Volume 9, No. 1, Jan. - 2018 (Special Issue)
International Journal of Mathematical Archive-9(1), 2018, 93-99
Available online through www.ijma.info ISSN 2229 – 5046

INTUITIONISTIC FUZZY CONTRA γ* GENERALIZED OPEN MAPPINGS

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ABSTRACT

In this paper, we have introduced the notion of intuitionistic fuzzy contra γ generalized closed mappings and intuitionistic fuzzy contra γ* generalized open mappings. Furthermore we have provided some properties of intuitionistic fuzzy contra γ* generalized closed mappings and discussed some fascinating theorems.*

Keywords: *Intuitionistic fuzzy sets, intuitionistic fuzzy topology, intuitionistic fuzzy γ* generalized closed sets, intuitionistic fuzzy contra γ* generalized closed mappings, intuitionistic fuzzy contra γ* generalized open mappings and intuitionistic fuzzy almost contra γ* generalized open mappings.*

Subject classification code: *54A40, 03F55.*

1. INTRODUCTION

Atanassov [1] introduced intuitionistic fuzzy sets. Coker [2] introduced intuitionistic fuzzy topological spaces. Seak Jong Lee and Eun Pyo Lee [8] have introduced intuitionistic fuzzy closed mappings in intuitionistic fuzzy topological spaces. Riya V. M and D. Jayanthi [7] introduced intuitionistic fuzzy γ* generalized closed mappings and γ* generalized open mappings. In this paper we have introduced intuitionistic fuzzy contra γ* generalized open mappings, intuitionistic fuzzy almost contra γ* generalized open mappings, intuitionistic fuzzy contra Mγ* generalized open mappings and investigated some of their properties. Also we have provided some characterization of intuitionistic fuzzy contra γ* generalized open mappings and intuitionistic fuzzy almost contra γ* generalized open mappings.

2. PRELIMINARIES

Definition 2.1: [1] An *intuitionistic fuzzy set* (IFS for short) A is an object having the form

$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$

where the functions $\mu_A: X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.
An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$.

Definition 2.2: [1] Let A and B be two IFSs of the form

$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and
$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle: x \in X\}$,
(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle: x \in X\}$,
(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle: x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ and $1_\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

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Definition 2.3: [2] An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

i) $0_\sim, 1_\sim \in \tau$,
ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \in \tau$.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS in short) in X.

Definition 2.4: [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the ***intuitionistic fuzzy interior*** and ***intuitionistic fuzzy closure*** are defined by

$int(A) = \cup \{G / G \text{ is an IFOS in X and } G \subseteq A\}$,
$cl(A) = \cap \{K / K \text{ is an IFCS in X and } A \subseteq K\}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 2.5: [8] Two IFSs A and B are said to be *q-coincident* ($A_q B$ in short) if and only if there exits an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.6: [8] Two IFSs A and B are said to be *not q-coincident* ($A_q B$ in short) if and only if $A \subseteq B^c$.

Definition 2.7: [3] An *intuitionistic fuzzy point* (IFP for short), written as $p_{(\alpha, \beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha,\beta) & if \quad x = p, \\ (0,1) & otherwise \end{cases}$$

An IFP $p_{(\alpha, \beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 2.7: Let (X,τ) be an IFTS and IFS $A = \{(x, \mu_B(x), \nu_B(x)) : x \in X\}$ is said to be an

(a) intuitionistic fuzzy semi closed set [4] (IFSCS in short) if $int(cl(A)) \subseteq A$,
(b) intuitionistic fuzzy α-closed set [4] (IFαCS in short) if $cl(int(cl(A))) \subseteq A$,
(c) intuitionistic fuzzy pre-closed set [4] (IFPCS in short) if $cl(int(A)) \subseteq A$,
(d) intuitionistic fuzzy regular closed set [4] (IFRCS in short) if $cl(int(A)) = A$,
(e) intuitionistic fuzzy generalized closed set [8] (IFGCS in short) if $cl(A) \subseteq U$, whenever $A \subseteq U$, and U in an IFOS,
(f) intuitionistic fuzzy γ closed set [5] (IFγCS in short) if $cl(int(A)) \cap int(cl(A)) \subseteq A$

An IFS A is called intuitionistic fuzzy semi open set, intuitionistic fuzzy α open set, intuitionistic fuzzy pre open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set and intuitionistic fuzzy γ open set (IFSOS, IFαOS, IFPOS, IFROS, IFGOS and IFγOS) if the complement A^c is an IFSCS, IFαCS, IFPCS, IFRCS, IFGCS and IFγCS respectively.

