, we can prove that D*(x, y, z, t + s) ≥ D*(x, y, a, t) * D*(a, z, z, s) and
F*(x, y, z, t + s) ≤ F*(x, y, a, t) ◊ (F*(a, z, z, s). Whenever,
D*(x, y, a, t) = 1, D*(a, z, z, s) = 1 or ½ or 0 and F*(x, y, a, t) = 0, F*(a, z, z, s) = 0 or ½ or 1.

Case-V: Suppose D*(x, y, a, t) = ½; D*(a, z, z, s) = ½ and F*(x, y, a, t) = ½; F*(a, z, z, s) = ½.

Then 0 < t = D*(x, y, a), 0 < s ≤ D*(a, z, z) and 1 > t = F*(x, y, a), 1 > s ≥ F*(a, z, z). Thus t + s >0.

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Now$D^*(x, y, z, t + s) \geq ½ = ½ * 1 \geq ½ * ½ = D^*(x, y, z, t) * D^*(a, z, z, s)$ and
$F^*(x, y, z, t + s) \leq ½ = ½ \lozenge 0 \leq ½ \lozenge ½ = F^*(x, y, a, t) \lozenge F^*(a, z, z, s)$.

So,for all $t, s \in [0, \infty]$ and for all x, y, z, a, X.
$D^*(x, y, z, t + s) \geq D^*(x, y, a, t) * D^*(a, z, z, s)$ and $F^*(x, y, z, t + s) \leq F^*(x, y, a, t) \lozenge F^*(a, z, z, s)$.

Hence $(X, D^*, F^*, *, \lozenge)$ is a generalized intuitionistic fuzzy metric space.

Definition 2.3: A sequence $\{x_n\}$ in a generalized intuitionistic fuzzy metric space X is said to be convergent to a point $x \in X$ if $\lim_{n\to\infty} D^*(x_n, x_n, x, t) = \lim_{n\to\infty} D^*(x, x, x_n, t) = 1$ and $\lim_{n\to\infty} F^*(x_n, x_n, x, t) = \lim_{n\to\infty} F^*(x, x, x_n, t) = 0$, for all $t > 0$.

Definition 2.4: A sequence $\{x_n\}$ in a generalized intuitionistic fuzzy metric space X is said to be a Cauchy sequence if $\lim_{n\to\infty} D^*(x_n, x_n, x_{n+p}, t) =1$ and $\lim_{n\to\infty} F^*(x_n, x_n, x_{n+p}, t) = 0$, for all $t>0$ and $p = 1, 2, \ldots$

Definition 2.5: Let X be a generalized intuitionistic fuzzy metric space. A non empty set A of X is said to be complete if every Cauchy sequence $\{x_n\}$ in A converges to some point in A.

Example 2.6: Let X be a non empty set and D and F be a D* and F* metric on X and (X, D, F) is complete. Choose $a* b = ab$ and $a\lozenge b = a +b - ab$, for all $a,b \in [0, 1]$ for each $t \in [0, \infty)$. We define$D^*(x, y, z, t) = \frac{t}{t+D(x,y,z)}$ and $F^*(x, y, z, t) = \frac{F(x,y,z)}{t+F(x,y,z)}$ for all $x, y, z \in X$. Then $(X, D^*, F^*, *,\lozenge)$ is a complete generalized intuitionistic fuzzy metric space.

Definition 2.7: Let $(X, D^*, F^*, *,\lozenge)$ be a generalized intuitionistic fuzzy metric space, where * is a continuous t- norm and $\lozenge$ is a continuous t- conorm. Then limit of a convergent sequence is unique.

Proof: Let $\{x_n\}$ be a sequence in generalized intuitionistic fuzzy metric space X and suppose $x_n \to x$ and $x_n \to y$ for some $x, y \in X$. We shall show that $x = y$.

We have$D^*(x, x, y, t+s) \geq D^*(x, x, x_n, t) * D^*(x_n, y, y, s)$ and
$F^*(x, x, y, t+s) \leq F^*(x, x, x_n, t) \lozenge F^*(x_n, y, y, s)$, for all $t, s \in (0, \infty)$, $n = 1, 2, \ldots$.

