Volume 9, No. 1, Jan. - 2018 (Special Issue)
International Journal of Mathematical Archive-9(1), 2018, 80-86
Available online through www.ijma.info ISSN 2229 – 5046

CONSIGNMENT AND VENDOR MANAGED INVENTORY IN SINGLE-VENDOR MULTIPLE BUYERS SUPPLY CHAIN WITH FUZZY DEMAND

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ABSTRACT

A policy for a single vendor with multiple buyers supply chain, as to be worked out based on a consignment (CS) and vendor managed inventory (VMI) policy. There are two types of partnerships between the vendor and buyers: independently acting vendor and buyers ii) the room to enter vendor managed inventory consignment partnership with the buyers by the vendor. Fuzzy mathematical model is also developed in which demand rate and production rate of the vendor are taken as trapezoidal fuzzy numbers. Modified graded mean integration representation method is used for defuzzifying the total cost. The benefits of the VMI&CS agreement can be studied by using (i) and (ii) relationship analytical and numerical results can also be provided. If the vendor has a capacity of flexibility, such an agreement will be more beneficial. Buyers find it more attractive while the significant order costs or the vendor's setup cost is not large. By such a method, the vendor will find it convenient in making more frequent shipments with smaller costs.

Keywords: *Consignment, Vendor Managed Inventory, supply chain, trapezoidal fuzzy number, Modified graded mean integration representation method.*

AMS Mathematics Subject Classification: *03E72, 90B05.*

1. INTRODUCTION

Blackstone and Cox (2008) define consignment as 'the process of a supplier placing goods at a customer location without receiving payment until after the goods are used or sold'. The main advantage of a consignment program is that the buyer does not have to tie its capital in the inventories and the vendor can have detailed access to the product stock levels and sales pattern at the buyer's site. Under VMI, the vendor is responsible for managing the inventory for the buyer, including initiating orders on behalf of the buyer. The vendor in return gets more visibility about the product's demand.

The vendor decides on both the order quantity and the number of shipments under both VMI and VMI&CS. However, under CS it is the buyer who assumes the responsibility of deciding about the order quantity and shipment frequency. The ordering cost is shared between the vendor and the buyer under VMI and VMI & CS programmes. The holding cost is the sole responsibility of the buyer under VMI but it is shared under both CS and VMI&CS. The nature of cost sharing will be specified later when we present the mathematical models.

In this paper, a policy for a single vendor with multiple buyers supply chain, as to be worked out based on a consignment (CS) and vendor managed inventory (VMI) policy. There are two types of partnerships between the vendor and buyers: independently acting vendor and buyers' ii) the room to enter vendor managed inventory consignment partnership with the buyers by the vendor. Fuzzy mathematical model is also developed in which demand rate and production rate of the vendor are taken as trapezoidal fuzzy numbers. Modified graded mean integration representation method is used for defuzzifying the total cost. The benefits of the VMI&CS agreement can

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International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

be studied by using (i) and (ii) relationship analytical and numerical results can also be provided. If the vendor has a capacity of flexibility, such an agreement will be more beneficial. Buyers find it more attractive while the significant order costs or the vendor's setup cost is not large. By such a method, the vendor will find it convenient in making more frequent shipments with smaller costs.

2. CONSIGNMENT MODELS

We discuss this model in following two cases.

Case-1: No partnership, vendor and buyers act independently, because their decisions are not coordinated.

Case-2: VMI & CS partnership; the vendor decides on the timing and quantities of the orders on behalf of the buyers and the vendor synchronizes the system such that these deliveries reach the buyers only when their existing stock gets exhausted. Prior to that, the product is stored at the vendor's site.

In all cases we analytically solve the optimal number of shipments and order quantities. We omit the convexity and optimality proves as we believe they are common in the related inventory literature.

