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COMMON FIXED POINT THEOREMS FOR WEAKLY COMMUTING MAPPINGS IN GENERALIZED INTUITIONISTIC FUZZY METRIC SPACES

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ABSTRACT

In this paper, we prove a common fixed point theorems for compatible and weakly commuting maps in generalized intuitionistic fuzzy metric spaces.

Keywords: *Intuitionistic fuzzy metric spaces, S- Fuzzy metric spaces, Compatible and weakly commuting maps.*

AMS Subject Classification: *47H10, 54H25.*

1. INTRODUCTION

Park introduced and discussed in [6] a notion of intuitionistic fuzzy metric space which is based both on the idea of intuitionistic fuzzy set due to Atanassov [2], and the concept of a fuzzy metric space given George and Veeramani [3]. In 1997, Singh and Chauhan [10] introduced the concept of generalized fuzzy metric spaces known S- fuzzy metric space. In 2000, Bijendra Singh and Chauhan [11] introduced the concept of compatibility in fuzzy metric space. They established the Banach contraction principle in this space. Sessa [12], introduced the weak commutativity for a pair of self maps. Further Jungck [13,14] extended these facts via the concept of compatible maps, Pant [15] introduced the notion of R – weakly commutativity of mappings in metric spaces and proved some common fixed point theorems. Later in Vasuki [16] defined R – weakly commuting maps in fuzzy metric spaces.

In this paper, we define weakly commuting and compatible maps in generalized intuitionistic fuzzy metric spaces and prove common fixed point theorem for weakly commuting and compatible maps in generalized intuitionistic fuzzy metric spaces.

2. PRELIMINARIES

Definition 2.1: The 5- tuple (X, S, T, *, ◊) is said to be generalized intuitionistic fuzzy metric space if X is an arbitrary set. * is a continuous t- norm, ◊ is a continuous t-conorm and S, T are fuzzy sets on $X^3 \times (0, \infty)$ satisfying the following conditions: for every x, y, z, a ∈ X and t, s > 0.

(i) $S(x, y, z, t) + T(x, y, z, t) \leq 1$,
(ii) $S(x, y, z, t) > 0$,
(iii) $S(x, y, z, t) = 1$ iff $x = y = z$,
(iv) $S(x, y, z, t) = S(y, z, x, t) = S(z, y, x, t) = \ldots$,
(v) $S(x, y, z, r + s + t) \geq S(x, y, w, r) * S(x, w, z, s) * S(w, y, z, t)$,
(vi) $S(x, y, z, .): (0, \infty) \rightarrow [0, 1]$ is continuous,
(vii) $T(x, y, z, t) < 0$,
(viii) $T(x, y, z, t) = 0$ iff $x = y = z$,
(ix) $T(x, y, z, t) = T(y, z, x, t) = T(z, y, x, t) = \ldots$,
(x) $T(x, y, z, r + s + t) \leq T(x, y, w, r) \lozenge T(x, w, z, s) \lozenge T(w, y, z, t)$,
(xi) $T(x, y, z, .): (0, \infty) \rightarrow [0, 1]$ is continuous.

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Definition 2.2: Let (X, S, T, *, ◊) be a generalized intuitionistic fuzzy metric space, then

i) A sequence $\{x_n\}$ in X is said to be convergent to x if $\lim_{n\to\infty} S(X_n, X_n, X, t) = 1$ and $\lim_{n\to\infty} T(x_n, x_n, x, t) = 0$.

ii) A sequence $\{x_n\}$ in X is said to be a Cauchy sequence if $\lim_{n,m,p\to\infty} S(X_n, X_m, X_p, t) = 1$ and $\lim_{n,m,p\to\infty} T(X_n, X_m, X_p, t) = 0$, that is, for any ε > 0 and for each t > 0, there exists $n_0 \in N$ such that $S(X_n, X_m, X_p, t) > 1-\varepsilon$ and $T(X_n, X_m, X_p, t) < \varepsilon$ for n, m, p ≥ n_0.

iii) A generalized intuitionistic fuzzy metric space (X, S, T, *, ◊) is said to be complete if every Cauchy sequence in X is convergent.

Definition 2.3: Two self maps A and B of a generalized intuitionistic fuzzy metric space (X, S, T, *,◊) are said to be weakly commuting if S (ABx, BAx, y, t)≥ S (Ax, Bx, z, t) and T(ABx, BAx, y, t)≤ T(Ax, Bx, z, t) where y = ABx or BAx and z = Ax or Bx for all x ∈ X.

Definition 2.4: Two self mappings A and B of a generalized intuitionistic fuzzy metric space (X, S, T, *◊) are said to be compatible if $\lim_{n\to\infty} S(ABx_n, BAx_n, z, t) = 1$ and $\lim_{n\to\infty} T(ABx_n, BAx_n, z, t) = 0$, where z = ABx_n or BAx_n, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = y$ for some y in X.

Clearly, commutativity implies weak commutativity and weak commutativityimplies compatibility, but neither implication is always reversible. This can be seen in following examples.

Example 2.5: Let X = [0, 1]. Define S(x, y, z, t) = min {M(x, y, t), M(y, z, t), M(z, x, t)} and T(x, y, z, t) = max{N(x, y, t), N(y, z, t), N(z, x, t)},where $M(x, y, t) = \frac{t}{t+d(x,y)}$, $N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$ and d(x, y) = | x – y | for all x, y ∈ X. Also define self maps A and B of X, by $Ax = x^2$, $Bx = x^2/2$ for all x ∈ X.
Then we see that AB ≠ BA and S(ABx, BAx, ABx, t) ≥ S(Ax, Bx, Ax, t) and T(ABx, BAx, ABx, t) ≤ T(Ax, Bx, Ax, t), for x ∈ [0, 1]. This shows weak commutativity does not imply commutativity.

