

**ON INTUITIONISTIC  $b$  - OPEN SETS**

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**ABSTRACT**

*In this paper, we introduce a new types of sets called intuitionistic  $b$  -open sets in intuitionistic topological space. Also intuitionistic  $b$  -interior and intuitionistic  $b$  -closure of an intuitionistic sets are defined and their properties are investigated with other existing forms of intuitionistic open sets.*

*Key words and phrases:* Intuitionistic  $b$  -open (closed) sets, intuitionistic  $b$  -interior and intuitionistic  $b$  -closure.

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**1. INTRODUCTION**

Ever since the introduction of fuzzy sets by Zadeh [8], the fuzzy concept has invaded almost all branches of mathematics, Atanassov [2], introduced the notion of intuitionistic fuzzy sets. Using this notion Coker [[3], [4], [5]] defined and studied intuitionistic topological spaces, intuitionistic open sets, intuitionistic closed sets and compactness on intuitionistic topological spaces. Also, he defined the closure and interior operators in intuitionistic topological spaces and established their properties. Many different forms of open sets have been introduced over the years in general topology. Andrijevic [1] introduced and studied about  $b$  -open sets in general topology. Palaniswamy *et al.*, [7] introduced the concept of intuitionistic semi-open, intuitionistic  $\alpha$  -open, intuitionistic pre-open and intuitionistic  $\beta$  -open sets in 2016. In this paper, is to define and study the intuitionistic  $b$  -open sets in intuitionistic topological spaces. Also some properties of these are discussed.

**2. PRELIMINARIES**

The following definitions and results are essential to proceed further.

**Definition 2.1:** [3] Let  $X$  be a non empty fixed set. An intuitionistic set (briefly,  $IS$ )  $A$  is an object of the form  $A = (X, A_1, A_2)$ , where  $A_1$  and  $A_2$  are subsets of  $X$  satisfying  $A_1 \cap A_2 = \emptyset$ . The set  $A_1$  is called the set of members of  $A$ , while  $A_2$  is called the set of non-members of  $A$ .

The family of all  $IS$  's in  $X$  will be denoted by  $IS(X)$ . Every crisp set  $A$  on a non-empty set  $X$  is obviously an intuitionistic set.

**Definition 2.2:** [3] Let  $X$  be a non-empty set,  $A = (X, A_1, A_2)$  and  $B = (X, B_1, B_2)$  be intuitionistic sets on  $X$ , then

1.  $A \subseteq B$  if and only if  $A_1 \subseteq B_1$  and  $B_2 \subseteq A_2$ .

2.  $A = B$  if and only if  $A \subseteq B$  and  $B \subseteq A$ .
3.  $A \subset B$  if and only if  $A_1 \cup A_2 \supseteq B_1 \cup B_2$ .
4.  $\overline{A} = (X, A_2, A_1)$ .
5.  $A \cup B = (X, A_1 \cup B_1, A_2 \cap B_2)$ .
6.  $A \cap B = (X, A_1 \cap B_1, A_2 \cup B_2)$ .
7.  $A - B = A \cap \overline{B}$
8.  $\tilde{\phi} = (X, \phi, X)$  and  $\tilde{X} = (X, X, \phi)$

**Corollary 2.1:** [3] Let  $A, B, C$  and  $A_i$  be IS 's in  $X$ . Then

1.  $A_i \subseteq B$  for each  $i$  implies that  $\bigcup A_i \subseteq B$ .
2.  $B \subseteq A_i$  for each  $i$  implies that  $B \subseteq \bigcap A_i$ .
3.  $\overline{\bigcup A_i} = \bigcap \overline{A_i}$  and  $\overline{\bigcap A_i} = \bigcup \overline{A_i}$ .
4.  $A \subseteq B \Leftrightarrow \overline{B} \subseteq \overline{A}$ .
5.  $\overline{(\overline{A})} = A, \overline{\tilde{\phi}} = \tilde{X}$  and  $\overline{\tilde{X}} = \tilde{\phi}$

**Definition 2.3:** [5] An intuitionistic topology (briefly  $IT$ ) on a non-empty set  $X$  is a family  $\tau$  of IS 's in  $X$  satisfying the following axioms

1.  $\tilde{\phi}, \tilde{X} \in \tau$ .
2.  $A \cap B \in \tau$  for any  $A, B \in \tau$ .
3.  $\bigcup A_i \in \tau$  for an arbitrary family in  $\tau$ .