In the following diagram, we provide the relations between various types of intuitionistic fuzzy closedness (openness).

$$\begin{array}{ccccccc} \text{regular closed} & \Rightarrow & \text{closed} & \Rightarrow & \alpha \text{ closed} & \Rightarrow & \text{pre-closed} \\ & & \Downarrow & & \Downarrow & & \\ & & \text{g closed} & & \text{semi closed} & & \end{array}$$

The reverse implications are not true in general [4].

Definition 2.8: [5] Let A be an IFS in an IFTS (X, τ). Then the γ-interior and γ-closure of A are defined as

$\gamma int(A) = \cup\{G / G \text{ is an IF}\gamma\text{OS in X and } G \subseteq A\}$
$\gamma cl(A) = \cap\{K / K \text{ is an IF}\gamma\text{CS in X and } A \subseteq K\}$

Note that for any IFS A in (X, τ), we have $\gamma cl(A^c) = (\gamma int(A))^c$ and $\gamma int(A)^c = (\gamma cl(A))^c$.

Corollary 2.9: [3] Let A, $A_i (i \in J)$ be intuitionistic fuzzy sets in X and B, $B_j (j \in K)$ be intuitionistic fuzzy sets in Y and $f: X \rightarrow Y$ be a function. Then

a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$

b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$
c) $A \subseteq f^{-1}(f(A))$ [If f is injective, then $A = f^{-1}(f(A))$]
d) $f(f^{-1}(B)) \subseteq B$ [If f is surjective, then $B = f(f^{-1}(B))$]
e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$
f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$
g) $f^{-1}(0_\sim) = 0_\sim$
h) $f^{-1}(1_\sim) = 1_\sim$
i) $f^{-1}(B^c) = (f^{-1}(B))^c$

Definition 2.10: [6] An IFS A of an IFTS (X, τ) is said to be an intuitionistic fuzzy γ* generalized closed set (briefly IFγ*GCS) if cl(int(A)) ∩ int(cl(A)) ⊆ U whenever A ⊆ U and U is an IFOS in (X, τ).

Result 2.11: [6] Every IFRCS, IFCS, IFSCS, IFPCS, IFγCS, IFGCS is an IFγ*GCS but the converses need not be true in general.

Definition 2.12: [6] If every IFγ*GCS in (X, τ) is an IFγCS in (X, τ), then the space can be called as an intuitionistic fuzzy γ* $T_{1/2}$ (IFγ*$T_{1/2}$ in short) space.

Definition 2.13: [6] If every IFγ*GCS in (X, τ) is an IFCS in (X, τ), then the space can be called as an intuitionistic fuzzy γ*c $T_{1/2}$ (IFγ*c$T_{1/2}$ in short) space.

Definition 2.14: [6] If every IFγ*GCS in (X, τ) is an IFGCS in (X, τ), then the space can be called as an intuitionistic fuzzy γ*g$T_{1/2}$ (IFγ*g$T_{1/2}$ in short) space.

Definition 2.15: [7] A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy γ* generalized closed mapping (IFγ*G closed mapping for short) if f (V) IFγ*GCS in Y for every IFCS V of X.

Definition 2.16: [7] A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy γ* generalized open mapping (IFγ*G open mapping for short) if f (V) IFγ*GOS in Y for every IFOS V of X.

3. Intuitionistic fuzzy contra γ* generalized open mappings

In this section we have introduced intuitionistic fuzzy contra γ* generalized open mappings and investigated some of their properties.

Definition 3.1: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy contra γ* generalized open(IF contra γ*G open for short) mapping if f(V) is an IFγ*GCS in (Y, σ) for every IFOS V of (X, τ).