Example 2.6: Let X = R. Define S(x, y, z, t) = min {M(x, y, t), M(y, z, t), M(z, x, t)} and T(x, y, z, t) = max{N(x, y, t), N(y, z, t), N(z, x, t)},where $M(x, y, t) = \frac{t}{t+d(x,y)}$, $N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$ and d(x, y) = | x – y | for all x, y ∈ X.Also define self maps A and B of X, by $Ax = x^2$, $Bx = x^3/3$ for all x∈ R and $x_n = 1/n$, n = 1,2,3 ....

Here $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = 0 \in X$. $S(ABx_n, BAx_n, ABx_n, t) \to 1$ and $T(ABx_n, BAx_n, ABx_n, t) \to 0$ as $n \to\infty$.

But S(ABx, BAx, ABx, t) ≥ S(Ax, Bx, Ax, t) andT(ABx, BAx, ABx, t) ≤ T(Ax, Bx, Ax, t) are not true for x∈R and AB ≠ BA. Thus we see that A and B are compatible, but neither commutative nor weakly commutative.

3. MAIN RESULTS

Theorem 3.1: Let A, B, P and T be self maps of a complete generalized intuitionistic fuzzy metric space (X, S, T, *,◊) with t - norm * defined by a * b = min{a, b} and t - conorm ◊ defined by a ◊ b = max {a, b}, a, b ∈ [0, 1] satisfying the conditions.

(3.1.1) A(X) ⊆T(X), B(X) ⊆ P(X),
(3.1.2) One of A, B, P or T is continuous,
(3.1.3) (A, P) and (B, T) is weakly commuting pairs of maps,
(3.1.4) For all x, y, z ∈ X, 0 < k < 1, t > 0
S(Ax, By, z, kt) ≥ min{S(Px, Ty, z, t), S(Ax, Ty, z, t), S(By, Px, z, t)} and
T(Ax, By, z, kt) ≤ max{T(Px, Ty, z, t), T(Ax, Ty, z, t), T(By, Px, z, t)}
(3.1.5) S(x, y, z, t) →1 and T(x, y, z, t) → 0 as t → ∞.
Then A, B, P and T have a unique common fixed point in X.

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Proof: Let $x_0 \in X$ be arbitrary, construct a sequence $\{y_n\}$ in X such that $y_{2n+1} = Tx_{2n+1} = Ax_{2n}$ and $y_{2n} = Px_{2n} = Bx_{2n-1}$; n = 0, 1, 2, . . . using (3.1.4), we have

$$\begin{aligned} S(y_1, y_2, y_m, kt) &= S(Ax_0, Bx_1, y_m, kt) \\ &\geq \min\{S(Px_0, Tx_1, y_m, t), S(Ax_0, Tx_1, y_m, t), S(Bx_1, Px_0, y_m, t)\} \\ &= \min\{S(y_0, y_1, y_m, t), S(y_1, y_1, y_m, t), S(y_2, y_0, y_m, t)\} \\ &\geq \min\{S(y_0, y_1, y_m, t), S(y_1, y_2, y_m, t), S(y_0, y_2, y_m, t)\} \end{aligned}$$

$$\begin{aligned} T(y_1, y_2, y_m, kt) &= T(Ax_0, Bx_1, y_m, kt) \\ &\leq \max\{T(Px_0, Tx_1, y_m, t), T(Ax_0, Tx_1, y_m, t), T(Bx_1, Px_0, y_m, t)\} \\ &= \max\{T(y_0, y_1, y_m, t), T(y_1, y_1, y_m, t), T(y_2, y_0, y_m, t)\} \\ &\leq \max\{T(y_0, y_1, y_m, t), T(y_1, y_2, y_m, t), T(y_0, y_2, y_m, t)\}. \end{aligned}$$

This implies that
$S(y_1, y_2, y_m, kt) \geq S(y_0, y_1, y_m, t)$ or $S(y_0, y_2, y_m, t)$ and $T(y_1, y_2, y_m, kt) \leq T(y_0, y_1, y_m, t)$ or $T(y_0, y_2, y_m, t)$.

Further using (3.1.4), we have,

$$\begin{aligned} S(y_2, y_3, y_m, kt) &= S(Bx_1, Ax_2, y_m, kt) = S(Ax_2, Bx_1, y_m, kt) \\ &\geq \min\{S(Px_2, Tx_1, y_m, t), S(Ax_2, Tx_1, y_m, t), S(Bx_1, Px_2, y_m, t)\} \\ &= \min\{S(y_2, y_1, y_m, t), S(y_3, y_1, y_m, t), S(y_2, y_2, y_m, t)\} \\ &\geq \min\{S(y_1, y_2, y_m, t), S(y_1, y_3, y_m, t), S(y_2, y_3, y_m, t)\} \text{ and} \end{aligned}$$

$$\begin{aligned} T(y_2, y_3, y_m, kt) &= T(Bx_1, Ax_2, y_m, kt) = T(Ax_2, Bx_1, y_m, kt) \\ &\leq \max\{T(Px_2, Tx_1, y_m, t), T(Ax_2, Tx_1, y_m, t), T(Bx_1, Px_2, y_m, t)\} \\ &= \max\{T(y_2, y_1, y_m, t), T(y_3, y_1, y_m, t), T(y_2, y_2, y_m, t)\} \\ &\leq \max\{T(y_1, y_2, y_m, t), T(y_1, y_3, y_m, t), T(y_2, y_3, y_m, t)\}. \end{aligned}$$

Which implies that,

$S(y_2, y_3, y_m, kt) \geq S(y_1, y_2, y_m, t)$ (or) $S(y_1, y_3, y_m, t)$ and $T(y_2, y_3, y_m, kt) \leq T(y_1, y_2, y_m, t)$ (or) $T(y_1, y_3, y_m, t)$.