In this case the pair  $(X, \tau)$  is called intuitionistic topological space (briefly  $ITS$ ) and the IS 's in  $\tau$  are called the intuitionistic open set in  $X$  denoted by  $I^{(\tau)}O$  and the complement of an  $I^{(\tau)}O$  is called Intuitionistic closed set in  $X$  denoted by  $I^{(\tau)}C$ . The family of all  $I^{(\tau)}O$  (resp.  $I^{(\tau)}C$ ) sets in  $X$  will be denoted by  $I^{(\tau)}O(X)$  (resp.  $I^{(\tau)}C(X)$ .)

**Definition 2.4:** [5] Let  $(X, \tau)$  be an  $ITS$  and  $A \in IS(X)$ . Then the intuitionistic interior (resp. intuitionistic closure) of  $A$  are defined by  $int(A) = \bigcup \{K : K \in I^{(\tau)}O(X) \text{ and } K \subseteq A\}$  (resp.  $cl(A) = \bigcap \{K : K \in I^{(\tau)}C(X) \text{ and } A \subseteq K\}$ )

In this study we use  $I^{(\tau)}i(A)$  (resp.  $I^{(\tau)}c(A)$ ) instead of  $int(A)$  (resp.  $cl(A)$ ).

**Definition 2.5:** Let  $(X, \tau)$  be an  $ITS$  and an IS  $A$  in  $X$  is said to be

1. intuitionistic regular-open [6] (briefly  $I^{(\tau)}RO$ ) if  $A = I^{(\tau)}i(I^{(\tau)}c(A))$  and intuitionistic regular-closed (briefly  $I^{(\tau)}RC$ ) if  $I^{(\tau)}c(I^{(\tau)}i(A)) = A$ .
2. intuitionistic pre-open [6] (briefly  $I^{(\tau)}PO$ ) if  $A \subseteq I^{(\tau)}i(I^{(\tau)}c(A))$  and intuitionistic pre-closed (briefly  $I^{(\tau)}PC$ ) if  $I^{(\tau)}c(I^{(\tau)}i(A)) \subseteq A$ .
3. intuitionistic semi-open [6] (briefly  $I^{(\tau)}SO$ ) if  $A \subseteq I^{(\tau)}c(I^{(\tau)}i(A))$  and intuitionistic semi-closed (briefly  $I^{(\tau)}SC$ ) if  $I^{(\tau)}i(I^{(\tau)}c(A)) \subseteq A$ .
4. intuitionistic  $\alpha$ -open [7] (briefly  $I^{(\tau)}\alpha O$ ) if  $A \subseteq I^{(\tau)}i(I^{(\tau)}c(I^{(\tau)}i(A)))$  and intuitionistic  $\alpha$ -closed (briefly  $I^{(\tau)}\alpha C$ ) if  $I^{(\tau)}c(I^{(\tau)}i(I^{(\tau)}c(A))) \subseteq A$ .

5. intuitionistic  $\beta$  -open [7] (briefly  $I^{(\tau)}\beta O$ ) if  $A \subseteq I^{(\tau)}c(I^{(\tau)}i(I^{(\tau)}c(A)))$  and intuitionistic  $\beta$  -closed (briefly  $I^{(\tau)}\beta C$ ) if  $I^{(\tau)}i(I^{(\tau)}c(I^{(\tau)}i(A))) \subseteq A$ .

The family of all  $I^{(\tau)}RO$  (resp.  $I^{(\tau)}RC$ ,  $I^{(\tau)}PO$ ,  $I^{(\tau)}PC$ ,  $I^{(\tau)}SO$ ,  $I^{(\tau)}SC$ ,  $I^{(\tau)}\alpha O$ ,  $I^{(\tau)}\alpha C$ ,  $I^{(\tau)}\beta O$  and  $I^{(\tau)}\beta C$ ) sets in  $X$  will be denoted by  $I^{(\tau)}RO(X)$  (resp.  $I^{(\tau)}RC(X)$ ,  $I^{(\tau)}PO(X)$ ,  $I^{(\tau)}PC(X)$ ,  $I^{(\tau)}SO(X)$ ,  $I^{(\tau)}SC(X)$ ,  $I^{(\tau)}\alpha O(X)$ ,  $I^{(\tau)}\alpha C(X)$ ,  $I^{(\tau)}\beta O(X)$  and  $I^{(\tau)}\beta C(X)$ .)

