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COMPLETELY γ - GENERALIZED CONTINUOUS MAPPINGS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce intuitionistic fuzzy completely γ generalized continuous mappings. We investigate some of their properties. Also we provide some characterization of intuitionistic fuzzy completely γ generalized continuous mappings.

Keywords: *Intuitionistic fuzzy sets, intuitionistic fuzzy topology, intuitionistic fuzzy γ generalized continuous mappings, intuitionistic fuzzy completely γ generalized continuous mappings.*

Subject classification code: *03F55, 54A40.*

1. INTRODUCTION

Atanassov [1] introduced the idea of intuitionistic fuzzy sets. Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. Prema, S and Jayanthi, D [9] introduced intuitionistic fuzzy γ generalized continuous mappings. In this paper we introduce the notion of intuitionistic fuzzy completely γ generalized continuous mappings and study some of their properties. We provide some characterizations of intuitionistic fuzzy completely γ generalized continuous mappings.

2. PRELIMINARIES

Definition 2.1: [1] An intuitionistic fuzzy set (IFS for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \longrightarrow [0,1]$ and $\nu_A : X \longrightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X. An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2.2: [1] Let A and B be two IFSs of the form $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B=\{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,
(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$,
(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ and $1_\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3: [2] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

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In this case the pair (X, τ) is called the intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [8] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy γ generalized closed set (IFγGCS for short) if $\gamma cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFγOS in (X, τ).

The complement A^c of an IFγGCS A in an IFTS (X, τ) is called an intuitionistic fuzzy γ generalized open set (IFγGOS for short) in X.

Definition 2.5: [9] A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy γ generalized continuous (IFγG continuous for short) mapping if $f^{-1}(V)$ is an IFγGCS in (X, τ) for every IFCS V of (Y, σ).

Definition 2.6: [8] An IFTS (X,τ) is an intuitionistic fuzzy $\gamma_\gamma T_{1/2}$ ($IF\gamma_\gamma T_{1/2}$ in short) space if every IFγGCS is an IFγCS in X.

Definition 2.7: [3] An intuitionistic fuzzy point (IFP in short), written as $p_{(\alpha,\beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise.} \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha,\beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 2.8: [6] Let X and Y be two IFTSs. Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle y, \mu_B(y), \nu_B(y) : y \in Y\}$ be IFSs of X and Y respectively. Then A x B is an IFS of X x Y defined by

$$(A \times B)(x, y) = \langle (x, y), \min(\mu_A(x), \mu_B(y)), \max(\nu_A(x), \nu_B(y))\rangle.$$

Definition 2.9: [6] Let $f_1: X \to Y_1$ and $f_2: X_2 \to Y_2$. The product $f_1 \times f_2: X_1 \times X_2 \to Y_1 \times Y_2$ is defined by $(f_1 \times f_2)(x_1, x_2) = (f_1(x_1), f_2(x_2))$ for every $(x_1, x_2) \in X_1 \times X_2$.

Definition 2.10: [4] Let X and Y be two non empty sets and f: X → Y be a function. If $B = \{\langle y, (\mu_B(y), \nu_B(y) / y \in Y\rangle\}$ is an IFS in Y, then the ***preimage*** of B under f is denoted and defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) / x \in X\rangle\} \text{ where } f^{-1}(\mu_B)(x) = \mu_B(f(x)) \text{ for every } x \in X.$$

3. Completely γ generalized continuous mappings in intuitionistic fuzzy topological spaces

In this section we introduce intuitionistic fuzzy completely γ generalized continuous mappings and study some of their properties.

Definition 3.1: A mapping f: (X, τ) → (Y, σ) is said to be an intuitionistic fuzzy completely γ generalized continuous (IF completely γG continuous for short) mapping if $f^{-1}(V)$ is an IFRCS in X for every IFγGCS V in Y.

Theorem 3.2: Every IF completely γG continuous mapping is an IF continuous mapping[4] but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely γG continuous mapping . Let V be an IFCS in Y. Since every IFCS is an IFγGCS [8], V is an IFγGCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFCS, $f^{-1}(V)$ is an IFCS in X. Hence f is an IF continuous mapping.

Example 3.3: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.2_a, 0.3_b), (0.8_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively.

Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.
$IF\gamma O(X) = \{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] /$ either $\nu_a < 0.2$ or $\nu_b < 0.3$, $\nu_a \geq 0.5$ whenever $\nu_b \geq 0.5$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1\}$.

Then $IF\gamma C(X) = \{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] /$ either $\mu_a < 0.2$ or $\mu_b < 0.3$, $\mu_a \geq 0.5$ whenever $\mu_b \geq 0.5$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1\}$.

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IFγO(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0, 1], \mu_v \in [0, 1], \nu_u \in [0, 1], \nu_v \in [0, 1] / 0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1\}$
IFγC(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0, 1], \mu_v \in [0, 1], \nu_u \in [0, 1], \nu_v \in [0, 1] / 0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1\}$

Then f is an IF continuous mapping, but not an IF completely γG continuous mapping, since G_3^c is an IFγGCS in Y, but $f^{-1}(G_3^c) = \langle x, (0.8_a, 0.7_b), (0.2_a, 0.3_b)\rangle$ is not an IFRCS in X as $cl(int((f^{-1}(G_3^c)) = cl(G_1) = G_1^c \neq f^{-1}(G_3^c)$.

Theorem 3.4: Every IF completely γG continuous mapping is an IF semi continuous mapping [7] but not conversely in general.

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an IF completely γG continuous mapping . Let V be an IFCS in Y. Since every IFCS is an IFγGCS, V is an IFγGCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFSCS, $f^{-1}(V)$ is an IFSCS in X. Hence f is an IF semi continuous mapping.

Example 3.5: In Example 3.3, f is an IF semi continuous mapping, but not an IF completely γG continuous mapping.

Theorem 3.6: Every IF completely γG continuous mapping is an IF pre continuous mapping [7] but not conversely in general.

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an IF completely γG continuous mapping. Let V be an IFCS in Y. Since every IFCS is an IFγGCS, V is an IFγGCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFPCS, $f^{-1}(V)$ is an IFPCS in X. Hence f is an IF pre continuous mapping.

Example 3.7: In Example 3.3, f is an IF pre continuous mapping, but not an IF completely γG continuous mapping.

Theorem 3.8: Every IF completely γG continuous mapping is an IFα continuous mapping [7] but not conversely in general.

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an IF completely γG continuous mapping . Let V be an IFCS in Y. Since every IFCS is an IFγGCS, V is an IFγGCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFαCS, $f^{-1}(V)$ is an IFαCS in X. Hence f is an IFα continuous mapping.

Example 3.9: In Example 3.3, f is an IFα continuous mapping, but not an IF completely γG continuous mapping.

Theorem 3.10: Every IF completely γG continuous mapping is an IFγG continuous mapping [9] but not conversely in general.

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an IF completely γG continuous mapping . Let V be an IFCS in Y. Since every IFCS is an IFγGCS, V is an IFγGCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IFγG continuous mapping.

Example 3.11: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$, $G_3 = \langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v)\rangle$ and $G_4 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, G_4, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v.
IFγO(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] /$ either $\nu_a < 0.5$ or $\nu_b < 0.3$, $\nu_a \geq 0.6$ whenever $\nu_b \geq 0.6$, $\nu_a = 0.5$ whenever $\nu_b > 0.3$, $0 \leq \mu_{a+}\nu_a \leq 1$ and $0 \leq \mu_{b+}\nu_b \leq 1\}$.
IFγC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0, 1], \mu_b \in [0, 1], \nu_a \in [0,1], \nu_b \in [0,1] /$ either $\mu_a < 0.5$ or $\mu_b < 0.3$, $\mu_a \geq 0.6$ whenever $\mu_b \geq 0.6$, $\mu_a = 0.5$ whenever $\mu_b > 0.3$, $0 \leq \mu_{a+}\nu_a \leq 1$ and $0 \leq \mu_{b+}\nu_b \leq 1\}$.
IFγO(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0, 1], \mu_v \in [0, 1], \nu_u \in [0, 1], \nu_v \in [0, 1] / 0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1\}$
IFγC(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0, 1], \mu_v \in [0, 1], \nu_u \in [0, 1], \nu_v \in [0, 1] / 0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1\}$
Then f is an IFγG continuous mapping, but not an IF completely γG continuous mapping, since G_3^c is an IFγGCS in Y, but $f^{-1}(G_3^c) = \langle x, (0.8_a, 0.7_b), (0.2_a, 0.3_b)\rangle$ is not an IFRCS in X as $cl(int((f^{-1}(G_3^c)) = cl(G_1) = G_1^c \neq f^{-1}(G_3^c)$.