Proceeding in the same way we get,

$$\begin{aligned} S(y_n, y_{n+1}, y_m, kt) &\geq S(y_{n-1}, y_n, y_m, t) \text{ or } S(y_{n-1}, y_{n+1}, y_m, t) \\ &\geq S(y_{n-2}, y_{n-1}, y_m, t/k) \text{ or } S(y_{n-2}, y_{n+1}, y_m, t/k) \\ &\ldots \\ &\geq S(y_0, y_1, y_m, t/k^{n-1}) \text{ or } S(y_0, y_{n+1}, y_m, t/k^{n-1}) \end{aligned}$$

$S(y_n, y_{n+1}, y_m, t) \geq S(y_0, y_1, y_m, t/k^n)$ or $S(y_0, y_{n+1}, y_m, t/k^n)$ and

$$\begin{aligned} T(y_n, y_{n+1}, y_m, kt) &\leq T(y_{n-1}, y_n, y_m, t) \text{ or } T(y_{n-1}, y_{n+1}, y_m, t) \\ &\leq T(y_{n-2}, y_{n-1}, y_m, t/k) \text{ or } T(y_{n-2}, y_{n+1}, y_m, t/k) \\ &\ldots \\ &\leq T(y_0, y_1, y_m, t/k^{n-1}) \text{ or } T(y_0, y_{n+1}, y_m, t/k^{n-1}) \end{aligned}$$

$T(y_n, y_{n+1}, y_m, t) \leq T(y_0, y_1, y_m, t/k^n)$ or $T(y_0, y_{n+1}, y_m, t/k^n)$.

Case-I: When $S(y_n, y_{n+1}, y_m, t) \geq S(y_0, y_1, y_m, t/k^n)$ and $T(y_n, y_{n+1}, y_m, t) \leq T(y_0, y_1, y_m, t/k^n)$. Then for $p, q \in N$ and $t > 0$, we have

$$\begin{aligned} S(y_n, y_{n+p}, y_{n+p+q}, 3t) &\geq S(y_n, y_{n+1}, y_{n+p+q}, t) * S(y_n, y_{n+1}, y_{n+p}, t) * S(y_{n+1}, y_{n+p}, y_{n+p+q}, t) \\ &\geq \{S(y_0, y_1, y_{n+p+q}, t/k^n) * S(y_0, y_1, y_{n+p}, t/k^n) * S(y_{n+1}, y_{n+2}, y_{n+p+q}, t/3) \\ &\quad * S(y_{n+1}, y_{n+2}, y_{n+p}, t/3) * S(y_{n+2}, y_{n+p}, y_{n+p+q}, t/3)\} \\ &\geq \{S(y_0, y_1, y_{n+p+q}, t/k^n) * S(y_0, y_1, y_{n+p}, t/k^n) * S(y_0, y_1, y_{n+p+q}, t/3k^{n+1}) \\ &\quad * S(y_0, y_1, y_{n+p}, t/3k^{n+1}) * S(y_{n+2}, y_{n+p}, y_{n+p+q}, t/3)\} \\ &\ldots \\ &\geq \{S(y_0, y_1, y_{n+p+q}, t/k^n) * S(y_0, y_1, y_{n+p}, t/k^n) * S(y_0, y_1, y_{n+p+q}, t/3k^{n+1}) * S(y_0, y_1, y_{n+p}, t/3k^{n+1}) \\ &\quad * \ldots * S(y_0, y_1, y_{n+p+q}, t/k^{n+p-2}3^{p-2}) * S(y_0, y_1, y_{n+p}, t/k^{n+p-2}3^{p-2}) * S(y_{n+p-1}, y_{n+p}, y_{n+p+q}, t/3^{p-2})\} \\ &\geq \{S(y_0, y_1, y_{n+p+q}, t/k^n) * S(y_0, y_1, y_{n+p}, t/k^n) * S(y_0, y_1, y_{n+p+q}, t/3k^{n+1}) * S(y_0, y_1, y_{n+p}, t/3k^{n+1}) \\ &\quad * \ldots * S(y_0, y_1, y_{n+p+q}, t/k^{n+p-2}3^{p-2}) * S(y_0, y_1, y_{n+p}, t/k^{n+p-2}3^{p-2}) * S(y_0, y_1, y_{n+p+q}, t/k^{n+p-1}3^{p-2})\}, \end{aligned}$$