**Definition 2.6:** [6, 7] Let  $(X, \tau)$  be an ITS and  $A$  be an IS( $X$ ), then

1. intuitionistic regular-interior (resp. intuitionistic pre- interior, intuitionistic semi-interior, intuitionistic  $\alpha$  -interior and intuitionistic  $\beta$  -interior) of  $A$  is the union of all  $I^{(\tau)}RO(X)$  (resp.  $I^{(\tau)}PO(X)$ ,  $I^{(\tau)}SO(X)$ ,  $I^{(\tau)}\alpha O(X)$  and  $I^{(\tau)}\beta O(X)$ ) contained in  $A$ , and is denoted by  $I^{(\tau)}Ri(A)$  (resp.  $I^{(\tau)}Pi(A)$ ,  $I^{(\tau)}Si(A)$ ,  $I^{(\tau)}\alpha i(A)$  and  $I^{(\tau)}\beta i(A)$ .)

$$\begin{aligned} \text{i.e. } I^{(\tau)}Ri(A) &= \bigcup \{G : G \in I^{(\tau)}RO(X) \text{ and } G \subseteq A\}, \\ I^{(\tau)}Pi(A) &= \bigcup \{G : G \in I^{(\tau)}PO(X) \text{ and } G \subseteq A\}, \\ I^{(\tau)}Si(A) &= \bigcup \{G : G \in I^{(\tau)}SO(X) \text{ and } G \subseteq A\}, \\ I^{(\tau)}\alpha i(A) &= \bigcup \{G : G \in I^{(\tau)}\alpha O(X) \text{ and } G \subseteq A\}, \\ I^{(\tau)}\beta i(A) &= \bigcup \{G : G \in I^{(\tau)}\beta O(X) \text{ and } G \subseteq A\}. \end{aligned}$$

2. intuitionistic regular-closure (resp. intuitionistic pre-closure, intuitionistic semi-closure, intuitionistic  $\alpha$  -closure, intuitionistic  $\beta$  -closure) of  $A$  is the intersection of all  $I^{(\tau)}RC(X)$  (resp.  $I^{(\tau)}PC(X)$ ,  $I^{(\tau)}SC(X)$ ,  $I^{(\tau)}\alpha C(X)$ ,  $I^{(\tau)}\beta C(X)$ ) containing  $A$ , and is denoted by  $I^{(\tau)}Rc(A)$  (resp.  $I^{(\tau)}Pc(A)$ ,  $I^{(\tau)}Sc(A)$ ,  $I^{(\tau)}\alpha c(A)$ ,  $I^{(\tau)}\beta c(A)$ .)

$$\begin{aligned} \text{i.e. } I^{(\tau)}Rc(A) &= \bigcap \{G : G \in I^{(\tau)}RC(X) \text{ and } G \supseteq A\}, \\ I^{(\tau)}Pc(A) &= \bigcap \{G : G \in I^{(\tau)}PC(X) \text{ and } G \supseteq A\} \\ I^{(\tau)}Sc(A) &= \bigcap \{G : G \in I^{(\tau)}SC(X) \text{ and } G \supseteq A\}, \\ I^{(\tau)}\alpha c(A) &= \bigcap \{G : G \in I^{(\tau)}\alpha C(X) \text{ and } G \supseteq A\}, \\ I^{(\tau)}\beta c(A) &= \bigcap \{G : G \in I^{(\tau)}\beta C(X) \text{ and } G \supseteq A\}. \end{aligned}$$

**Proposition 2.1:** [7] Let  $A \in IS(X)$ , then

1.  $I^{(\tau)}S(\dot{A}) = A \cap I^{(\tau)}c(I^{(\tau)}i(A))$ .
2.  $I^{(\tau)}Pi(A) = A \cap I^{(\tau)}i(I^{(\tau)}c(A))$ .
3.  $I^{(\tau)}S(\check{A}) = A \cup I^{(\tau)}i(I^{(\tau)}c(A))$ .
4.  $I^{(\tau)}Pc(A) = A \cup I^{(\tau)}c(I^{(\tau)}i(A))$ .
5.  $I^{(\tau)}Pc(I^{(\tau)}Pi(A)) = I^{(\tau)}Pi(A) \cup I^{(\tau)}c(I^{(\tau)}i(A))$ .