Theorem 3.12: Every IF completely γG continuous mapping is an IFγ continuous mapping [5] but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely γG continuous mapping. Let V be an IFCS in Y. Since every IFCS is an IFγGCS, V is an IFγGCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFγCS, $f^{-1}(V)$ is an IFγCS in X. Hence f is an IFγ continuous mapping.

Example 3.13: In Example 3.3, f is an IFγ continuous mapping, but not an IF completely γG continuous mapping.

Theorem 3.14: Every IF completely γG continuous mapping is an IFSP continuous mapping [12] but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely γG continuous mapping. Let V be an IFCS in Y. Since every IFCS is an IFγGCS, V is an IFγGCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFSPCS, $f^{-1}(V)$ is an IFSPCS in X. Hence f is an IFSP continuous mapping.

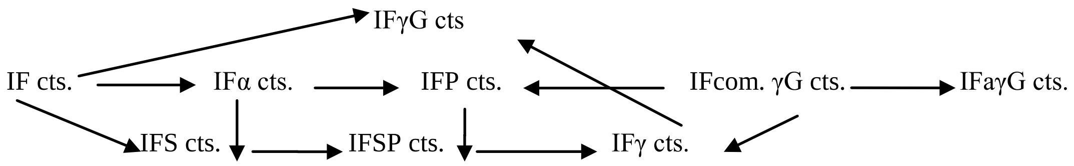
Example 3.15: In Example 3.3, f is an IFSP continuous mapping, but not an IF completely γG continuous mapping.

Theorem 3.16: Every IF completely γG continuous mapping is an IFaγG continuous mapping [10] but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely γG continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFγGCS, V is an IFγGCS in Y. Then $f^{-1}(V)$ is an IFRCS and hence is an IFγGCS in X. Thus f is an IFaγG continuous mapping.

Example 3.17: In Example 3.11, the mapping f is an IFaγG continuous mapping, but not an IF completely γG continuous mapping.

The relation between various types of intuitionistic fuzzy continuity is given in the following diagram. In this diagram 'cts.' means continuous and IFcom. γG cts. means IF completely γG continuous.



The reverse implications are not true in general in the above diagram.

Theorem 3.18: If f: (X, τ) → (Y, σ) is an IF completely γG continuous mapping then $\gamma cl(f^{-1}(A)) \subseteq f^{-1}(cl(A))$ for every IFγOS $A \subseteq Y$.

Proof: Let A be an IFγOS in Y. Then cl(A) is an IFRCS in Y. Hence cl(A) is an IFγGCS in Y. By hypothesis, $f^{-1}(cl(A))$ IFRCS in X and thus an IFγCS in X. Therefore $\gamma cl(f^{-1}(A)) \subseteq \gamma cl(f^{-1}(cl(A))) = f^{-1}(cl(A))$.

Theorem 3.19: Let f: (X, τ) → (Y, σ) be a mapping. Then the following are equivalent:

(i) f is an IF completely γG continuous mapping
(ii) $f^{-1}(V)$ is an IFROS in X for every IFγGOS V in Y
(iii) for every IFP $p_{(\alpha,\beta)} \in X$ and for every IFγGOS B in Y such that $f(p_{(\alpha,\beta)}) \in B$ there exists an IFROS in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$

Proof:
(i) ⇔ (ii): is obvious as $f^{-1}(A^c) = (f^{-1}(A))^c$.

(ii) ⇒ (iii): Let $p_{(\alpha,\beta)} \in X$ and $B \subseteq Y$ such that $f(p_{(\alpha,\beta)}) \in B$. This implies $p_{(\alpha,\beta)} \in f^{-1}(B)$. Since B is an IFγGOS in Y, by hypothesis $f^{-1}(B)$ is an IFROS in X. Let $A = f^{-1}(B)$. Then $p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \in f^{-1}(B) = A$. Therefore $p_{(\alpha,\beta)} \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$. This implies $f(A) \subseteq B$.