$$\begin{aligned}T(y_n, y_{n+p}, y_{n+p+q}, 3t) &\le T(y_n, y_{n+1}, y_{n+p+q}, t) \Diamond T(y_n, y_{n+1}, y_{n+p}, t) \Diamond T(y_{n+1}, y_{n+p}, y_{n+p+q}, t)\\ &\le \{T(y_0, y_1, y_{n+p+q}, t/k^n) \Diamond T(y_0, y_1, y_{n+p}, t/k^n) \Diamond T(y_{n+1}, y_{n+2}, y_{n+p+q}, t/3) \Diamond\\ &\quad T(y_{n+1}, y_{n+2}, y_{n+p}, t/3) \Diamond T(y_{n+2}, y_{n+p}, y_{n+p+q}, t/3)\}\\ &\le \{T(y_0, y_1, y_{n+p+q}, t/k^n) \Diamond T(y_0, y_1, y_{n+p}, t/k^n) \Diamond T(y_0, y_1, y_{n+p+q}, t/3k^{n+1}) \Diamond\\ &\quad T(y_0, y_1, y_{n+p}, t/3k^{n+1}) \Diamond T(y_{n+2}, y_{n+p}, y_{n+p+q}, t/3)\}\\ &\ \ \ldots\\ &\le \{T(y_0, y_1, y_{n+p+q}, t/k^n) \Diamond T(y_0, y_1, y_{n+p}, t/k^n) \Diamond T(y_0, y_1, y_{n+p+q}, t/3k^{n+1}) \Diamond T(y_0, y_1, y_{n+p}, t/3k^{n+1})\\ &\quad \Diamond \ldots \Diamond T(y_0, y_1, y_{n+p+q}, t/k^{n+p-2}3^{p-2}) \Diamond T(y_0, y_1, y_{n+p}, t/k^{n+p-2}\, 3^{p-2}) \Diamond T(y_{n+p-1}, y_{n+p}, y_{n+p+q}, t/3^{p-2})\}\\ &\le \{T(y_0, y_1, y_{n+p+q}, t/k^n) \Diamond T(y_0, y_1, y_{n+p}, t/k^n) \Diamond T(y_0, y_1, y_{n+p+q}, t/3k^{n+1}) \Diamond T(y_0, y_1, y_{n+p}, t/3k^{n+1})\\ &\quad \Diamond \ldots \Diamond T(y_0, y_1, y_{n+p+q}, t/k^{n+p-2}3^{p-2}) \Diamond T(y_0, y_1, y_{n+p}, t/k^{n+p-2}3^{p-2}) \Diamond T(y_0, y_1, y_{n+p+q}, t/k^{n+p-1}\, 3^{p-2})\}.\end{aligned}$$

Taking the limit as $n \to \infty$, we have,

$\lim_{n\to\infty} S(y_n, y_{n+p}, y_{n+p+q}, 3t) \ge 1 * 1 * 1 * \ldots * 1$ (2p – 1times) and

$\lim_{n\to\infty} T(y_n, y_{n+p}, y_{n+p+q}, 3t) \le 0 \Diamond 0 \Diamond 0 \Diamond \ldots \Diamond 0$ (2p – 1times),

which implies that $S(y_n, y_{n+p}, y_{n+p+q}, 3t) \to 1$ and $T(y_n, y_{n+p}, y_{n+p+q}, 3t) \to 0$ as $n \to \infty$.

Case-II:

When $S(y_n, y_{n+1}, y_m, t) \ge S(y_0, y_{n+1}, y_m, t/k^n)$ and $T(y_n, y_{n+1}, y_m, t) \le T(y_0, y_{n+1}, y_m, t/k^n)$.

Then on the lines of Case I, we have,

$$\begin{aligned}S(y_n, y_{n+p}, y_{n+p+q}, 3t) \ge \{&S(y_0, y_{n+1}, y_{n+p+q}, t/k^n) * S(y_0, y_{n+1}, y_{n+p}, t/k^n) * S(y_0, y_{n+2}, y_{n+p+q}, t/3k^{n+1}) *\\ &S(y_0, y_{n+2}, y_{n+p}, t/3k^{n+1}) * \ldots * S(y_0, y_{n+p-2}, y_{n+p+q}, t/k^{n+p-2}\, 3^{p-2}) *\\ &S(y_0, y_{n+p-2}, y_{n+p}, t/k^{n+p-2}\, 3^{p-2}) * S(y_0, y_{n+p}, y_{n+p+q}, t/k^{n+p-1}\, 3^{p-2})\},\end{aligned}$$

$$\begin{aligned}T(y_n, y_{n+p}, y_{n+p+q}, 3t) \le \{&T(y_0, y_{n+1}, y_{n+p+q}, t/k^n) \Diamond T(y_0, y_{n+1}, y_{n+p}, t/k^n) \Diamond T(y_0, y_{n+2}, y_{n+p+q}, t/3\, k^{n+1}) \Diamond\\ &T(y_0, y_{n+2}, y_{n+p}, t/3k^{n+1}) \Diamond \ldots \Diamond T(y_0, y_{n+p-2}, y_{n+p+q}, t/k^{n+p-2}\, 3^{p-2}) \Diamond\\ &T(y_0, y_{n+p-2}, y_{n+p}, t/k^{n+p-2}\, 3^{p-2}) \Diamond T(y_0, y_{n+p}, y_{n+p+q}, t/k^{n+p-1}\, 3^{p-2})\}.\end{aligned}$$

Taking the limit as $n \to \infty$, we have,

$\lim_{n\to\infty} S(y_n, y_{n+p}, y_{n+p+q}, 3t) \ge 1 * 1 * 1 * \ldots * 1$ (2p – 1times) and

$\lim_{n\to\infty} T(y_n, y_{n+p}, y_{n+p+q}, 3t) \le 0 \Diamond 0 \Diamond 0 \Diamond \ldots \Diamond 0$ (2p – 1times).

Which implies that $S(y_n, y_{n+p}, y_{n+p+q}, 3t) \to 1$ and $T(y_n, y_{n+p}, y_{n+p+q}, 3t) \to 0$ as $n \to \infty$.

Thus, in both cases $\{y_n\}$is a Cauchy sequence. By the completeness of X, sequence $\{y_n\}$ and its subsequences $\{Ax_{2n}\}$, $\{Bx_{2n-1}\}$, $\{Px_{2n}\}$ and $\{Tx_{2n+1}\}$ converge to some u in X.

Now, suppose that P is continuous then PAx_{2n}, $PPx_{2n} \to Pu$. Since (A, P) are S- weakly commuting, therefore $S(APx_{2n}, PAx_{2n}, APx_{2n}, t) \ge S(Ax_{2n}, Px_{2n}, Ax_{2n}, t)$ and$T(APx_{2n}, PAx_{2n}, APx_{2n}, t) \le S(Ax_{2n}, Px_{2n}, Ax_{2n}, t)$.