Example 3.2: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_1 = \langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)\rangle$ is an IFOS in X. Then $f(G_1) = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$ is an IFγ*GCS in Y as $f(G_1) \subseteq G_2$ and $cl(int(f(G_1))) \cap int(cl(f(G_1))) = 0_\sim \subseteq G_2$, where G_2 is an IFOS in Y. Hence $f(G_1)$ is an IFγ*GCS in Y. Therefore f is an IF contra γ*G open mapping.

Definition 3.3: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy contra γ* generalized closed(IF contra γ*G closed for short) mapping if f(V) is an IFγ*GOS in (Y, σ) for every IFCS V of (X, τ).

Theorem 3.4: If f : X → Y is an IF contra γ*G closed mapping and Y is an IFγ*g$T_{1/2}$ space, then f(A) is an IFGOS in Y for every IFCS A in X.

Proof: Let f : (X, τ) → (Y, σ) be an IF contra γ*G closed mapping and let A be an IFCS in X. Then by hypothesis f(A) is an IFγ*GOS in Y. Since Y is an IFγ*g$T_{1/2}$ space, f(A) is an IFGOS in Y.

Theorem 3.5: Let f: (X, τ) → (Y, σ) be a bijective mapping, suppose that one of the following properties hold:
i) $f(cl(B)) \subseteq int(\gamma cl(f(B)))$ for each IFS B in X
ii) $cl(\gamma int(f(B))) \subseteq f(int(B))$ for each IFS B in X
iii) $f^{-1}(cl(\gamma int(A))) \subseteq int(f^{-1}(A))$ for each IFS A in Y
iv) $f^{-1}(cl(A)) \subseteq int(f^{-1}(A))$ for each IFγOS A in Y.

Then f is an IF contra γ*G open mapping.

Proof:
(i) ⇒ (ii): is obvious by taking complement in (i).

(ii) ⇒ (iii): Let A ⊆ Y. Put B = f^{-1}(A) in X. This implies A = f(f^{-1}(A)) = f(B) in Y. Now cl(γint(A)) = cl(γint(f(B))) ⊆ f(int(B)) by (ii). Therefore f^{-1}(cl(γint(A))) ⊆ f^{-1}(f(int(B))) = int(B) = int(f^{-1}(A)).

(iii) ⇒ (iv): Let A ⊆ Y be an IFγOS. Then γ(int(A)) = A. By hypothesis, f^{-1}(cl(γint(A))) ⊆ int(f^{-1}(A)). Therefore f^{-1}(cl(A)) = f^{-1}(cl(γint(A))) ⊆ int(f^{-1}(A)).

Suppose (iv) holds: Let A be an IFOS in X. Then f(A) is an IFS in Y an γint(f(A)) is an IFγOS in Y. Hence by hypothesis, f^{-1}(cl(γint(f(A)))) ⊆ int(f^{-1}(γint(f(A)))) ⊆ int(f^{-1}(f(A))) = int(A) = A. Therefore cl(γint(f(A))) = f(f^{-1}(cl(γint(f(A))))) ⊆ f(A). Now cl(int(f(A))) ⊆ cl(γint(f(A))) ⊆ f(A). This implies f(A) is an IFPCS in Y and hence an IFγ*GCS in Y [6]. Thus f is an IF contra γ*G open mapping.

Theorem 3.6: Let f: (X, τ) → (Y, σ) be a bijective mapping. Suppose that one of the following properties are hold:

i) f^{-1}(γcl(A)) ⊆ int(f^{-1}(A)) for each IFS A in Y
ii) γcl(f(B)) ⊆ f(int(B)) for each IFS B in X
iii) f (cl(B)) ⊆ γint(f(B)) for each IFS B in X

Then f is an IF contra γ*G closed mapping.

Proof:
(i) ⇒ (ii): Let B ⊆ X. Then f(B) is an IFS in Y. By hypothesis, f^{-1}(γcl(f(B))) ⊆ int(f^{-1}(f(B))) = int(B). Now γcl(f(B)) = f(f^{-1}(γcl(f(B)))) ⊆ f(int(B)).

(ii) ⇒ (iii): is obvious by taking complement in (ii).

Suppose (iii) holds: Let A be an IFCS in X. Then cl(A) = A and f(A) is an IFS in Y. Now f(A) = f(cl(A)) ⊆ γint(f(A)) ⊆ f(A), by hypothesis. This implies f(A) is an IFγOS in Y and hence IFγ*GOS in Y. Therefore f is an IF contra γ*G closed mapping.