Notations:

k_{vs}	Vendor's setup cost ($ per order)
k_{vri}	Vendor's shipment release cost to the i[th] buyer ($ per order)
A_{bpi}	The cost of placing an order by the i[th] buyer ($ per order)
A_{bri}	The cost of receiving a shipment by the i[th] buyer ($ per order)
A_{bi}	The i[th] buyer's total ordering cost ($A_{bi} = A_{bpi} + A_{bri}$) ($ per order)
h_{boi}	The i[th] buyer's opportunity cost of holding one unit in stock for one unit of time ($/unit/unit time)
h_{bsi}	The i[th] buyer's physical storage cost for mone unit of stock held for one unit of time ($/unit/unit time)
h_{bi}	The i[th] buyer's total holding cost per unit of stock per unit of time ($h_{bi} = h_{boi} + h_{bsi}$)
h_v	Vendor's total cost of holding one unit in stock for one unit of time ($/unit/ unit time)
N	Number of buyers
$\widetilde{d_i}$	Fuzzy demand from buyer i (units)
$\tilde{d}$	Total fuzzy demand rate of buyers (units) (i.e., $\tilde{d} = \sum_{i=1}^{N} \widetilde{d_i}$)
$\tilde{p}$	Fuzzy production rate for the vendor (units/unit time)
q_i	Shipment size for the buyer i
T	Replenishment cycle
$\widetilde{TC}_s$	Fuzzy total system cost ($ per year)
$P[\widetilde{TC}_s]$	Defuzzified total system cost of $\widetilde{TC}_s$ ($ per year)

Table-1: Share of cost and decisions in a supply chain under VMI, CS and VMI&CS inventory management programmes.

	Supply chain Structure			
	Independent Parties costs		VMI&CS	
SC Partner	Ordering	Holding	Ordering	Holding
Vender	k_{vs} k_{vri}	h_v	A_{bpi} k_{vs} k_{vri}	h_{boi} h_v
Buyer	A_{bi}	h_{bi}	A_{bri}	h_{bsi}

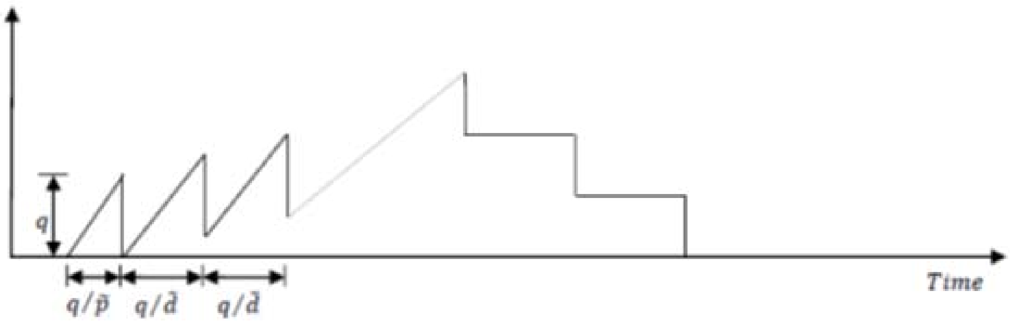

Figure-7.1: Inventory level for vendor

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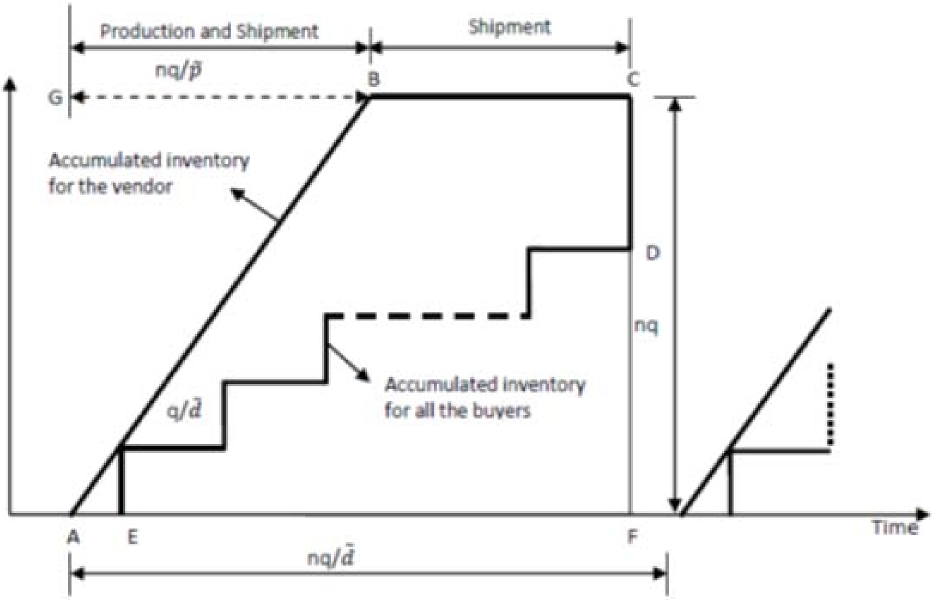