(iii) ⇒ (ii): Let $B \subseteq Y$ be an IFγGOS. Let $p_{(\alpha,\beta)} \in X$ and $f(p_{(\alpha,\beta)}) \in B$. By hypothesis, there exists an IFROS C in X such that $p_{(\alpha,\beta)} \in C$ and $f(C) \subseteq B$. This implies $C \subseteq f^{-1}(f(C)) \subseteq f^{-1}(B)$. Therefore $p_{(\alpha,\beta)} \in C \subseteq f^{-1}(B)$. That is

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$f^{-1}(B) = \bigcup_{p_{(\alpha,\beta)} \in f^{-1}(B)} p_{(\alpha,\beta)} \subseteq \bigcup_{p_{(\alpha,\beta)} \in f^{-1}(B)} C \subseteq f^{-1}(B)$. This implies $f^{-1}(B) = \bigcup_{p_{(\alpha,\beta)} \in f^{-1}(B)} C$. Since the union IFROSs is an IFROS, $f^{-1}(B)$ is an IFROS in X. Hence f is an IF completely γG continuous mapping.

Theorem 3.20: If a mapping f: (X, τ) → (Y, σ) is an IF completely γG continuous mapping then for every IFP $p_{(\alpha,\beta)} \in X$ and for every IFN [11] A of $f(p_{(\alpha,\beta)})$, there exists an IFROS $B \subseteq X$ such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$.

Proof: Let $p_{(\alpha,\beta)} \in X$ and let A be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFOS C in Y such that $f(p_{(\alpha,\beta)}) \in C \subseteq A$. Since every IFOS is an IFγGOS, C is an IFγGOS in Y. Hence by hypothesis, $f^{-1}(C)$ is an IFROS in X and $p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \subseteq f^{-1}(C) \subseteq f^{-1}(A)$ and therefore $p_{(\alpha,\beta)} \in f^{-1}(C)$. Now let $f^{-1}(C) = B$. Therefore $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$.

Theorem 3.21: A mapping f: (X, τ) → (Y, σ) is an IF completely γG continuous mapping then for every IFP $p_{(\alpha,\beta)} \in X$ and for every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFROS $B \subseteq X$ such that $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq A$.

Proof: Let $p_{(\alpha,\beta)} \in X$ and let A be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFOS C in Y such that $f(p_{(\alpha,\beta)}) \in C \subseteq A$. Since every IFOS is an IFγGOS, C is an IFγGOS in Y. Hence by hypothesis, $f^{-1}(C)$ is an IFROS in X and $p_{(\alpha,\beta)} \in f^{-1}(C)$. Now let $f^{-1}(C) = B$. Therefore $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$. Thus $f(B) \subseteq f(f^{-1}(A)) \subseteq A$. That is $f(B) \subseteq A$.

Theorem 3.22: A mapping f: (X, τ) → (Y, σ) is an IF completely γG continuous mapping then $int(cl(f^{-1}(int(B)))) \subseteq f^{-1}(B)$ for every IFS B in Y.

Proof: Let $B \subseteq Y$. Then int(B) is an IFOS in Y and hence an IFγGOS in Y. By hypothesis, $f^{-1}(int(B))$ is an IFROS in X. Hence $int(cl(f^{-1}(int(B)))) = f^{-1}(int(B)) \subseteq f^{-1}(B)$.

Theorem 3.23: For any two IF completely γG continuous functions f_1, f_2 : (X, τ) → (Y, σ), the function (f_1, f_2) : (X, τ) → (Y x Y, σ x σ) is also an IF completely γG continuous function where $(f_1, f_2)(x) = (f_1(x), f_2(x))$ for every x ∈ X.