Theorem 3.7: Let f: (X, τ) → (Y, σ) be a bijective mapping. Then f is an IF contra γ*G closed mapping if cl(f^{-1}(A)) ⊆ f^{-1}(γint(A)) for every IFS A in Y.

Proof: Let A be an IFCS in X. Then cl(A) = A and f(A) is an IFS in Y. By hypothesis cl(f^{-1}(f(A))) ⊆ f^{-1}(γint(f(A))). Since f is bijective, f^{-1}(f(A)) = A. Therefore A = cl(A) = cl(f^{-1}(f(A))) ⊆ f^{-1}(γint(f(A))). Now f(A) ⊆ f(f^{-1}(γint(f(A)))) = γint(f(A)) ⊆ f(A). Hence f(A) is an IFγOS in Y and hence an IFγ*GOS in Y. Thus f is an IF contra γ*G closed mapping.

Theorem 3.8: If f: (X, τ)→ (Y, σ) is a bijective mapping where Y is an IF γ*$T_{1/2}$ space, then the following are equivalent:

i) f is an IF contra γ*G closed mapping
ii) for each IFP $p_{(\alpha, \beta)}$ ∈ Y and for each IFCS B containing f^{-1}($p_{(\alpha, \beta)}$), there exists an IFγOS A in Y and $p_{(\alpha, \beta)}$ ∈ A such that A⊆ f(B)
iii) For each IFP $p_{(\alpha, \beta)}$ ∈Y and for each IFCS B containing f^{-1}($p_{(\alpha, \beta)}$), there exists an IFγOS A in Y and $p_{(\alpha, \beta)}$ ∈ A such that f^{-1}(A) ⊆ B

Proof:
(i) ⇒ (ii): Let B be an IFCS in X. Let $p_{(\alpha, \beta)}$ be an IFP in Y such that f^{-1}($p_{(\alpha, \beta)}$) ∈ B. Then $p_{(\alpha, \beta)}$ ∈ f(f^{-1}($p_{(\alpha, \beta)}$) ∈ f(B). By hypothesis f(B) is an IFγ*GOS in Y. Since Y is an IFγ*$T_{1/2}$ space, f(B) is an IFγOS in Y. Now let A = γint(f(B)) ⊆ f(B). Therefore A ⊆ f(B).
(ii) ⇒ (iii): Let B be an IFCS in X. Let $p_{(\alpha, \beta)}$ be an IFP in Y such that f^{-1}($p_{(\alpha, \beta)}$) ∈ B. Then $p_{(\alpha, \beta)}$ ∈ f(f^{-1}($p_{(\alpha, \beta)}$)) ∈ f(B). By hypothesis f(B) is an IFγOS in Y and A ⊆ f(B). This implies f^{-1}(A) ⊆ f^{-1}(f(B)) ⊆ B.
(iii) ⇒ (i): Let B be an IFCS in X and let $p_{(\alpha, \beta)}$ ∈ Y. Let f^{-1}($p_{(\alpha, \beta)}$) ∈ B. By hypothesis there exists an IFγOS A in Y such that $p_{(\alpha, \beta)}$ ∈ A and f^{-1}(A) ⊆ B. This implies $p_{(\alpha, \beta)}$ ∈ A ⊆ f(f^{-1}(A)) ⊆ f(B). That is $p_{(\alpha, \beta)}$ ∈ f(B). Since A is an IFγOS, A = γint(A) ⊆ γint(f(B)). Therefore $p_{(\alpha, \beta)}$ ∈ γint(f(B)). But f(B) = ∪{ $p_{(\alpha, \beta)}$ / $p_{(\alpha, \beta)}$ ∈ f(B) ⊆ γint(f(B)) ⊆ f(B). Hence f(B) is an IFγOS in Y and hence f(B) is an IFγ*GOS in Y. Thus f is an IF contra γ*G closed mapping.

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Remark 3.9: The composition of two IF contra γ*G open mapping is not an IF contra γ*G open mapping in general as seen in the following example.