On letting $n \to \infty$, we have,

$S(\lim_{n\to\infty} APx_{2n}, Pu, \lim_{n\to\infty} APx_{2n}, t) \ge S(u, u, u, t) = 1$and $T(\lim_{n\to\infty} APx_{2n}, Pu, \lim_{n\to\infty} APx_{2n}, t) \ge T(u, u, u, t) = 0$.

Which implies that $APx_{2n} \to Pu$. Now using (3.1.4) we have,

$S(APx_{2n}, Bx_{2n+1}, u, kt) \ge \min\{S(PPx_{2n}, Tx_{2n+1}, u, t), S(APx_{2n}, Tx_{2n+1}, u, t), S(Bx_{2n+1}, PPx_{2n}, u, t)\}$

$T(APx_{2n}, Bx_{2n+1}, u, kt) \le \max\{T(PPx_{2n}, Tx_{2n+1}, u, t), T(APx_{2n}, Tx_{2n+1}, u, t), T(Bx_{2n+1}, PPx_{2n}, u, t)\}$.

On letting $n \to \infty$, we have,

$S(Pu, u, u, kt) \ge \min\{S(Pu, u, u, t), S(Pu, u, u, t), S(u, Pu, Pu, t)\}$or $S(Pu, u, u, kt) \ge S(Pu, u, u, t)$

$T(Pu, u, u, kt) \le \max\{T(Pu, u, u, t), T(Pu, u, u, t), T(u, Pu, Pu, t)\}$or $T(Pu,u, u, kt) \ge T(Pu, u, u, t)$.

Which implies that $Pu = u$. Further using (3.1.4) we have,

$S(Au, Bx_{2n+1}, u, kt) \ge \min\{S(Pu,Tx_{2n+1}, u, t), S(Au, Tx_{2n+1}, u, t), S(Bx_{2n+1}, Pu, u, t)\}$ and

$T(Au, Bx_{2n+1}, u, kt) \le \max\{T(Pu,Tx_{2n+1}, u, t), T(Au, Tx_{2n+1}, u, t), T(Bx_{2n+1}, Pu, u, t)\}$

On letting $n \to \infty$, we have,
$S(Au, u, u, kt) \geq \min\{S(u, u, u, t), S(Au, u, u, t), S(u, u, u, t)\}$ or $S(Au, u, u, kt) \geq S(Au, u, u, t)$
$T(Au, u, u, kt) \leq \max\{T(u, u, u, t), T(Au, u, u, t), T(u, u, u, t)\}$or $T(Au, u, u, kt) \leq T(Au, u, u, t)$.

Which implies that $Au = u$. Since $A(X) \subseteq T(X)$, there exists $v \in X$ such that $u = Tv = Pu$.

Using (3.1.4) we have,
$S(u, Bv, u, kt) = S(Au, Bv, u, kt)$
$\geq \min\{S(Pu, Tv, u, t), S(Au, Tv, u, t), S(Bv, Pu, u, t)\}$
$= \min\{S(u, u, u, t), S(u, u, u, t), S(Bv, u, u, t)\}$ or $S(u, Bv, u, kt) \geq S(u, Bv, u, t)$ and

$T(u, Bv, u, kt) = T(Au, Bv, u, kt)$
$\leq \max\{T(Pu, Tv, u, t), T(Au, Tv, u, t), T(Bv, Pu, u, t)\}$
$= \max\{T(u, u, u, t), T(u, u, u, t), T(Bv, u, u, t)\}$ or $T(u, Bv, u, kt) \leq T(u, Bv, u, t)$.

Which implies that $Bv = u$. Thus $u = Bv = Tv$. Since (T, B) are weakly commuting, therefore
$S(TBv, BTv, TBv, t) \geq S(Tv, Bv, Tv, t) = 1$ and $T(TBv, BTv, TBv, t) \leq T(Tv, Bv, Tv, t) = 0$.

Which implies that $TBv = BTv$ and so $Tu = Bu$. Using (3.1.4) we have,
$S(u, Tu, u, kt) = S(Au, Bu, u, kt)$
$\geq \min\{S(Pu, Tu, u, t), S(Au, Tu, u, t), S(Bu, Pu, u, t)\}$
$= \min\{S(u, Tu, u, t), S(u, Tu, u, t), S(Tu, u, u, t)\}$

$S(u, Tu, u, kt) \geq S(u, Tu, u, t)$ and

$T(u, Tu, u, kt) = T(Au, Bu, u, kt)$
$\leq \max\{T(Pu, Tu, u, t), T(Au, Tu, u, t), T(Bu, Pu, u, t)\}$
$= \max\{T(u, Tu, u, t), T(u, Tu, u, t), T(Tu, u, u, t)\}$

$T(u, Tu, u, kt) \leq T(u, Tu, u, t)$.

Which implies that $u = Tu = Bu$. Hence $u = Tu = Bu = Au = Pu$. Shows u is a common fixed point of A, B, P and T.

Now to prove uniqueness of u, let w be another common fixed point of A, B, P and T.

Then from (3.1.4) we have,
$S(u, w, u, kt) = S(Au, Bw, u, kt)$
$\geq \min\{S(Pu, Tw, u, t), S(Au, Tw, u, t), S(Bw, Pu, u, t)\}$
$= \min\{S(u, w, u, t), S(u, w, u, t), S(w, u, u, t)\}$
$= S(u, w, u, t)$ or $S(u, w, u, kt) \geq S(u, w, u, t)$ and

$T(u, w, u, kt) = T(Au, Bw, u, kt)$
$\leq \max\{T(Pu, Tw, u, t), T(Au, Tw, u, t), T(Bw, Pu, u, t)\}$
$= \max\{T(u, w, u, t), T(u, w, u, t), T(w, u, u, t)\}$
$= T(u, w, u, t)$ or $T(u, w, u, kt) \leq T(u, w, u, t)$, which implies that $u = w$.