Let $n \to \infty$. Then$D^*(x, x, y, t + s) \geq \lim_{n\to\infty} D^*(x, x, x_n, t) * \lim_{n\to\infty} D^*(x_n, y, y, s) = 1 *1 = 1$ and
$F^*(x, x, y, t + s) \leq \lim_{n\to\infty} F^*(x, x, x_n, t) \lozenge \lim_{n\to\infty} F^*(x_n, y, y, s) = 0 \lozenge 0 = 0$.
Thus $D^*(x, x, y, t+s) = 1$ and$F^*(x, x, y, t+s) = 0$, for all $t, s >0$. So, $x = y$.

Proposition 2.8: Every convergent sequence isa Cauchy sequence.

Proof: Let $\{x_n\}$ be a sequence in X and $x_n \to x$ for some $x \in X$.

Now $D^*(x, x_n, x_{n+p}, t+s) \geq D^*(x_n, x_n, x, t) * D^*(x, x, x_{n+p}, s)$ and
$F^*(x, x_n, x_{n+p}, t+s) \leq F^*(x_n, x_n, x, t) \lozenge F^*(x, x, x_{n+p}, s)$, for all $t, s \in (0, \infty)$ and $p \in N$.Let $n \to \infty$.

Then $\lim_{n\to\infty} D^*(x_n, x_n, x_{n+p}, t+s) \geq \lim_{n\to\infty} D^*(x_n, x_n, x, t) * \lim_{n\to\infty} D^*(x, x_{n+p}, x_{n+p}, s) = 1*1 =1$,
$\lim_{n\to\infty} F^*(x_n, x_n, x_{n+p}, t+s) \leq \lim_{n\to\infty} F^*(x_n, x_n, x, t) \lozenge \lim_{n\to\infty} F^*(x, x_{n+p}, x_{n+p}, s) = 0 \lozenge 0 = 0$.

Thus $\lim_{n\to\infty} D^*(x_n, x_n, x_{n+p}, t + s) = 1$ and $\lim_{n\to\infty} F^*(x_n, x_n, x_{n+p}, t + s) = 0$, for $p = 1, 2, \ldots$ and for all $t, s \in (0, \infty)$. So, $\{x_n\}$ is a Cauchy sequence in X.

3. SOME FIXED POINT THEOREMS IN GENERALIZED INTUITIONISTIC FUZZY METRIC SPACES

Theorem 3.1: Let $(X, D^*, F^*, *,\lozenge)$ be a complete generalized intuitionistic fuzzy metric space and $T_1, T_2, T_3 : X \to X$ be three mappings satisfying that$D^*(T_1x, T_2y, T_3z, t) \geq D^*(x, y, z, t/a)$ and $F^*(T_1x, T_2y, T_3z, t) \leq F^*(x, y, z, t/a)$, for all $t > 0$, for all $x, y, z, \in X$ and $0 < a < 1$. Then T_1, T_2 andT_3 have a unique fixed point in X.

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Proof: Let $x_0 \in X$ be a fixed arbitrary point. Define sequence $\{x_n\}$ in X.
$T_1x_n = x_{n+1}$, $T_2x_{n+1} = x_{n+2}$, $T_3x_{n+2} = x_{n+3}$, . . .Then

$$\begin{aligned}D^*(x_n, x_n, x_{n+1}, t) &= D^*(T_1x_{n-1}, T_2x_{n-1}, T_3x_n, t), \text{ for all } t > 0\\ &\geq D^*(x_{n-1}, x_{n-1}, x_n, t/a), a \in (0, 1)\\ &\dots\\ &\geq D^*(x_0, x_0, x_1, t/a^n) \text{ and}\end{aligned}$$

$$\begin{aligned}F^*(x_n,x_n, x_{n+1}, t) &= F^*(T_1x_{n-1}, T_2x_{n-1}, T_3x_n, t) \text{ for all } t > 0\\ &\leq F^*(x_{n-1}, x_{n-1}, x_n, t/a), a \in (0, 1)\\ &\dots\\ &\leq F^*(x_0, x_0, x_1, t/a^n).\end{aligned}$$

Thus, $\lim_{n\to\infty} D^*(x_0, x_0, x_1, t/a^n) = 1$ and $\lim_{n\to\infty} F^*(x_0, x_0, x_1, t/a^n) = 0$, for all $t > 0$. So, $\lim_{n\to\infty} D^*(x_n, x_n, x_{n+1}, t) = 1$ and $\lim_{n\to\infty} F^*(x_n, x_n, x_{n+p}, t) = 0$, for all $t > 0$.Similarly, we can prove that $\lim_{n\to\infty} D^*(x_n, x_n, x_{n+p}, t) = 1$ and $\lim_{n\to\infty} F^*(x_n, x_n, x_{n+p}, t) = 0$, for p = 1, 2, . . . and for all $t > 0$.