Figure-7.2: Inventory level for a system

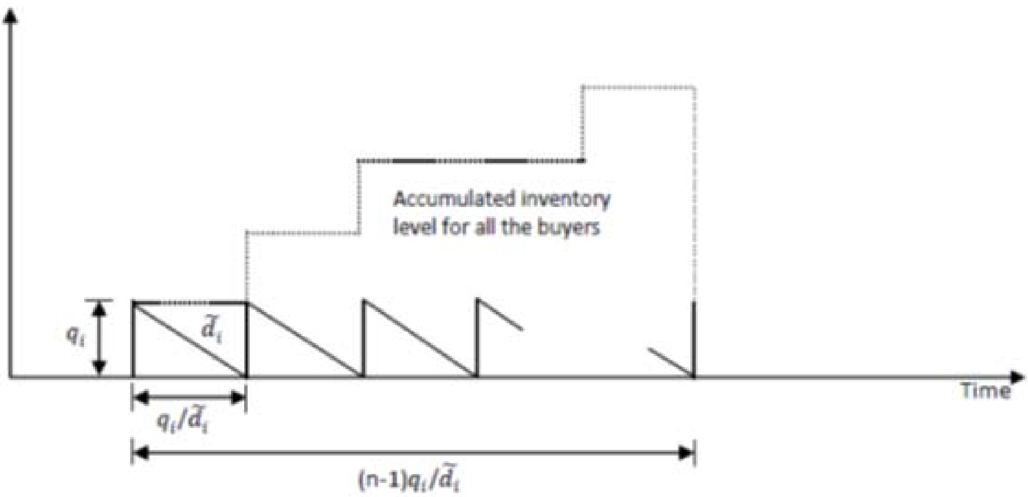

Figure-7.3: Inventory level for all the buyers i

2.1 No Partnership

In this case, there is no coordination between the vendor and buyers and all parties act independently and attempt to optimize their own cost without taking into consideration the decision of the other parties. It is optimal for them to operate according to their economic order quantity.

The total fuzzy holding cost for ith buyer is

$$\frac{1}{2}\sum_{i=1}^{N} h_{bi}\widetilde{d_i}T_i$$

Then the total fuzzy total cost incurred by the ith buyer is the sum of the total fuzzy holding cost and the fuzzy ordering cost:

$$\widetilde{TC_{bi}} = \frac{1}{2}\sum_{i=1}^{N} h_{bi}\widetilde{d_i}T_i + \sum_{i=1}^{N}\frac{A_{bi}}{T_i}$$

Let $\tilde{d} = (d_1, d_2, d_3, d_4)$, $\widetilde{d_i} = (d_{i_1}, d_{i_2}, d_{i_3}, d_{i_4})$ and $\tilde{p} = (p_1, p_2, p_3, p_4)$ be trapezoidal fuzzy numbers.