Hence u is a unique common fixed point of A, B, P and T.

Proposition 3.2: Let A and B be compatible self mappings of a generalized intuitionistic fuzzy metric space X. If $Ay = By$ then $ABy = BAy$.

Proof: Let $Ay = By$ and $\{x_n\}$ be a sequence in X, such that $x_n = y$ for all n. Then $Ax_n, Bx_n \to Ay$.

Now by the compatibility of A and B. We have $S(ABy, BAy, ABy, t) = S(ABx_n, BAx_n, ABx_n, t) \to 1$ and
$T(ABy, BAy, ABy, t) = T(ABx_n, BAx_n, ABx_n, t) \to 0$ as $n \to \infty$, which yields $ABy = BAy$.

Theorem 3.3: Let A, B, P and T be self maps of a complete generalized intuitionistic fuzzy metric space (X, S, T, *,◊) with t - norm * defined by a * b = min{a, b}and t – conorm ◊ defined by a ◊ b = max{a, b}, a,b ∈ [0, 1], satisfying the conditions,

(3.3.1) $A(X) \subseteq T(X)$, $B(X) \subseteq P(X)$,

(3.3 2) One of A, B, P or T is continuous,

(3.3.3) (A, P) and (B, T) are compatible pairs of maps,

(3.3.4) For all x, y, z ∈ X, $0 < k < 1$, $t > 0$

$$S(Ax, By, z, kt) \geq \min\{S(Px, Ty, z, t), S(Ax, Ty, z, t), S(By, Px, z, t), S(Ax, Px, z, t), S(By, Ty, z, t)\} \text{ and}$$
$$T(Ax, By, z, kt) \leq \max\{T(Px, Ty, z, t), T(Ax, Ty, z, t), T(By, Px, z, t), T(Ax, Px, z, t), T(By, Ty, z, t)\},$$

(3.3.5) $S(x, y, z, t) \to 1$ and $T(x, y, z, t) \to 0$ as $t \to \infty$.

Then A, B, P and T have a unique common fixed point in X.

Proof: Let $x_0 \in X$ be arbitrary. Construct a sequence $\{y_n\}$ in X such that $y_{2n+1} = Tx_{2n+1} = Ax_{2n}$ and $y_{2n} = Px_{2n} = Bx_{2n-1}$, n = 0, 1, 2, . . . using (3.3.4) we have,

$$\begin{aligned} S(y_1, y_2, y_m, kt) &= S(Ax_0, Bx_1, y_m, kt) \\ &\geq \min\{S(Px_0, Tx_1, y_m, t), S(Ax_0, Tx_1, y_m, t), S(Bx_1, Px_0, y_m, t), S(Ax_0, Px_0, y_m, t), S(Bx_1, Tx_1, y_m, t)\} \\ &= \min\{S(y_0, y_1, y_m, t), S(y_1, y_1, y_m, t), S(y_2, y_0, y_m, t), S(y_1, y_0, y_m, t), S(y_2, y_1, y_m, t)\} \\ &\geq \min\{S(y_0, y_1, y_m, t), S(y_1, y_2, y_m, t), S(y_0, y_2, y_m, t), S(y_0, y_1, y_m, t), S(y_1, y_2, y_m, t)\} \\ &= \min\{S(y_0, y_1, y_m, t), S(y_1, y_2, y_m, t), S(y_0, y_2, y_m, t)\} \end{aligned}$$

$$\begin{aligned} T(y_1, y_2, y_m, kt) &= T(Ax_0, Bx_1, y_m, kt) \\ &\leq \max\{T(Px_0, Tx_1, y_m, t), T(Ax_0, Tx_1, y_m, t), T(Bx_1, Px_0, y_m, t), T(Ax_0, Px_0, y_m, t), T(Bx_1, Tx_1, y_m, t)\} \\ &= \max\{T(y_0, y_1, y_m, t), T(y_1, y_1, y_m, t), T(y_2, y_0, y_m, t), T(y_1, y_0, y_m, t), T(y_2, y_1, y_m, t)\} \\ &\leq \max\{T(y_0, y_1, y_m, t), T(y_1, y_2, y_m, t), T(y_0, y_2, y_m, t), T(y_0, y_1, y_m, t), T(y_1, y_2, y_m, t)\} \\ &= \max\{T(y_0, y_1, y_m, t), T(y_1, y_2, y_m, t), T(y_0, y_2, y_m, t)\}. \end{aligned}$$

Which implies that,

$S(y_1, y_2, y_m, kt) \geq S(y_0, y_1, y_m, t)$ or $S(y_0, y_2, y_m, t)$ and $T(y_1, y_2, y_m, kt) \leq T(y_0, y_1, y_m, t)$ or $T(y_0, y_2, y_m, t)$.

Further using (3.3.4) we have,

$$\begin{aligned} S(y_2, y_3, y_m, kt) &= S(Bx_1, Ax_2, y_m, kt) = S(Ax_2, Bx_1, y_m, kt) \\ &\geq \min\{S(Px_2, Tx_1, y_m, t), S(Ax_2, Tx_1, y_m, t), S(Bx_1, Px_2, y_m, t), S(Ax_2, Px_2, y_m, t), S(Bx_1, Tx_1, y_m, t)\} \\ &= \min\{S(y_2, y_1, y_m, t), S(y_3, y_1, y_m, t), S(y_2, y_2, y_m, t), S(y_3, y_2, y_m, t), S(y_2, y_1, y_m, t)\} \end{aligned}$$

$$\begin{aligned} T(y_2, y_3, y_m, kt) &= T(Bx_1, Ax_2, y_m, kt) = T(Ax_2, Bx_1, y_m, kt) \\ &\leq \max\{T(Px_2, Tx_1, y_m, t), T(Ax_2, Tx_1, y_m, t), T(Bx_1, Px_2, y_m, t), T(Ax_2, Px_2, y_m, t), T(Bx_1, Tx_1, y_m, t)\} \\ &= \max\{T(y_2, y_1, y_m, t), T(y_3, y_1, y_m, t), T(y_2, y_2, y_m, t), T(y_3, y_2, y_m, t), T(y_2, y_1, y_m, t)\}, \end{aligned}$$

which implies that,

$S(y_2, y_3, y_m, kt) \geq S(y_1, y_2, y_m, t)$ or $S(y_1, y_3, y_m, t)$ and $T(y_2, y_3, y_m, kt) \leq T(y_1, y_2, y_m, t)$ or $T(y_1, y_3, y_m, t)$.

