Volume 9, No. 1, Jan. - 2018 (Special Issue)
International Journal of Mathematical Archive-9(1), 2018, 123-126
IJMA Available online through www.ijma.info ISSN 2229 – 5046

SOME NEW GENERALIZED AGGREGATION OPERATORS FOR TRIANGULAR INTUITIONISTIC FUZZY NUMBERS AND APPLICATION TO MULTI-ATTRIBUTE GROUP DECISION MAKING: SUGGESTED MODIFICATIONS

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ABSTRACT

Wan et al. (Computers & Industrial Engineering, 93 (2016) 286-301) proposed triangular intuitionistic fuzzy generalized ordered weighted averaging (TIFGOWA) operator and proved some of its basic properties. In this paper, it is pointed out that the statement and proof of boundedness property of TIFGOWA operator, proposed by Wan et al. is not valid. Also, a valid statement and proof for the same is proposed.

Keywords: *Multi-attribute group decision making, Triangular intuitionistic fuzzy number (TIFN), Generalized ordered weighted averaging operator.*

1. INTRODUCTION

If $\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n$ are triangular intuitionistic fuzzy numbers (TIFNs) then,

$$TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = g^{-1}(\sum_{i=1}^{n} w_i g(\tilde{a}_i))$$

where g is continuous strictly monotone increasing function n, $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector associated with TIFGOWA operator, satisfying $0 \leq w_i \leq 1$ $(i = 1, 2, \ldots, n)$ and $\sum_{i=1}^{n} w_i = 1$, $((1), (2), \ldots, (n))$ is a permutation of $(1, 2, \ldots, n)$ such that $\tilde{a}_{(i)} \geq \tilde{a}_{(i+1)}$) for all i, is called TIFGOWA operator [1, Sec. 4, Definition 15, pp. 291]. Wan et al. [1, Sec. 4, pp 291], stated and proved some useful properties (idempotent, monotonicity, boundedness etc.) of TIFGOWA operator. In this paper, it is pointed out that the statement and proof of boundedness property of TIFGOWA operator, proposed by Wan et al. [1, Sec. 4, Proposition 4, pp. 292], is not valid. Also, a valid statement and proof of the same is proposed.

2. INVALIDITY OF THE BOUNDEDNESS PROPERTY

Wan et al. [1, Sec. 4, Proposition 4, pp. 292] stated the boundedness property of TIFGOWA operator in following manner.

"Let $\tilde{a}_i = ((\underline{a}_i, a_i, \bar{a}_i); w_{\tilde{a}_i}, u_{\tilde{a}_i})$ $(i = 1, 2, \ldots, n)$ be a series of TIFNs. If $\tilde{a}^- = ((\min_i\{\underline{a}_i\}, \min_i\{a_i\}, \min_i\{\bar{a}_i\}); \wedge_i w_{\tilde{a}_i}, \vee_i u_{\tilde{a}_i})$, $\tilde{a}^+ = ((\max_i\{\underline{a}_i\}, \max_i\{a_i\}, \max_i\{\bar{a}_i\}); \vee_i w_{\tilde{a}_i}, \wedge_i u_{\tilde{a}_i})$ then $\tilde{a}^- \leq TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq \tilde{a}^+$."

It is obvious from this statement that according to Wan et al. [1], there will exit at least one weight vector $w = (w_1, w_2, \ldots, w_n)^T$, satisfying $w_1, w_2, \ldots, w_n \geq 0$ and $w_1 + w_2 + \cdots + w_n = 1$, such that

$$TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \tilde{a}^- \text{ and } TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \tilde{a}^+.$$

In this section, two TIFNs $\tilde{a}_1$ and $\tilde{a}_2$ are chosen and shown that there will not exist any weight vector $w = (w_1, w_2)^T$, satisfying $w_1, w_2 \geq 0$ and $w_1 + w_2 = 1$, such that $TIFGOWA_w(\tilde{a}_1, \tilde{a}_2) = \tilde{a}^+$.