$$\widetilde{TC_{bi}} = \left[\left\{\frac{1}{2}\sum_{i=1}^{N} h_{bi}d_{i_1}T_i\right\}, \left\{\frac{1}{2}\sum_{i=1}^{N} h_{bi}d_{i_2}T_i\right\}, \left\{\frac{1}{2}\sum_{i=1}^{N} h_{bi}d_{i_3}T_i\right\}, \left\{\frac{1}{2}\sum_{i=1}^{N} h_{bi}d_{i_4}T_i\right\}\right] + \sum_{i=1}^{N}\frac{A_{bi}}{T_i}$$

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Applying Modified mean integration representation method to defuzzify the above fuzzy value, we get

$$P[\widetilde{TC_{bi}}] = \frac{1}{6}\left[\left\{\frac{1}{2}\sum_{i=1}^{N} h_{bi} d_{i_1} T_i\right\} + 2\left\{\frac{1}{2}\sum_{i=1}^{N} h_{bi} d_{i_2} T_i\right\} + 2\left\{\frac{1}{2}\sum_{i=1}^{N} h_{bi} d_{i_3} T_i\right\} + \left\{\frac{1}{2}\sum_{i=1}^{N} h_{bi} d_{i_4} T_i\right\}\right] + \sum_{i=1}^{N} \frac{A_{bi}}{T_i}$$

$$P[\widetilde{TC_{bi}}] = \frac{1}{12}\sum_{i=1}^{N} h_{bi} T_i (d_{i_1} + 2d_{i_2} + 2d_{i_3} + d_{i_4}) + \sum_{i=1}^{N} \frac{A_{bi}}{T_i} \qquad (1)$$

The buyers fuzzy total cost is taken as function of one decision variable T_i, whatever the value of T_i will be determined by equality $\frac{\partial (P[\widetilde{TC_{bi}}])}{\partial T_i} = 0$ gives the optimal replenishment cycle for the ith buyer is

$$T_i^* = \sum_{i=1}^{N} \sqrt{\frac{12A_{bi}}{h_{bi}(d_{i_1} + 2d_{i_2} + 2d_{i_3} + d_{i_4})}} \qquad (2)$$

Using (2) in (1), we get the optimal total cost incurred by the ith buyer:

$$\widetilde{TC}_{bi}^* = \sum_{i=1}^{N} \sqrt{\frac{1}{3} A_{bi} h_{bi} (d_{i_1} + 2d_{i_2} + 2d_{i_3} + d_{i_4})} \qquad (3)$$

The vendor receives orders from the buyers as per their respective optimal replenishment cycles. In this situation, it becomes difficult to determine vendor's cost. As explained in Chan and Kingsman [5], in order to avoid stockouts, the vendor should have an extra stock of $\sum_{i=1}^{r} q_i$ at the start of each production run which adds an additional fuzzy holding cost per cycle. Hence the vendor's fuzzy inventory holding cost at a time interval T_v will be

$$h_v\left[\left\{\frac{1}{2}\left(\frac{\tilde{p} - \tilde{d}}{\tilde{p}} \tilde{d} T_v\right)\right\} + \left\{\sum_{i=1}^{N} \tilde{d}_i T_i\right\}\right]$$

Then the vendor's fuzzy total cost is the sum of total setup cost, total holding cost (including safety stock)

$$\widetilde{TC}_v = \frac{k_{vs}}{T_v} + h_v\left[\left\{\frac{\tilde{d}T_v}{2}\left(1 - \frac{\tilde{d}}{\tilde{p}}\right)\right\} + \left\{\sum_{i=1}^{N} \tilde{d}_i T_i\right\}\right] + \sum_{i=1}^{N} \frac{k_{vri}}{T_i}$$

$$\widetilde{TC}_v = \frac{k_{vs}}{T_v} + \sum_{i=1}^{N} \frac{k_{vri}}{T_i} + h_v\left[\left[\left\{\frac{d_1 T_v}{2}\left(1 - \frac{d_4}{p_1}\right)\right\} + \left\{\sum_{i=1}^{N} d_{i_1} T_i\right\}\right], \left[\left\{\frac{d_2 T_v}{2}\left(1 - \frac{d_3}{p_2}\right)\right\} + \left\{\sum_{i=1}^{N} d_{i_2} T_i\right\}\right], \left[\left\{\frac{d_3 T_v}{2}\left(1 - \frac{d_2}{p_3}\right)\right\} +\right.\right.$$