Again with the similar process as in Theorem (3.1) we can show $\{y_n\}$ is a Cauchy sequence.

By the completeness of X, sequence $\{y_n\}$ and its subsequences $\{Ax_{2n}\}$, $\{Bx_{2n-1}\}$, $\{Px_{2n}\}$ and $\{Tx_{2n+1}\}$ converge to some u in X. Now if we suppose that P is continuous then $PAx_{2n}, PPx_{2n} \to Pu$.

Since (A, P) are compatible, therefore $\lim_{n\to\infty} S(PAx_{2n}, APx_{2n}, PAx_{2n}, t) = 1$ and $\lim_{n\to\infty} T(PAx_{2n}, APx_{2n}, PAx_{2n}, t) = 0$, where $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} Ax_{2n} = \lim_{n\to\infty} Px_{2n} = u$.

Thus, we have $S(Pu, \lim_{n\to\infty} APx_{2n}, Pu, t) = 1$and $T(Pu, \lim_{n\to\infty} APx_{2n}, Pu, t) = 0$.

Which implies that $\lim_{n\to\infty} APx_{2n} = Pu$. Now using (3.3.4) we have,

$$S(APx_{2n}, Bx_{2n+1}, u, kt) \geq \min\{S(PPx_{2n}, Tx_{2n+1}, u, t), S(APx_{2n}, Tx_{2n+1}, u, t), S(Bx_{2n+1}, PPx_{2n}, u, t), S(APx_{2n}, PPx_{2n}, u, t), S(Bx_{2n+1}, Tx_{2n+1}, u, t)\} \text{ and}$$

$$T(APx_{2n}, Bx_{2n+1}, u, kt) \leq \max\{T(PPx_{2n}, Tx_{2n+1}, u, t), T(APx_{2n}, Tx_{2n+1}, u, t), T(Bx_{2n+1}, PPx_{2n}, u, t), T(APx_{2n}, PPx_{2n}, u, t), T(Bx_{2n+1}, Tx_{2n+1}, u, t)\}.$$

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On letting $n \to \infty$ we have,
$S(Pu, u, u, kt) \geq \min\{S(Pu, u, u, t), S(Pu, u, u, t), S(u, Pu, u, t), S(Pu, Pu, u, t), S(u, u, u, t)\}$
$= S(Pu, u, u, t)$ and

$T(Pu, u, u, kt) \leq \max\{T(Pu, u, u, t), T(Pu, u, u, t), T(u, Pu, u, t), T(Pu, Pu, u, t), T(u, u, u, t)\}$
$= T(Pu, u, u, t)$, which implies that,

$S(Pu, u, u, kt) \geq S(Pu, u, u, t)$ and $T(Pu, u, u, kt) \leq T(Pu, u, u, t)$.

Hence $Pu = u$. Further using (3.3.4) we have,
$S(Au, Bx_{2n+1}, u, kt) \geq \min\{S(Pu, Tx_{2n+1}, u, t), S(Au, Tx_{2n+1}, u, t), S(Bx_{2n+1}, Pu, u, t), S(Au, Pu, u, t), S(Bx_{2n+1}, Tx_{2n+1}, u, t)\}$
$T(Au, Bx_{2n+1}, u, kt) \leq \max\{T(Pu, Tx_{2n+1}, u, t), T(Au, Tx_{2n+1}, u, t), T(Bx_{2n+1}, Pu, u, t), T(Au, Pu, u, t), T(Bx_{2n+1}, Tx_{2n+1}, u, t)\}$.

On letting $n \to \infty$ we have,
$S(Au, u, u, kt) \geq \min\{S(u, u, u, t), S(Au, u, u, t), S(u, u, u, t), S(Au, u, u, t), S(u, u, u, t)\}$ and
$T(Au, u, u, kt) \leq \max\{T(u, u, u, t), T(Au, u, u, t), T(u, u, u, t), T(Au, u, u, t), T(u, u, u, t)\}$.

This implies that,
$S(Au, u, u, kt) \geq S(Au, u, u, t)$ and $T(Au, u, u, kt) \leq T(Au, u, u, t)$.

Hence $Au = u$. Since $A(X) \subseteq T(X)$, there exists $v \in X$ such that $u = Tv = Pu$, using (3.3.4) we have,
$S(u, Bv, u, kt) = S(Au, Bv, u, kt)$
$\geq \min\{S(Pu, Tv, u, t), S(Au, Tv, u, t), S(Bv, Pu, u, t), S(Au, Pu, u, t), S(Bv, Tv, u, t)\}$

$T(u, Bv, u, kt) = T(Au, Bv, u, kt)$
$\leq \max\{T(Pu, Tv, u, t), T(Au, Tv, u, t), T(Bv, Pu, u, t), T(Au, Pu, u, t), T(Bv, Tv, u, t)\}$.

This implies that,
$S(u, Bv, u, kt) \geq S(u, Bv, u, t)$ and $T(u, Bv, u, kt) \leq T(u, Bv, u, t)$, which implies that $Bv = u$. Thus, $u = Bv = Tv$.