Example 3.10: Let X = {a, b}, Y= {u, v} and Z= {p, q}. Then τ = {$0_{\sim}$, G_1, $1_{\sim}$}, σ = {$0_{\sim}$, G_2, $1_{\sim}$} and δ = {$0_{\sim}$, G_3, G_4, $1_{\sim}$} are IFTs on X, Y and Z respectively, where G_1 = ⟨x, (0.5_a, 0.8_b), (0.2_a, 0.2_b)⟩ , G_2 = ⟨y, (0.2_u, 0.2_v), (0.5_u, 0.7_v)⟩, G_3 = ⟨z, (0.6_p, 0.8_q), (0.2_p, 0.2_q)⟩ and G_4 = ⟨z, (0.5_p, 0.6_q), (0.5_p, 0.4_q)⟩. Then (X, τ), (Y, σ) and (Z, δ) are IFTSs. Now define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v and g: (Y, σ) → (Z, δ) by g(u)= p and g(v) = q. Here f and g are IF contra γ*G open mappings but their composition g o f: (X, τ) → (Z, δ) defined by g(f(a)) =p and g(f(b)) = q is not an IF contra γ*G open mapping, since G_1 = ⟨x, (0.5_a, 0.8_b), (0.2_a, 0.2_b)⟩ is an IFOS in X but g(f(G_1)) = ⟨z, (0.5_p, 0.8_q), (0.2_p, 0.2_q)⟩ is not an IFγ*GCS in Z as g(f(G_1)) ⊆ G_3 but cl(int(g(f(G_1)))) ∩ int(cl(g(f(G_1)))) = $1_{\sim}$ ⊈ G_3 .

Theorem 3.11: For a mapping f: (X, τ) → (Y, σ), where Y is an IFγ*$T_{1/2}$ space, the following are equivalent:

i) f is an IF contra γ*G closed mapping
ii) For every IFCS A in X and for every IFP $p_{(\alpha, \beta)}$ ∈ Y, if $f^{-1}(p_{(\alpha, \beta)})$ q A then $p_{(\alpha, \beta)}$ q int(f(A))

Proof:
(i) ⇒ (ii): Let f be an IF contra γ*G closed mapping. Let A ⊆ X be an IFCS and let $p_{(\alpha, \beta)}$ ∈ Y. Also let $f^{-1}(p_{(\alpha, \beta)})$ q A then $p_{(\alpha, \beta)}$ q f (A). By hypothesis f (A) is an IFγ*GOS in Y. Since Y is an IFγ*c$T_{1/2}$space, f(A) is an IFOS in Y. Hence int(f(A)) = f(A). This implies $p_{(\alpha, \beta)}$ q int(f(A)).

(ii) ⇒ (i): Let A ⊆ X be an IFCS then f(A) is an IFS in Y. Let $p_{(\alpha, \beta)}$ ∈ Y and let $f^{-1}(p_{(\alpha, \beta)})$ q A then $p_{(\alpha, \beta)}$ q f(A). By hypothesis this implies $p_{(\alpha, \beta)}$ q int(f(A)). That is f(A) ⊆ int(f (A)). But int(f(A)) ⊆ f(A). Therefore int(f(A)) = f(A). Thus f(A) is an IFOS in Y and hence an IFγ*GOS in Y. This implies f is an IF contra γ*G closed mapping.

Theorem 3.12: Let f: (X, τ)→ (Y, σ) be an IF contra γ*G open mapping, where Y is an IFγ*$T_{1/2}$ space, then the following conditions hold:

i) γcl(f(B)) ⊆ f(int(γcl(B))) for each IFOS B in X
ii) f(cl(γint(B))) ⊆ γint(f(B)) for each IFCS B in X

Proof: Let B ⊆ X be an IFOS. Then int(B) = B. By hypothesis f(B) is an IFγ*GCS in Y. Since Y is an IFγ*$T_{1/2}$ space, f(B) is an IFγCS in Y. This implies γcl(f(B)) = f(B) = f(int(B)) ⊆ f(int(γcl(B))).
(ii) can be proved by taking complement in (i).

Theorem 3.13: A mapping f: (X, τ)→ (Y, σ) is an IF contra γ*G closed mapping, where Y is an IFγ*$T_{1/2}$ space if and only if f(γcl(B)) ⊆ γint(f(cl(B))) for every IFS B in X.