i=1Ndi3Ti,d4Tv21−d1p4+i=1Ndi4Ti

$$P[\widetilde{TC}_v] = \frac{k_{vs}}{T_v} + \sum_{i=1}^{N} \frac{k_{vri}}{T_i} + \frac{h_v}{6}\left\{\frac{T_v}{2}\left[d_1\left(1 - \frac{d_4}{p_1}\right) + 2d_2\left(1 - \frac{d_3}{p_2}\right) + 2d_3\left(1 - \frac{d_2}{p_3}\right) + d_4\left(1 - \frac{d_1}{p_4}\right)\right] + \sum_{i=1}^{N}(d_{i_1} + 2d_{i_2} +\right.$$

2di3+di4)Ti

(4)

The vendors' fuzzy total cost is taken as a function of one decision variable T_v will be determine by equating $\frac{\partial [\widetilde{TC}_v]}{\partial T_v} = 0$ which gives optimal time interval is given by:

$$T_v^* = \sqrt{\frac{12k_{vs}}{h_v\left[d_1\left(1-\frac{d_4}{p_1}\right)+2d_2\left(1-\frac{d_3}{p_2}\right)+2d_3\left(1-\frac{d_2}{p_3}\right)+d_4\left(1-\frac{d_1}{p_4}\right)\right]}} \qquad (5)$$

Using (5) in (4), we get the vendor's optimal total cost:

$$\widetilde{TC}_v^* = \sqrt{\frac{k_{vs} h_v\left[d_1\left(1-\frac{d_4}{p_1}\right)+2d_2\left(1-\frac{d_3}{p_2}\right)+2d_3\left(1-\frac{d_2}{p_3}\right)+d_4\left(1-\frac{d_1}{p_4}\right)\right]}{3}} + \sum_{i=1}^{N} \frac{k_{vri}}{T_i} + \frac{h_v}{6}\sum_{i=1}^{N}(d_{i_1} + 2d_{i_2} + 2d_{i_3} + d_{i_4})T_i \qquad (6)$$

Thus, the fuzzy total system cost is the sum of vendor fuzzy total cost and buyer's fuzzy cost

$$\widetilde{TC}_s = \widetilde{TC}_v^* + \widetilde{TC}_{ri}^* \qquad (7)$$

2.2 VMI & CS Partnership

In this partnership, the vendor decides on the timing and quantity of the orders on behalf of the buyers and the vendor synchronizes the system such that these deliveries reach the buyers only when their existing stock gets exhausted. Prior to that, the product is stored at the vendor's site.

In Fig. 3, the total area under the curve can be written as a sum of area of triangles for all the buyers in the system for all n delivery cycles in a given replenishment cycle length T as

$$\sum_{i=1}^{N}(n\frac{1}{2}q_i\frac{q_i}{\tilde{d}_i})$$

Noting that $nq_i = \tilde{d}_i T$, the average inventory will be

$$\frac{T}{2n}\sum_{i=1}^{N}\tilde{d}_i h_{bsi}$$

Then the total cost of the buyers can be written as:

$$\widetilde{TC_{bi}} = \sum_{i=1}^{N}\frac{nA_{bri}}{T} + \frac{T}{2n}\sum_{i=1}^{N}\tilde{d}_i h_{bsi}$$

The first term represents the buyers ordering and receiving a shipment cost. The next term is a buyer holding for a physical storage cost. Then,

$$P[\widetilde{TC_{bi}}] = \sum_{i=1}^{N}\frac{nA_{bri}}{T} + \frac{T}{12n}\sum_{i=1}^{N}h_{bsi}(d_{i_1} + 2d_{i_2} + 2d_{i_3} + d_{i_4}) \quad (8)$$

In Fig. 2, the total inventory for the vendor is given by the area ABCDEA. Pan and Yang (2002) to developed the similar approach as follows:

Area (ABCDEA) = Area (AGCFA) – Area (AGBA) – Area (EDFE)

In Fig. 2, the inventory accumulation across all the buyers happens for $(n-1)$ sub-cycles and for producing the first sub-batch, the time taken will be $q/\tilde{p}$ and the side CF is nq is equal to the side AG for a rectangle AGCF.