Proof: Let A x B be an IFγGOS in Y x Y.

$$\begin{aligned} \text{Then } (f_1, f_2)^{-1}(A \times B)(x) &= (A \times B)(f_1(x), f_2(x)) \\ &= \langle x, \min(\mu_A(f_1(x)), \mu_B(f_2(x))), \max(\nu_A(f_1(x)), \nu_B(f_2(x)))\rangle \\ &= \langle x, \min(f_1^{-1}(\mu_A)(x), f_2^{-1}(\mu_B)(x)), \max(f_1^{-1}(\nu_A)(x), f_2^{-1}(\nu_B)(x))\rangle \\ &= (f_1^{-1}(A) \cap f_2^{-1}(B))(x). \end{aligned}$$

Since f_1 and f_2 are IF completely γG continuous functions, $f_1^{-1}(A)$ and $f_2^{-1}(B)$ are IFROSs in X. Since intersection of IFROSs is an IFROS, $f_1^{-1}(A) \cap f_2^{-1}(B)$ is an IFROS in X. Hence (f_1, f_2) is an IF completely γG continuous mappings.

Theorem 3.24: A mapping f: (X, τ) → (Y, σ) is an IF completely γG continuous mapping then the following are equivalent:

(i) For any IFγGOS A in Y and for any IFP $p_{(\alpha,\beta)} \in X$, if $f(p_{(\alpha,\beta)})\ _q\ A$ then $p_{(\alpha,\beta)}\ _q\ int(f^{-1}(A))$.

(ii) For any IFγGOS A in Y and for any IFP $p_{(\alpha,\beta)} \in X$, if $f(p_{(\alpha,\beta)})\ _q\ A$ then there exists an IFOS B such that $p_{(\alpha,\beta)}\ _q\ B$ and $f(B) \subseteq A$.

Proof:

(i) ⇒ (ii): Let $A \subseteq Y$ be an IFγGOS and let $p_{(\alpha,\beta)} \in X$. Let $f(p_{(\alpha,\beta)})\ _q\ A$. Then (i) implies that $p_{(\alpha,\beta)}\ _q\ int(f^{-1}(A))$, where $int(f^{-1}(A))$ is an IFOS in X. Let $B = int(f^{-1}(A))$. Since $int(f^{-1}(A)) \subseteq f^{-1}(A)$, $B \subseteq f^{-1}(A)$. Then $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

(ii) ⇒ (i): Let $A \subseteq Y$ be an IFγGOS and let $p_{(\alpha,\beta)} \in X$. Suppose $f(p_{(\alpha,\beta)})\ _q\ A$, then by (ii) there exists an IFOS B in X such that $p_{(\alpha,\beta)}\ _q\ B$ and $f(B) \subseteq A$. Now $B \subseteq f^{-1}(f(B)) \subseteq f^{-1}(A)$. That is $B = int(B) \subseteq int(f^{-1}(A))$. Therefore $p_{(\alpha,\beta)}\ _q\ B$ implies $p_{(\alpha,\beta)}\ _q\ int(f^{-1}(A))$.

Theorem 3.25: The composition of any two IF completely γG continuous mapping is an IF completely γG continuous mapping in general.

Proof: Let f: X→ Y and g: Y → Z be any two IF completely γG continuous mappings. Let B be an IFγGOS in Z. Since g is an IF completely γG continuous mapping, $g^{-1}(B)$ is an IFROS in Y. Since every IFROS is an IFγGOS, $g^{-1}(B)$ is an IFγGOS in Y. Since f is an IF completely γG continuous mapping, $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$ is an IFROS in X. Hence $g \circ f$ is an IF completely γG continuous mapping.

Theorem 3.26: Let f: X → Y and g: Y → Z be any two functions. Then

(i) g∘ f is an IF completely γG continuous mapping if f is an IF completely γG continuous mapping and g is an IFγG irresolute mapping.

(ii) g∘ f is an IFγG continuous mapping if f is an IF completely γG continuous mapping and g is an IFγG continuous mapping.

Proof:

(i) Let B be an IFγGOS in Z. Snce g is an IFγG irresolute mapping, $g^{-1}(B)$ is an IFγGOS in Y. Also, since f is an IF completely γG continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFROS in X. Since $(g\circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, g ∘ f is an IF completely γG continuous mapping.

(ii) Let B be an IFOS in Z. Since g is an IFγG continuous mapping, $g^{-1}(B)$ is an IFγGOS in Y. Also, since f is an IF completely γG continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFROS in X. Hence $f^{-1}(g^{-1}(B))$ is an IFγGOS in X. From the fact that $(g\circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, it follows that (g∘ f) is an IFγG continuous mapping.

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