### 3. ON INTUITIONISTIC $b$ -OPEN SETS

Here intuitionistic  $b$  -open sets is defined and its relation with existing intuitionistic sets and basic properties are studied.

**Definition 3.1:** A subset  $A$  of an intuitionistic topological space  $X$  is said to be

1. intuitionistic (briefly  $I^{(\tau)}bO$ )  $b$  -open set if  $A \subseteq I^{(\tau)}i(I^{(\tau)}c(A)) \cup I^{(\tau)}c(I^{(\tau)}i(A))$ .
2. intuitionistic  $b$  -closed (briefly  $I^{(\tau)}bC$ ) set if  $I^{(\tau)}i(I^{(\tau)}c(A)) \cap I^{(\tau)}c(I^{(\tau)}i(A)) \subseteq A$ .

The family of all intuitionistic  $b$  -open (resp.  $b$  -closed) in  $X$  will be denoted by  $I^{(\tau)}bO(X)$  (resp.  $I^{(\tau)}bC(X)$ ).

**Remark 3.1** The complement of intuitionistic  $b$  -open sets are intuitionistic  $b$  -closed.

**Theorem 3.1:**

1. Every intuitionistic regular open (resp. open, semi-open, pre-open and  $\alpha$  -open) set is intuitionistic  $b$  -open.
2. Every intuitionistic  $b$  -open set is intuitionistic  $\beta$  -open.

**Proof:**

1. Let  $A \in I^{(\tau)}RO(X)$ , this implies that  $A = I^{(\tau)}i(I^{(\tau)}c(A))$  which gives

$$A \subseteq I^{(\tau)}i(I^{(\tau)}c(A)) \cup I^{(\tau)}c(I^{(\tau)}i(A)).$$

Let  $A \in I^{(\tau)}O(X)$ , this implies that  $A = I^{(\tau)}i(A)$  which gives  $A \subseteq I^{(\tau)}c(I^{(\tau)}i(A))$  and also

$$A \subseteq I^{(\tau)}i(I^{(\tau)}c(A)) \cup I^{(\tau)}c(I^{(\tau)}i(A)).$$

Let  $A \in I^{(\tau)}SO(X)$ , this implies that  $A \subseteq I^{(\tau)}c(I^{(\tau)}i(A))$  which gives

$$A \subseteq I^{(\tau)}i(I^{(\tau)}c(A)) \cup I^{(\tau)}c(I^{(\tau)}i(A)).$$

Let  $A \in I^{(\tau)}PO(X)$ , this implies that  $A \subseteq I^{(\tau)}i(I^{(\tau)}c(A))$  which gives

$$A \subseteq I^{(\tau)}i(I^{(\tau)}c(A)) \cup I^{(\tau)}c(I^{(\tau)}i(A)).$$

Let  $A \in I^{(\tau)}\alpha O(X)$ , this implies that  $A \subseteq I^{(\tau)}i(I^{(\tau)}c(I^{(\tau)}i(A)))$  which gives

$$A \subseteq I^{(\tau)}i(I^{(\tau)}c(A)) \cup I^{(\tau)}c(I^{(\tau)}i(A)).$$

2. Let  $A \in I^{(\tau)}bO(X)$ , this implies that  $A \subseteq I^{(\tau)}i(I^{(\tau)}c(A)) \cup I^{(\tau)}c(I^{(\tau)}i(A))$ , that is either  $A \subseteq I^{(\tau)}i(I^{(\tau)}c(A))$  or  $A \subseteq I^{(\tau)}c(I^{(\tau)}i(A))$ , in the first case, it is direct that  $A \in I^{(\tau)}\beta O(X)$  and in the other case using  $A \subseteq I^{(\tau)}c(A)$  implies the remaining notion.