Proof:
Necessity: Let B ⊆ X be an IFS. Then cl(B) is an IFCS in X. By hypothesis, f(cl(B)) is an IFγ*GOS in Y. Since Y is an IFγ*$T_{1/2}$ space, f(cl(B)) is an IFγOS in Y. Therefore f(γcl(B)) ⊆ f(cl(B)) = γint(f(cl(B))).

Sufficiency: Let B ⊆ X be an IFCS. Then cl(B) = B. By hypothesis, f(γcl(B)) ⊆ γint(f(cl(B))) = γint(f(B)). But γcl(B) = B. Therefore f(B) = f(γcl(B)) ⊆ γint(f(B)) ⊆ f(B). This implies f(B) is an IFγOS in Y and hence an IFγ*GOS in Y. Hence f is an IF contra γ*G closed mapping.

Theorem 3.14: Let f : X → Y be a bijective mapping. Then the following conditions are equivalent if Y is an IFγ*c$T_{1/2}$ space:

(i) f is an IF contra γ*G closed mapping
(ii) f is an IF contra γ*G open mapping
(iii) int(cl(f(A))) ⊆ (f(A)) for every IFOS A in X.

Proof:
(i) ⇔ (ii): is obviously true.
(ii) ⇒ (iii): Let A be an IFOS in X. Then f(A) is an IFγ*GCS in Y. Since Y is an IFγ*c$T_{1/2}$ space, f(A) is an IFCS in Y. Therefore cl(f(A)) = f(A). This implies int(cl(f(A))) = int(f(A)) ⊆ f(A).
(iii) ⇒ (i): Let A be an IFCS in X. Then its complement A^c is an IFOS in X. By hypothesis, int(cl(f(A^c))) ⊆ f(A^c). Hence f(A^c) is an IFSCS in Y. Since every IFSCS [4] is an IFγ*GCS, f(A^c) is an IFγ*GCS in Y. Therefore f(A) is an IFγ*GOS in Y. Hence f is an IF contra γ*G closed mapping.

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International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

Definition 3.15: A mapping f: (X, τ) → (Y, σ) is said to be an intuitionistic fuzzy almost contra γ* generalized open mapping (IF almost contra γ*G open mapping for short) if f(A) is an IFγ*GCS in Y for every IFROS A in X.

Example 3.16: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.5_b), (0.5_a, 0.5_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ be IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

Now the IFS $G_1 = \langle x, (0.5_a, 0.5_b), (0.5_a, 0.5_b)\rangle$is an IFROS in X, since $int(cl(G_1)) = int(G_1^c) = G_1$. We have $f(G_1) = \langle y, (0.5_u, 0.5_v), (0.5_u, 0.5_v)\rangle$ is an IFγ*GCS as $f(G_1) \subseteq G_2$ and $int(cl(f(G_1))) \cap cl(int(f(G_1))) = 0_\sim \cap 1_\sim = 0_\sim \subseteq G_2$. Thus f is an IF almost contra γ*G open mapping.

Theorem 3.17: Every IF contra γ*G open mapping is an IF almost contra γ*G open mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF contra γ*G open mapping. Let A be an IFROS in X. Since every IFROS is an IFOS, A is an IFOS in X. Then f(A) is an IFγ*GCS in Y, by hypothesis. Therefore f is an IF almost contra γ*G open mapping.

Example 3.18: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, $G_2 = \langle x, (02_a, 0.2_b), (0.5_a, 0.6_b)\rangle$, $G_3 = \langle y, (0.6_u, 0.8_v), (0.2_u, 0.2_v)\rangle$ and $G_4 = \langle y, (0.5_u, 0.7_v), (0.2_u, 0.2_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, G_4, 1_\sim\}$ be IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

Now $G_2 = \langle x, (02_a, 0.2_b), (0.5_a, 0.6_b)\rangle$ is an IFROS in X, Since $int(cl(G_2)) = int(G_2^c) = G_2$. We have $f(G_2) = \langle y, (0.2_u, 0.2_v), (0.5_u, 0.6_v)\rangle \subseteq G_3, G_4$. As $int(cl(f(G_2))) \cap cl(int(f(G_2))) = 1_\sim \cap 0_\sim = 0_\sim \subseteq G_3, G_4$, f is an IF almost contra γ*G open mapping, but not an IF contra γ*G open mapping, as $G_1 = \langle x, (0.5_a, 0.8_b), (0.2_a, 0.2_b)\rangle$ is an IFOS in X and $f(G_1)) \subseteq G_3$ but $int(cl(f(G_1))) \cap cl(int(f(G_1))) = 1_\sim \cap 1_\sim = 1_\sim \not\subseteq G_3$.