Area (AGCFA) = $nq\left(\frac{q}{\tilde{p}} + (n-1)\frac{q}{\tilde{d}}\right)$

Area (AGBA) = $\frac{1}{2}\, nq\frac{nq}{\tilde{p}}$

Area (EDFE) = $q\frac{q}{\tilde{d}}\left(1 + 2 + \cdots (n-1)\right)$

Then, the area ABCDEA will be

$$= \left\{nq\left(\frac{q}{\tilde{p}} + (n-1)\frac{q}{\tilde{d}}\right)\right\} - \left\{\frac{1}{2}\, nq\frac{nq}{\tilde{p}}\right\} - \left\{q\frac{q}{\tilde{d}}\left(1 + 2 + \cdots (n-1)\right)\right\}$$

Average inventory at the vendor can be written as:

$$= \frac{1}{(nq/\tilde{d})}\left[\left\{nq\left(\frac{q}{\tilde{p}} + (n-1)\frac{q}{\tilde{d}}\right)\right\} - \left\{\frac{1}{2}\, nq\frac{nq}{\tilde{p}}\right\} - \left\{q\frac{q}{\tilde{d}}\left(1 + 2 + \cdots (n-1)\right)\right\}\right]$$

$$= \frac{q}{2}\left[n\left(1 - \frac{\tilde{d}}{\tilde{p}}\right) - 1 + \frac{2\tilde{d}}{\tilde{p}}\right]$$

The total inventory holding cost for the vendor can be written as:

$$= \frac{qh_v}{2}\left[n\left(1 - \frac{\tilde{d}}{\tilde{p}}\right) - 1 + \frac{2\tilde{d}}{\tilde{p}}\right]$$

Using $T = nq/\tilde{d}$, then the total inventory holding cost for the vendor is:

$$\frac{\tilde{d}Th_v}{2n}\left[n\left(1 - \frac{\tilde{d}}{\tilde{p}}\right) - 1 + \frac{2\tilde{d}}{\tilde{p}}\right]$$

Then the total vendor's cost is:

$$\widetilde{TC_v} = \frac{k_{vs} + n\sum_{i=1}^{N}k_{vri}}{T} + \frac{h_v T}{2n}\tilde{d}\left[n\left(1 - \frac{\tilde{d}}{\tilde{p}}\right) - 1 + \frac{2\tilde{d}}{\tilde{p}}\right] + \sum_{i=1}^{N}\frac{nA_{bpi}}{T} + \frac{T}{2n}\sum_{i=1}^{N}\tilde{d}_i h_{boi}$$

The first term represents the vendor's setup and shipment release cost. The second term is the vendor's inventory holding cost. The last two terms are the extra cost components charged to the vendor as a result of the partnership. Then the vendor's cost function can be rewritten in fuzzy demand as follows:

Let $\tilde{d} = (d_1, d_2, d_3, d_4)$, $\tilde{d}_i = (d_{i_1}, d_{i_2}, d_{i_3}, d_{i_4})$ and $\tilde{p} = (p_1, p_2, p_3, p_4)$ be trapezoidal fuzzy numbers.