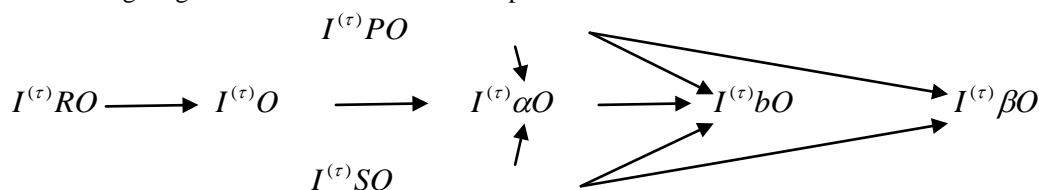
The converse of above theorem need not be true. It can be seen in the following examples.

**Example 3.1:** Let  $X = \{a, b, c\}$  and  $\tau = \{(X, \phi, X), (X, X, \phi), (X, \phi, \{a, b\}), (X, \phi, \{b\}), (X, \phi, \{b, c\}), (X, \{c\}, \{a, b\}), (X, \{c\}, \{b\}), (X, \{a, c\}, \{b\})\}$ . Then the IS

1.  $(X, \{b\}, \{c\}) \in I^{(\tau)}bO(X)$  but  $(X, \{b\}, \{c\}) \notin I^{(\tau)}RO(X)$ .
2.  $(X, \{b\}, \{c\}) \in I^{(\tau)}bO(X)$  but  $(X, \{b\}, \{c\}) \notin I^{(\tau)}OS(X)$ .
3.  $(X, \{b, c\}, \{a\}) \in I^{(\tau)}bO(X)$  but  $(X, \{b\}, \{c\}) \notin I^{(\tau)}PO(X)$ .
4.  $(X, \{b, c\}, \phi) \in I^{(\tau)}bO(X)$  but  $(X, \{b\}, \{c\}) \notin I^{(\tau)}\alpha O(X)$ .

**Example 3.2:** Let  $X = \{a, b\}$  and  $\tau = \{(X, \phi, X), (X, X, \phi), (X, \{a\}, \phi)\}$  then the IS  $(X, \{a\}, \{b\}) \in I^{(\tau)}bO(X)$  but  $(X, \{b\}, \{c\}) \notin I^{(\tau)}SO(X)$ .

The following diagram shows that the relationship between  $I^{(\tau)}bO$  and some other intuitionistic sets.



$A \rightarrow B$  represents A implies B, but not conversely.

**Theorem 3.2:** Let  $(X, \tau)$  be an ITS and  $\{A_i\}_{i \in J}$  be any arbitrary family of members from  $I^{(\tau)}bO(X)$  then  $\bigcup A_i \in I^{(\tau)}bO(X)$ .

**Proof.** Let  $A_i \in I^{(\tau)}bO(X)$  then  $A_i \subseteq I^{(\tau)}c(I^{(\tau)}i(A_i)) \cup I^{(\tau)}i(I^{(\tau)}c(A_i))$ . Now we decompose the family into two sub families such that  $B_i = \{K : K \text{ is a member in } A_i \text{ and } K \subseteq I^{(\tau)}c(I^{(\tau)}i(K))\}$  and  $C_i = \{G : G \text{ is a member in } A_i \text{ and } G \subseteq I^{(\tau)}i(I^{(\tau)}c(G))\}$  Clearly  $\bigcup A_i = (\bigcup B_i) \cup (\bigcup C_i)$  and we have  $\bigcup B_i \in I^{(\tau)}SO(X)$  and  $\bigcup C_i \in I^{(\tau)}PO(X)$ . Union of an  $I^{(\tau)}SO$  and  $I^{(\tau)}PO$  is in  $I^{(\tau)}bO(X)$ .

**Corollary 3.1:** Let  $(X, \tau)$  be an ITS and  $\{A_i\}_{i \in J}$  be any arbitrary family of members from  $I^{(\tau)}bC(X)$  then  $\bigcap A_i \in I^{(\tau)}bC(X)$ .

**Remark 3.2:** The intersection of two  $I^{(\tau)}bO(X)$  need not be  $I^{(\tau)}bO(X)$  as shown by the following example.