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

Let $\tilde{a}_1 = \big((1,4,7); 0.4, 0.2\big)$ and $\tilde{a}_2 = \big((2,3,8); 0.3, 0.8\big)$ be two TIFNs then

$$\tilde{a}^- = \big((\min\{1,2\}, \min\{4,3\}, \min\{7,8\}); \min\{0.4, 0.3\}, \max\{0.2, 0.8\}\big)$$
$$= \big((1,3,7); 0.3, 0.8\big)$$
$$\tilde{a}^+ = \big((\max\{1,2\}, \max\{4,3\}, \max\{7,8\}); \max\{0.4, 0.3\}, \min\{0.2, 0.8\}\big)$$
$$= \big((2,4,8); 0.4, 0.2\big)$$

And for $g(x) = x$,

$$TIFGOWA_w(\tilde{a}_1, \tilde{a}_2) = g^{-1}\big(w_1 g(\tilde{a}_1) + w_2 g(\tilde{a}_2)\big)$$
$$= g^{-1}(w_1\tilde{a}_1 + w_2\tilde{a}_2)$$
$$= w_1\tilde{a}_1 + w_2\tilde{a}_2$$
$$= w_1\big((1,4,7); 0.4, 0.2\big) + w_2\big((2,3,8); 0.3, 0.8\big)$$
$$= \big((w_1 + 2w_2, 4w_1 + 3w_2, 7w_1 + 8w_2); 0.3, 0.8\big)$$

Now, the aim is to find a weight vector $= (w_1, w_2)^T$, satisfying $w_1, w_2 \geq 0$ and $w_1 + w_2 = 1$, such that $TIFGOWA_w(\tilde{a}_1, \tilde{a}_2) = \tilde{a}^+$

According to existing result [1, Sec. 2, Definition 10, pp. 289],
$TIFGOWA_w(\tilde{a}_1, \tilde{a}_2) = \tilde{a}^+$

$$\Rightarrow m_\mu(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2), \gamma) = m_\mu(\tilde{a}^+, \gamma),$$
$$m_\nu(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2), \gamma) = m_\nu(\tilde{a}^+, \gamma)\ \forall\ 0 \leq \gamma \leq 1.$$

$$\Rightarrow m_\mu\Big(\big((w_1 + 2w_2, 4w_1 + 3w_2, 7w_1 + 8w_2); 0.3, 0.8\big), \gamma\Big) = m_\mu\Big(\big((2,4,8); 0.4, 0.2\big), \gamma\Big),$$
$$m_\nu\Big(\big((w_1 + 2w_2, 4w_1 + 3w_2, 7w_1 + 8w_2); 0.3, 0.8\big), \gamma\Big) = m_\nu\Big(\big((2,4,8); 0.4, 0.2\big), \gamma\Big)\ \forall\ 0 \leq \gamma \leq 1.$$

$$\Rightarrow \tfrac{1}{3}\big[(1-\gamma)\big(w_1 + 2w_2 + 2(4w_1 + 3w_2)\big) + \gamma\big(2(4w_1 + 3w_2) + 7w_1 + 8w_2\big)\big](0.3) = \tfrac{1}{3}\big[(1-\gamma)(2 + 2\times 4) + \gamma(2\times 4 + 8)\big](0.4)$$

,

$$\Rightarrow \tfrac{1}{3}\big[(1-\gamma)\big(w_1 + 2w_2 + 2(4w_1 + 3w_2)\big) + \gamma\big(2(4w_1 + 3w_2) + 7w_1 + 8w_2\big)\big](1 - 0.8) = \tfrac{1}{3}\big[(1-\gamma)(2 + 2\times 4) + \gamma(2\times 4 + 8)\big](1 - 0.2)$$

$$[(1-\gamma)(9w_1 + 8w_2) + \gamma(15w_1 + 14w_2)] \times 0.3 = [(1-\gamma)10 + \gamma 16] \times 0.4 \quad (1)$$
$$[(1-\gamma)(9w_1 + 8w_2) + \gamma(15w_1 + 14w_2)] \times 0.2 = [(1-\gamma)10 + \gamma 16] \times 0.8 \quad (2)$$

Equations (1) and (2) should be satisfied for all values of $\gamma \in [0,1]$. Assuming $\gamma = 1$, Equations (1) and (2) are transformed into Equations (3) and (4) respectively.

$$4.5w_1 + 4.2w_2 = 6.4 \quad (3)$$
$$3w_1 + 2.8w_2 = 1.28 \quad (4)$$

It can be easily verified that it is not possible to find any values of w_1 and $w_2 \geq 0$, which satisfy Equations (3), (4) and the condition $w_1 + w_2 = 1$, simultaneously i.e. ,

$m_\mu(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2), \gamma) \neq m_\mu(\tilde{a}^+, \gamma)$ and $m_\nu(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2), \gamma) \neq m_\nu(\tilde{a}^+, \gamma)\ \forall\ 0 \leq \gamma \leq 1$

$\Rightarrow TIFGOWA_w(\tilde{a}_1, \tilde{a}_2) \neq \tilde{a}^+$ for any weight vector $w = (w_1, w_2)^T$.