$$\widetilde{TC_v} = \frac{k_{vs} + n\sum_{i=1}^{N}(k_{vri} + A_{bpi})}{T} + \frac{T}{2n}\left[\left\{h_v d_1\left[n\left(1 - \frac{d_4}{p_1}\right) - 1 + \frac{2d_1}{p_4}\right] + \sum_{i=1}^{N}d_{i_1}h_{boi}\right\}, \left\{h_v d_2\left[n\left(1 - \frac{d_3}{p_2}\right) - 1 + \frac{2d_2}{p_3}\right] + \right.\right.$$

i=1Ndi2hboi,hvd3n1− d2p3−1+2d3p2+i=1Ndi3hboi,hvd4n1− d1p4−1+2d4p1+i=1Ndi4hboi

$P[\widetilde{TC}_v] = \frac{k_{vs}+n\sum_{i=1}^{N}(k_{vri}+A_{bpi})}{T} + \frac{T}{2n}\frac{1}{6}\Big[\Big\{h_v d_1\left[n\left(1-\frac{d_4}{p_1}\right)-1+\frac{2d_1}{p_4}\right]+\sum_{i=1}^{N} d_{i_1}h_{boi}\Big\}+2\Big\{h_v d_2\left[n\left(1-\frac{d_3}{p_2}\right)-1+\right.$
2d2p3+i=1Ndi2hboi+2hvd3n1− d2p3− 1+2d3p2+i=1Ndi3hboi+hvd4n1− d1p4− 1+2d4p1+ i=1Ndi4hboi (9)

The vendors' fuzzy total cost is taken as a function of one decision variable T_v will be determine by equating $\frac{\partial[\widetilde{TC}_v]}{\partial T}=0$ which gives optimal cycle length is given by:

$$T^* = \sqrt{\frac{12n\left[k_{vs}+n\sum_{i=1}^{N}(k_{vri}+A_{bpi})\right]}{h_v\left[d_1\left[n\left(1-\frac{d_4}{p_1}\right)-1+\frac{2d_1}{p_4}\right]+2d_2\left[n\left(1-\frac{d_3}{p_2}\right)-1+\frac{2d_2}{p_3}\right]+2d_3\left[n\left(1-\frac{d_2}{p_3}\right)-1+\frac{2d_3}{p_2}\right]+d_4\left[n\left(1-\frac{d_1}{p_4}\right)-1+\frac{2d_4}{p_1}\right]\right] + \sum_{i=1}^{N} h_{boi}(d_{i_1}+2d_{i_2}+2d_{i_3}+d_{i_4})}} \tag{10}$$

Substituting (10) in (9), we obtain an expression for the vendor's cost as a function of the number of shipments:

$$P[\widetilde{TC}_v(n)] = \sqrt{\frac{k_{vs}+n\sum_{i=1}^{N}(k_{vri}+A_{bpi})}{3n}\left\{h_v\begin{bmatrix} d_1\left[n\left(1-\frac{d_4}{p_1}\right)-1+\frac{2d_1}{p_4}\right]+2d_2\left[n\left(1-\frac{d_3}{p_2}\right)-1+\frac{2d_2}{p_3}\right] \\ +2d_3\left[n\left(1-\frac{d_2}{p_3}\right)-1+\frac{2d_3}{p_2}\right]+d_4\left[n\left(1-\frac{d_1}{p_4}\right)-1+\frac{2d_4}{p_1}\right]\end{bmatrix} + \sum_{i=1}^{N} h_{boi}\,(d_{i_1}+2d_{i_2}+2d_{i_3}+d_{i_4})\right\}}$$

Minimizing the above cost function is equivalent to minimize

$$\frac{k_{vs}}{n}\left\{h_v\left[d_1\left(\frac{2d_1}{p_4}-1\right)+2d_2\left(\frac{2d_2}{p_3}-1\right)+2d_3\left(\frac{2d_3}{p_2}-1\right)+d_4\left(\frac{2d_4}{p_1}-1\right)\right]+\sum_{i=1}^{N} h_{boi}\,(d_{i_1}+2d_{i_2}+2d_{i_3}+d_{i_4})\right\}$$
$$+nh_v\sum_{i=1}^{N}(k_{vri}+A_{bpi})\left[d_1\left(1-\frac{d_4}{p_1}\right)+2d_2\left(1-\frac{d_3}{p_2}\right)+2d_3\left(1-\frac{d_2}{p_3}\right)+d_4\left(1-\frac{d_1}{p_4}\right)\right] \tag{11}$$