By the compatibility of (T, B) and from propositions (3.2), we have $TBv = BTv$ and so $Tu = Bu$.

Using (3.3.4) we have,
$S(u, Tu, u, kt) = S(Au, Bu, u, kt)$
$\geq \min\{S(Pu, Tu, u, t), S(Au, Tu, u, t), S(Bu, Pu, u, t), S(Au, Pu, u, t), S(Bu, Tu, u, t)\}$
$= \min\{S(u, Tu, u, t), S(u, Tu, u, t), S(Tu, u, u, t), S(u, u, u, t), S(Tu, Tu, u, t)\}$ and

$T(u, Tu, u, kt) = T(Au, Bu, u, kt)$
$\leq \max\{T(Pu, Tu, u, t), T(Au, Tu, u, t), T(Bu, Pu, u, t), T(Au, Pu, u, t), T(Bu, Tu, u, t)\}$
$= \max\{T(u, Tu, u, t), T(u, Tu, u, t), T(Tu, u, u, t), T(u, u, u, t), T(Tu, Tu, u, t)\}$.

This implies that $S(u, Tu, u, kt) \geq S(u, Tu, u, t)$ and $T(u, Tu, u, kt) \leq T(u, Tu, u, t)$,
which implies that $u = Tu = Bu$. Hence $u = Tu = Bu = Au = Pu$. Shows u is a common fixed point of A, B, P and T.

Now to prove uniqueness of u, let w be another common fixed point of A, B, P and T.

Then from (3.3.4) we have,
$S(u, w, u, kt) = S(Au, Bw, u, kt)$
$\geq \min\{S(Pu, Tw, u, t), S(Au, Tw, u, t), S(Bw, Pu, u, t), S(Au, Pu, u, t), S(Bw, Tw, u, t)\}$
$= \min\{S(u, w, u, t), S(u, w, u, t), S(w, u, u, t), S(u, u, u, t), S(w, w, u, t)\}$and

$T(u, w, u, kt) = T(Au, Bw, u, kt)$
$\leq \max\{T(Pu, Tw, u, t), T(Au, Tw, u, t), T(Bw, Pu, u, t), T(Au, Pu, u, t), T(Bw, Tw, u, t)\}$
$= \max\{T(u, w, u, t), T(u, w, u, t), T(w, u, u, t), T(u, u, u, t), T(w, w, u, t)\}$.

This implies that, $S(u, w, u, kt) \geq S(u, w, u, t)$ and $T(u, w, u, kt) \leq T(u, w, u, t)$. Hence $u = w$.

Thus, u is a unique common fixed point of A,B, P and T.

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4. REFERENCES

1. Alaca. C, Turkoglu. D and Yildiz. C "Fixed points in intuitionistic fuzzy metric spaces", Chaos, solutions and Fractals, 29(2006), 1073-1078.
2. Atanassov. K, "Intuitionistic fuzzy Sets" Fuzzy sets and Systems, 20 (1986), 87-96.
3. George. A and Veeramani. P, "On Some results in fuzzy metric spaces", Fuzzy sets and Systems, 64(1994), 395-399.
4. Kramosil. O and Michalek. J, "Fuzzy metric and statistical metric spaces", Ky-bernetics, 11 (1975), 330-334.
5. Mehra. S and Gugnani. M, "A Common fixed point for six mappings in an intuitionistic $\mathcal{M}$-fuzzy metric space", Indian Journal of Mathematics, Vol.51 No.1, (2009), 23-47.
6. Park. J. H, "Intuitionistic fuzzy metric spaces", Chaos, Solitons and Fractals, 22 (2004), 1039-1046.
7. Ranjeeta Jain and Bajaj.N, "A Common Fixed Point for Eight Mappings in an Intuitionistic $\mathcal{M}$-fuzzy metric space with Property 'E'", Global journal of science frontier research Mathematics and decision science, Vol.13(2) version 1.0 year 2013.
8. Rathor. M.S., Deepak Singh and Naval Singh, " Common Fixed Point Theorems for S- weakly commuting, S-Compatible and RS- weakly commuting Mappings of complete S- Fuzzy metric spaces", Int. Journal of Math. Analysis, Vol.4, 2010 (2) 75- 87.
9. Renu Chugh and Sanjay Kumar, "Fixed point theorem in generalized fuzzy metric space", Bull. Cal. Math. Soc., (2001) 93 (2) 93 - 98.
10. Singh. B and Chughan. M.S., "Generalized fuzzy metric space and Fixed point theorem", Bull. Cal. Math. Soc.,(1997) 89, 457 - 460.
11. Singh. B and Chughan. M. S., "Common fixed points of compatible maps in fuzzy metric Spaces", Fuzzy sets and systems, 115 (2000), 471 - 475.
12. Sessa.S, On weak commutativity condition of mappings in fixed point considerations, Publ. Inst. Math, 32(1982), 149-153.
13. Jungck. G, Compatible mappings and common fixed points, Intern. J. Math. Math. Sci, 9(1986), 771-779.
14. Jungck. G, Compatible mappings and common fixed points(2), Intern. J. Math. Math. Sci, 11(2) (1988), 285-288.
15. Pant. R.P, R-Weakly commuting maps in metric space, J.Math.Anal. Appl, 188(2)(1994),436 - 440.
16. Vasuki. R, Common fixed points for R-Weakly commuting maps in fuzzy metric spaces, Indian Jour. Pure. Appl. Math, 30(4)(1999), 419 – 423.
17. Veerapandi. T and Jeyaraman. M., "Fixed point theorem in generalized fuzzy metric space", Varahmihir Journal of Math. Sci., (2004) No.4, 121-127.
18. Zadeh, L.A., "Fuzzy sets", Information and Control, 8 (1965), 338-353.

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