Hence, the statement $TIFGOWA_w(\tilde{a}_1, \tilde{a}_2) \leq \tilde{a}^+$ is mathematically incorrect. Similarly, it can be verified that the statement $\tilde{a}^- \leq TIFGOWA_w(\tilde{a}_1, \tilde{a}_2)$ is also mathematically incorrect.

3. PROPOSED STATEMENT AND PROOF OF BOUNDEDNESS PROPERTY

In Section 2, it is shown that the statement and proof of boundedness property of TIFGOWA operator of TIFNs is not valid. In this section, a valid statement and proof of the same is proposed.

Let $\tilde{a}_i = \left((\underline{a}_i, a_i, \bar{a}_i);\ w_{\tilde{a}_i}, u_{\tilde{a}_i}\right)$ $(i = 1,2,\dots,n)$ be a series of TIFNs. If

$\tilde{a}^- = \Big(\Big(g^{-1}(\sum_{i=1}^n w_i g(\underline{a}_i)) + 2g^{-1}(\sum_{i=1}^n w_i g(a_i)), g^{-1}(\sum_{i=1}^n w_i g(\underline{a}_i)) + 2g^{-1}(\sum_{i=1}^n w_i g(a_i)), g^{-1}(\sum_{i=1}^n w_i g(\underline{a}_i)) + 2g^{-1}(\sum_{i=1}^n w_i g(a_i))\Big); \min_i\{w_{\tilde{a}_i}\}, \max_i\{u_{\tilde{a}_i}\}\Big)$

and

$\tilde{a}^+ = \Big(\Big(g^{-1}(\sum_{i=1}^n w_i g(\bar{a}_i)) + 2g^{-1}(\sum_{i=1}^n w_i g(a_i)), g^{-1}(\sum_{i=1}^n w_i g(\bar{a}_i)) + 2g^{-1}(\sum_{i=1}^n w_i g(a_i)), g^{-1}(\sum_{i=1}^n w_i g(\bar{a}_i)) + 2g^{-1}(\sum_{i=1}^n w_i g(a_i))\Big); \max_i\{w_{\tilde{a}_i}\}, \min_i\{u_{\tilde{a}_i}\}\Big)$

then $\tilde{a}^- \le TIFGOWA_w(\tilde{a}_1, \tilde{a}_2) \le \tilde{a}^+$.

Proof: Using the existing result [1, Sec. 4, Theorem 3, pp. 291],

$TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \Big(\Big(g^{-1}(\sum_{i=1}^n w_i g(\underline{a}_i)), g^{-1}(\sum_{i=1}^n w_i g(a_i)), g^{-1}(\sum_{i=1}^n w_i g(\bar{a}_i))\Big); \min_i\{w_{\tilde{a}_i}\}, \max_i\{u_{\tilde{a}_i}\}\Big)$

Furthermore, using the existing result [1, Sec. 2.2, Equation 16-17, pp. 289],

$m_\mu(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n), \gamma) = \frac{1}{3}\Big((1-\gamma)\Big(g^{-1}(\sum_{i=1}^n w_i g(\underline{a}_i)) + 2g^{-1}(\sum_{i=1}^n w_i g(a_i))\Big) + \gamma\big(g^{-1}(\sum_{i=1}^n w_i g(\bar{a}_i)) + 2g^{-1}(\sum_{i=1}^n w_i g(a_i))\big)\Big)\min_i\{w_{\tilde{a}_i}\}$

and

$m_\nu(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n), \gamma) = \frac{1}{3}\Big((1-\gamma)\Big(g^{-1}(\sum_{i=1}^n w_i g(\underline{a}_i)) + 2g^{-1}(\sum_{i=1}^n w_i g(a_i))\Big) + \gamma\big(g^{-1}(\sum_{i=1}^n w_i g(\bar{a}_i)) + 2g^{-1}(\sum_{i=1}^n w_i g(a_i))\big)\Big)\Big(1 - \max_i\{u_{\tilde{a}_i}\}\Big)$

.