Hence $\{x_n\}$ is a Cauchy sequence. Since, X is complete $\lim x_n = x$, for some $x \in X$.

Now, we prove that $T_1x = x$.

Clearly, $D^*(T_1x, T_2x, T_3x, t) \geq D^*(x, x, x, t/a)$ and $F^*(T_1x, T_2x, T_3x, t) \leq F^*(x, x, x, t/a)$ for all $t > 0$, $0 < a < 1$. Then $D^*(T_1x, T_2x, T_3x, t) = 1$ and $F^*(T_1x, T_2x, T_3x, t) = 0$, for all $t > 0$. Thus $T_1x = T_2x = T_3x$.

Again

$$\begin{aligned}D^*(T_1x, T_1x, x, t) &= D^*(T_1x, T_2x, x, t), \text{ for all } t > 0\\ &\geq D^*(T_1x, T_2x, x_{n+3}, t/2) * D^*(x_{n+3}, x, x, t/2), \text{ for all } t > 0\\ &= D^*(T_1x, T_2x, T_3x_{n+2}, t/2) * D^*(x_{n+3}, x, x, t/2), \text{ for all } t > 0\\ &\geq D^*(x, x, x_{n+2}, t/2) * D^*(x_{n+3}, x, x, t/2) \text{ and}\end{aligned}$$

$$\begin{aligned}F^*(T_1x, T_1x, x, t) &= F^*(T_1x, T_2x, x, t), \text{for all } t > 0\\ &\leq F^*(T_1x, T_2x, x_{n+3}, t/2) \lozenge F^*(x_{n+3}, x, x, t/2), \text{for all } t > 0\\ &= F^*(T_1x, T_2x, T_3x_{n+2}, t/2) \lozenge F^*(x_{n+3}, x, x, t/2), \text{for all } t > 0\\ &\leq F^*(x, x, x_{n+2}, t/2) \lozenge F^*(x_{n+3}, x, x, t/2).\end{aligned}$$

Let $n \to \infty$.Then $D^*(T_1x, T_1x, x, t) \geq 1 * 1 = 1$ and $F^*(T_1x, T_1x, x, t) \leq 0 \lozenge 0 = 0$.

Thus, $D^*(T_1x, T_1x, x, t) = 1$ and $F^*(T_1x, T_1x, x, t) = 0$, for all $t > 0$. So, $T_1x = T_1x = x$.

Hence x is a fixed point of T_1, T_2 and T_3.

Uniqueness:

Assume that there exists $(y \neq x)$ such that $T_1y = T_2y = T_3y = y$. Then

$$\begin{aligned}D^*(x, y, y, t) &= D^*(T_1x, T_2y, T_3y, t)\\ &\geq D^*(x, y, y, t/a)\\ &= D^*(T_1x, T_2y, T_3y, t/a)\\ &\geq D^*(x, y, y, t/a^2)\\ &\dots\\ &\geq D^*(x, y, y, t/ a^n), \text{ for some } n \in N \text{ and}\end{aligned}$$

$$\begin{aligned}F^*(x, y, y, t) &= F^*(T_1x, T_2y, T_3y, t)\\ &\leq F^*(x, y, y, t/a)\\ &= F^*(T_1x, T_2y, T_3y, t/a)\\ &\leq F^*(x, y, y, t/a^2)\\ &\dots\\ &\leq F^*(x, y, y, t/ a^n), \text{ for some } n \in N.\end{aligned}$$

Let $n \to \infty$. Then $D^*(x, y, y, t/a^n) = 1$ and $F^*(x, y, y, t/a^n) = 0$, for all $t > 0$.

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Thus, $0 < a < 1$. So, $D^*(x, y, y, t) = 1$ and $F^*(x, y, y, t) = 0$, for all $t > 0$.

Hence $x = y$ and thus, T_1, T_2 and T_3 have a unique and common fixed point in X.

Theorem 3.2: Let (X, D*, F*, *, ◊) be a complete generalized intuitionstic fuzzy metric space and T: X→ X be a mapping such that $D^*(Tx, T^2x, T^3x, t) \geq D^*(x, Tx, T^2x, t/a)$ and $F^*(Tx, T^2x, T^3x, t) \leq F^*(x, Tx, T^2x, t/a)$, for all $x \in X$ and $0 \leq a < 1$, for all $t > 0$. Then T has a unique fixed point.