**Example 3.3:** Let  $X = \{a, b, c\}$  and  $\tau = \{(X, \phi, X), (X, X, \phi), (X, \{\phi\}, \{b\}), (X, \{\phi\}, \{a, b\}), (X, \{\phi\}, \{b, c\}), (X, \{c\}, \{a, b\}), (X, \{c\}, \{b\}), (X, \{a, c\}, \{b\})\}$  then the IS  $(X, \{\phi\}, \{a\})$  and  $(X, \{\phi\}, \{c\})$  are  $I^{(\tau)}bO(X)$  but the intersection  $(X, \{\phi\}, \{a, c\})$  is not a b-open.

**Theorem 3.3:** Let  $(X, \tau)$  be an ITS and  $A \subset X$ , then

1.  $(I^{(\tau)}bc(A))^C = I^{(\tau)}bi(A^C)$ .
2.  $(I^{(\tau)}bi(A))^C = I^{(\tau)}bc(A^C)$ .

**Proof.**

(i) Let  $A \in IS(X)$  and  $(X, \tau)$  be an ITS. Now

$$\begin{aligned} I^{(\tau)}bc(A) &= \bigcap \{G : G \in I^{(\tau)}bC(X) \text{ and } G \supseteq A\} \\ (I^{(\tau)}bc(A))^C &= \left[ \bigcap \{G : G \in I^{(\tau)}bC(X) \text{ and } G \supseteq A\} \right]^C \\ &= \bigcup \{G^C : G^C \in I^{(\tau)}bO(X) \text{ and } G^C \subseteq A^C\} \\ (I^{(\tau)}bc(A))^C &= I^{(\tau)}bi(A^C). \end{aligned}$$

(ii) The similar proof can be given.

**Theorem 3.4** In an ITS  $(X, \tau)$ , for an  $A \in IS(X)$ , the following are equivalent:

1.  $A \in I^{(\tau)}bO(X)$ .
2.  $A = I^{(\tau)}Pi(A) \cup I^{(\tau)}Si(A)$ .
3.  $A \subseteq I^{(\tau)}Pc(I^{(\tau)}Pi(A))$ .

**Proof.** Let  $A \in IS(X)$  in an ITS  $(X, \tau)$

(i)  $\Rightarrow$  (ii): Suppose that  $A \in I^{(\tau)}bO(X)$ , so that  $A \subseteq I^{(\tau)}c(I^{(\tau)}i(A)) \cup I^{(\tau)}i(I^{(\tau)}c(A))$ . Now,

$$\begin{aligned} I^{(\tau)}Pi(A) \cup I^{(\tau)}Si(A) &= (A \cap I^{(\tau)}i(I^{(\tau)}c(A))) \cup (A \cap I^{(\tau)}c(I^{(\tau)}i(A))) \\ &= A \cap (I^{(\tau)}i(I^{(\tau)}c(I^{(\tau)}i(A)))) \\ &= A. \end{aligned}$$

(ii)  $\Rightarrow$  (iii): Consider

$$\begin{aligned} A &= I^{(\tau)}Pi(A) \cup I^{(\tau)}Si(A) \\ &= I^{(\tau)}Pi(A) \cup (A \cap I^{(\tau)}c(I^{(\tau)}i(A))) \\ &\subseteq I^{(\tau)}Pi(A) \cup I^{(\tau)}c(I^{(\tau)}i(A)) \\ &= I^{(\tau)}Pc(I^{(\tau)}Pi(A)) \\ \text{i.e., } A &\subseteq I^{(\tau)}Pc(I^{(\tau)}Pi(A)). \end{aligned}$$

(iii)  $\Rightarrow$  (i): Consider

$$\begin{aligned} A &\subseteq I^{(\tau)}Pc(I^{(\tau)}Pi(A)) \\ &= I^{(\tau)}Pi(A) \cup I^{(\tau)}c(I^{(\tau)}i(A)) \\ &= (A \cap I^{(\tau)}i(I^{(\tau)}c(A))) \cup I^{(\tau)}c(I^{(\tau)}i(A)) \\ &\subseteq I^{(\tau)}i(I^{(\tau)}c(A)) \cup I^{(\tau)}c(I^{(\tau)}i(A)) \\ A &\subseteq I^{(\tau)}i(I^{(\tau)}c(A)) \cup I^{(\tau)}c(I^{(\tau)}i(A)) \\ \text{i.e., } A &\in I^{(\tau)}bO(X). \end{aligned}$$

This completes the proof.