It is obvious that on putting $\gamma = 0$, the minimum values of $m_\mu(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n), \gamma)$ as well as $m_\nu(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n), \gamma)$ will be obtained and on putting $\gamma = 1$. Also, the maximum value of $m_\mu(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n), \gamma)$ as well as $m_\nu(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n), \gamma)$ will be obtained i.e.,

$$m_\mu(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n), \gamma = 0) \le m_\mu(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n), \gamma) \le m_\mu(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n), \gamma = 1) \quad (5)$$

and $$m_\nu(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n), \gamma = 0) \le m_\nu(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n), \gamma) \le m_\nu(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n), \gamma = 1) \quad (6)$$

Furthermore, using the existing result [1, Sec. 2.2, Equation 16-17, pp. 289],

$m_\mu(\tilde{a}^-, \gamma) =$

$m_\mu\Big(\Big(\Big(g^{-1}(\sum_{i=1}^n w_i g(\underline{a}_i)) + 2g^{-1}(\sum_{i=1}^n w_i g(a_i)), g^{-1}(\sum_{i=1}^n w_i g(\underline{a}_i)) + 2g^{-1}(\sum_{i=1}^n w_i g(a_i)), g^{-1}(\sum_{i=1}^n w_i g(\underline{a}_i)) + 2g^{-1}(\sum_{i=1}^n w_i g(a_i))\Big); \min_i\{w_{\tilde{a}_i}\}, \max_i\{u_{\tilde{a}_i}\}\Big), \gamma\Big)$

$= \frac{1}{3}\Big((1-\gamma)\Big(g^{-1}(\sum_{i=1}^n w_i g(\underline{a}_i)) + 2g^{-1}(\sum_{i=1}^n w_i g(a_i))\Big) + \gamma\Big(g^{-1}(\sum_{i=1}^n w_i g(\underline{a}_i)) + 2g^{-1}(\sum_{i=1}^n w_i g(a_i))\Big)\Big)\min_i\{w_{\tilde{a}_i}\}$

$$= \frac{1}{3}\Big(g^{-1}(\sum_{i=1}^n w_i g(\underline{a}_i)) + 2g^{-1}(\sum_{i=1}^n w_i g(a_i))\Big)\min_i\{w_{\tilde{a}_i}\}$$
$$= m_\mu(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n), \gamma = 0). \quad (7)$$

$m_{\mu}(\tilde{a}^{+},\gamma) =$
$m_{\mu}\left(\left(\left(g^{-1}(\sum_{i=1}^{n} w_i g(\bar{a}_i)) + 2g^{-1}(\sum_{i=1}^{n} w_i g(a_i)), g^{-1}(\sum_{i=1}^{n} w_i g(\bar{a}_i)) + 2g^{-1}(\sum_{i=1}^{n} w_i g(a_i)), g^{-1}(\sum_{i=1}^{n} w_i g(\bar{a}_i)) + 2g^{-1}(\sum_{i=1}^{n} w_i g(a_i)\right); \max_i\{w_{\tilde{a}_i}\}, \min_i\{u_{\tilde{a}_i}\}\right), \gamma\right)$

$= \frac{1}{3}\left((1-\gamma)\left(g^{-1}(\sum_{i=1}^{n} w_i g(\bar{a}_i)) + 2g^{-1}(\sum_{i=1}^{n} w_i g(a_i))\right) + \gamma\left(g^{-1}(\sum_{i=1}^{n} w_i g(\bar{a}_i)) + 2g^{-1}(\sum_{i=1}^{n} w_i g(a_i))\right)\right)(1 - \max_i\{u_{\tilde{a}_i}\})$

$= \frac{1}{3}\left(g^{-1}(\sum_{i=1}^{n} w_i g(\bar{a}_i)) + 2g^{-1}(\sum_{i=1}^{n} w_i g(a_i))\right)(1 - \max_i\{u_{\tilde{a}_i}\})$
$= m_{\mu}(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n), \gamma = 1)$ (8)

Similarly,
$m_{\nu}(\tilde{a}^{-},\gamma) = m_{\nu}(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n), \gamma = 0)$ (9)
$m_{\nu}(\tilde{a}^{+},\gamma) = m_{\nu}(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n), \gamma = 1)$ (10)

Using Equations (5), (7) and (8), we have
$m_{\mu}(\tilde{a}^{-},\gamma) \le m_{\mu}(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n), \gamma) \le m_{\mu}(\tilde{a}^{+},\gamma)$ (11)

Using Equations (6), (9) and (10), we have
$m_{\nu}(\tilde{a}^{-},\gamma) \le m_{\nu}(TIFGOWA_w(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n), \gamma) \le m_{\nu}(\tilde{a}^{+},\gamma)$ (12)

Using Equations (11) and (12), we have
$\tilde{a}^{-} \le TIFGOWA_w(\tilde{a}_1, \tilde{a}_2) \le \tilde{a}^{+}$.

4. CONCLUSION

It is shown that the statement and proof of boundedness property of TIFGOWA operator, proposed by Wan *et al.* [1], is not valid and a valid statement and proof of the same is proposed.

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Source of support: Proceedings of UGC Funded International Conference on Intuitionistic Fuzzy Sets and Systems (ICIFSS-2018), Organized by: Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