Proof: Let $x_0 \in X$ be a fixed arbitrary element.

Define a sequence $\{x_n\}$ in X as $x_{n+1} = Tx_n$, for n = 0, 1, 2, . . . Then for $n \geq 0$, we have

$$\begin{aligned} D^*(x_n, x_n, x_{n+1}, t) &= D^*(Tx_{n-1}, Tx_{n-1}, Tx_n, t) \\ &\geq D^*(x_{n-1}, x_{n-1}, x_n, t/a) \\ &\dots \\ &\geq D^*(x_0, x_0, x_1, t/a^n) \to 1 \text{ as } n \to \infty \end{aligned}$$

$$\begin{aligned} F^*(x_n, x_n, x_{n+1}, t) &= F^*(Tx_{n-1}, Tx_{n-1}, Tx_n, t) \\ &\leq F^*(x_{n-1}, x_{n-1}, x_n, t/a) \\ &\dots \\ &\leq F^*(x_0, x_0, x_1, t/a^n) \to 0 \text{ as } n \to \infty, \text{ since } 0 \leq a < 1. \end{aligned}$$

Thus, $\lim_{n\to\infty} D^*(x_n, x_n, x_{n+p}, t) = 1$ and $\lim_{n\to\infty} F^*(x_n, x_n, x_{n+p}, t) = 0$, for all $t > 0$ and $p = 1, 2, \dots$

So, $\{x_n\}$ is a Cauchy sequence in X. Since X is complete $x_n \to x$, for some $x \in X$. Then

$D^*(x_{n+1}, x_{n+1}, Tx, t) = D^*(Tx_n, Tx_n, Tx, t) \geq D^*(x_n, x_n, x, t/a)$ and

$F^*(x_{n+1}, x_{n+1}, Tx, t) = F^*(Tx_n, Tx_n, Tx, t) \leq F^*(x_n, x_n, x, t/a)$.

Thus, $\lim_{n\to\infty} D^*(x_{n+1}, x_{n+1}, Tx, t) \geq \lim_{n\to\infty} D^*(x_n, x_n, x, t/a) = 1$ and

$\lim_{n\to\infty} F^*(x_{n+1}, x_{n+1}, Tx, t) \leq \lim_{n\to\infty} F^*(x_n, x_n, x, t/a) = 0$, for all $t > 0$.

So, $\lim_{n\to\infty} D^*(x_{n+1}, x_{n+1}, Tx, t) = 1$ and $\lim_{n\to\infty} F^*(x_{n+1}, x_{n+1}, Tx, t) = 0$, for all $t > 0$. Hence $\{x_{n+1}\} \to Tx$.

Since limit of a sequence is unique, $Tx = x$.

Uniqueness:

Suppose there exists $y \in X$, $x \neq y$ such that $Ty = y$. Then

$$\begin{aligned} D^*(x, y, y, t) &= D^*(T^3x, T^2y, Ty, t), \text{ for all } t > 0 \\ &\geq D^*(T^2x, Ty, y, t/a), \text{ for all } t > 0, 0 < a < 1 \\ &= D^*(T^3x, T^2y, Ty, t/a) \\ &\geq D^*(T^2x, Ty, y, t/a^2) \\ &\geq D^*(T^2x, Ty, y, t/a^n) \text{ and} \end{aligned}$$

$$\begin{aligned} F^*(x, y, y, t) &= F^*(T^3x, T^2y, Ty, t) \text{ for all } t > 0 \\ &\leq F^*(T^2x, Ty, y, t/a) \text{ for all } t > 0, 0 < a < 1 \\ &= F^*(T^3x, T^2y, Ty, t/a) \\ &\leq F^*(T^2x, Ty, y, t/a^2) \\ &\leq F^*(T^2x, Ty, y, t/a^n). \end{aligned}$$

Let $n \to \infty$. Then $\lim_{n\to\infty} D^*(T^2x, Ty, y, t/a^n) = 1$ and $\lim_{n\to\infty} F^*(T^2x, Ty, y, t/a^n) = 0$, $(0 \leq a < 1)$ for all $t > 0$.

Thus, $D^*(x, y, y, t) = 1$ and $F^*(x, y, y, t) = 0$, for all $t > 0$. So, $x = y$.

Hence T has a unique fixed point in X.

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