**Theorem 3.5:** Let  $A$  be an  $IS$  in  $X$ , then

1.  $I^{(\tau)}bi(A) = I^{(\tau)}Si(A) \cup I^{(\tau)}Pi(A)$
2.  $I^{(\tau)}bc(A) = I^{(\tau)}Sc(A) \cap I^{(\tau)}Pc(A)$ .

**Proof:**

(a) Let  $A \in IS(X)$ ,  $(X, \tau)$  be an  $ITS$ . Since  $I^{(\tau)}bi(A) \in I^{(\tau)}bO(X)$ , we have,

$$\begin{aligned} I^{(\tau)}bi(A) &\subseteq I^{(\tau)}c(I^{(\tau)}i(I^{(\tau)}bi(A))) \cup I^{(\tau)}i(I^{(\tau)}c(I^{(\tau)}bi(A))) \\ &\subseteq I^{(\tau)}c(I^{(\tau)}i(A)) \cup I^{(\tau)}i(I^{(\tau)}c(A)) \\ A \cap I^{(\tau)}bi(A) &\subseteq A \cap [I^{(\tau)}c(I^{(\tau)}i(A)) \cup I^{(\tau)}i(I^{(\tau)}c(A))] \\ &= [A \cap I^{(\tau)}c(I^{(\tau)}i(A))] \cup [A \cap I^{(\tau)}i(I^{(\tau)}c(A))] \\ &= I^{(\tau)}Si(A) \cup I^{(\tau)}Pi(A) \quad (\text{by Proposition(2.1)}) \\ I^{(\tau)}bi(A) &\subseteq I^{(\tau)}Si(A) \cup I^{(\tau)}Pi(A). \end{aligned}$$

But  $I^{(\tau)}Si(A) \subseteq I^{(\tau)}bi(A)$  and  $I^{(\tau)}Pi(A) \subseteq I^{(\tau)}bi(A)$ , hence  $I^{(\tau)}Si(A) \cup I^{(\tau)}Pi(A) \subseteq I^{(\tau)}bi(A)$ .

Therefore  $I^{(\tau)}bi(A) = I^{(\tau)}Si(A) \cup I^{(\tau)}Pi(A)$ .

(b) Since  $I^{(\tau)}bc(A) \in I^{(\tau)}bC(X)$ , we have,

$$\begin{aligned} I^{(\tau)}c(I^{(\tau)}i(I^{(\tau)}bc(A))) \cap I^{(\tau)}i(I^{(\tau)}c(I^{(\tau)}bc(A))) &\subseteq I^{(\tau)}bc(A) \\ I^{(\tau)}c(I^{(\tau)}i(A)) \cap I^{(\tau)}i(I^{(\tau)}c(A)) &\subseteq I^{(\tau)}c(I^{(\tau)}i(I^{(\tau)}bc(A))) \cap I^{(\tau)}i(I^{(\tau)}c(I^{(\tau)}bc(A))) \subseteq I^{(\tau)}bc(A) \\ A \cup [I^{(\tau)}c(I^{(\tau)}i(A)) \cap I^{(\tau)}i(I^{(\tau)}c(A))] &\subseteq A \cup I^{(\tau)}bc(A) \cap [A \cup I^{(\tau)}i(I^{(\tau)}c(A))] \subseteq I^{(\tau)}bc(A) \\ I^{(\tau)}Sc(A) \cap I^{(\tau)}Pc(A) &\subseteq I^{(\tau)}bc(A). \quad (\text{by Proposition(2.1)}) \end{aligned}$$

But  $I^{(\tau)}bc(A) \subseteq I^{(\tau)}Sc(A)$  and  $I^{(\tau)}bc(A) \subseteq I^{(\tau)}Pc(A)$ , hence  $I^{(\tau)}bc(A) \subseteq I^{(\tau)}Sc(A) \cap I^{(\tau)}Pc(A)$ .

Therefore  $I^{(\tau)}bc(A) = I^{(\tau)}Sc(A) \cap I^{(\tau)}Pc(A)$ .

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