Definition 3.19: A mapping f : X → Y is said to be an intuitionistic fuzzy contra M γ* generalized open mapping (IF contra Mγ*G open mapping) if f(A) is an IFγ*GCS in Y for every IFγ*GOS A in X.

Example 3.20: Let X = {a, b} and Y= {u, v}. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$ and $G_2 = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

$IF\gamma^*GO(X) = \{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0, 1] / 0 \leq \mu_a + \nu_a \leq 1, 0 \leq \mu_b + \nu_b \leq 1\}$

$IF\gamma^*GC(Y) = \{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0, 1] / 0 \leq \mu_u + \nu_u \leq 1, 0 \leq \mu_v + \nu_v \leq 1\}$

We have every IFγ*GOS in X is an IFγ*GCS in Y. Therefore f is an IF contra Mγ*G open mapping.

Theorem 3.21: Let f : X → Y be a bijective mapping. Then the following are equivalent.

(i) f is an IF contra Mγ*GOM

(ii) f(A) is an IFγ*GOS in Y for every IFγ*GCS A in X

Proof: Proof is obvious for a bijective mapping as $f(A^c) = (f(A))^c$.

Theorem 3.22: Every IF contra Mγ*G open mapping is an IF contra γ*G open mapping but not conversely in general.

Proof: Let f : X → Y be an IF contra Mγ*G n open mapping. Let A ⊆ X be an IFOS. Then A is an IFγ*GOS in X. By hypothesis, f(A) is an IFγ*GCS in Y. Hence f is an IF contra γ*G open mapping.

Example 3.23: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.8_v), (0.4_u, 0.1_v)\rangle$ and $G_3 = \langle y, (0.3_u, 0.3_v), (0.5_u, 0.4_v)\rangle$, Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

Now $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ is an IFOS in X. We have $f(G_1) = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ is an IFγ*GCS. Hence f is an IF contra γ*G open mapping.

Now let $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ which is an IFγ*GOS in X. But it is not an IFγ*GCS in Y, since $f(G_1) \subseteq G_2$ but $int(cl(f(G_1))) \cap cl(int(f(G_1))) = 1_\sim \cap G_3^c = G_3^c \not\subseteq G_2$. Hence f is not an IF contra Mγ*G open mapping.

Theorem 3.24:

(i) (i) If f : X → Y is an IF γ*G open mapping and g: Y → Z is an IF contra γ*G open mapping, then g o f : X → Z is an IF contra γ*G open mapping.

(ii) If f : X → Y is an IF contra γ*G open mapping and g: Y → Z is an IF contra Mγ*G open mapping, then g o f : X → Z is an IF γ*G open mapping.

(iii) If f : X → Y is an IF almost contra γ*G open mapping and g: Y → Z is an IF contra Mγ*G open mapping, then g o f : X → Z is an IF almost γ*G open mapping.

Proof:

(i) Let A be an IFOS in X. Then f(A) is an IFγ*GOS in Y. Therefore g(f(A)) is an IFγ*GCS in Z. Hence g o f is an IF contra γ*G open mapping.

(ii) Let A be an IFOS in X. Then f(A) is an IFγ*GCS in Y, since f is an IF contra γ*GOM. Since g is an IF contra γ*G open mapping, g(f(A)) is an IFγ*GOS in Z. Therefore g o f is an IFγ*G open mapping.

(iii) Let A be an IFROS in X. Then f(A) is an IFγ*GCS in Y, since f is an IF almost contra γ*G open mapping. Since g is an IF contra γ*G open mapping, g(f(A)) is an IFγ*GOS in Z. Therefore g o f is an IF almost γ*G open mapping.

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Source of support: Proceedings of UGC Funded International Conference on Intuitionistic Fuzzy Sets and Systems (ICIFSS-2018), Organized by: Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