Applying the first difference approach to (11), it can be shown that the optimal number of shipments to be sent to each retailers belong to the interval $\big(0.5(-1+\alpha), 0.5(1+\alpha)\big)$, where

$$\alpha^2 = 1+4\frac{\left\{h_v\left[d_1\left(\frac{2d_1}{p_4}-1\right)+2d_2\left(\frac{2d_2}{p_3}-1\right)+2d_3\left(\frac{2d_3}{p_2}-1\right)+d_4\left(\frac{2d_4}{p_1}-1\right)\right]+\sum_{i=1}^{N} h_{boi}(d_{i_1}+2d_{i_2}+2d_{i_3}+d_{i_4})\right\}}{h_v\sum_{i=1}^{N}(k_{vri}+A_{bpi})\left[d_1\left(1-\frac{d_4}{p_1}\right)+2d_2\left(1-\frac{d_3}{p_2}\right)+2d_3\left(1-\frac{d_2}{p_3}\right)+d_4\left(1-\frac{d_1}{p_4}\right)\right]}$$

Given that there is only one integer belonging to this interval, it follows that

$$n^* = \left\lceil 0.5\left(-1+\sqrt{1+4\frac{\left\{h_v\left[d_1\left(\frac{2d_1}{p_4}-1\right)+2d_2\left(\frac{2d_2}{p_3}-1\right)+2d_3\left(\frac{2d_3}{p_2}-1\right)+d_4\left(\frac{2d_4}{p_1}-1\right)\right]+\sum_{i=1}^{N} h_{boi}(d_{i_1}+2d_{i_2}+2d_{i_3}+d_{i_4})\right\}}{h_v\sum_{i=1}^{N}(k_{vri}+A_{bpi})\left[d_1\left(1-\frac{d_4}{p_1}\right)+2d_2\left(1-\frac{d_3}{p_2}\right)+2d_3\left(1-\frac{d_2}{p_3}\right)+d_4\left(1-\frac{d_1}{p_4}\right)\right]}}\right)\right\rceil \tag{12}$$

3. NUMERICAL EXAMPLE

3.1 (no partnership): Consider with one vendor and two buyers, the details given below:
$\tilde{p} = (3100, 3150, 3250, 3300)$ item/year, $\tilde{d} = (1400, 1450, 1550, 1600)$ item/year, $\widetilde{d_1} = (480.500,500,520)$ item/year, $\widetilde{d_2} = (940,970,1030,1060)$ item/year, $k_{vs} = \$400$ per setup, $k_{vri} = 0$ per shipment, $A_{b1} = \$25$ per order, $A_{b2} = \$75$ per order, $h_{b1} = h_{b2} = \$5$ per item per year and $h_v = \$4$ per item per year.

Solution:

The cycle times are $T_v = 0.501$, $T_1 = 0.141$, $T_2 = 0.173$.
The costs are $TC_v = 2573.76$, $TC_{b1} = 353.55$, $TC_{b2} = 866.03$ and the total cost system $TC_s = 3793.336$

3.2 (efficient partnership): Consider the same data as Example 3.1 and $A_{bp1} = \$15$ per order, $A_{bp2} = \$50$ per order, $h_{bo1} = \$2.5$ per item per year and $h_{bo2} = \$2$

Solution: The costs are $TC_v = 2133.87$, $TC_{b1} = 169.98$, $TC_{b2} = 413.32$ and the total cost system $TC_s = 2717.167$

3.3 (potentially efficient partnership for vendor): Consider the same data as Example 3.1 and $A_{bp1} = \$20$ per order, $A_{bp2} = \$65$ per order, $h_{bo1} = \$4.5$ per item per year and $h_{bo2} = \$4.5$

Solution: The costs are $TC_v = 2502.19$, $TC_{b1} = 52.30$, $TC_{b2} = 104.59$ and the total cost system $TC_s = 2659.08$

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

Table-2: Summary of models results

	n^*	T^*	Vendor's cost TC_v	Buyer 1 cost TC_{b1}	Buyer 2 cost TC_{b2}	Total cost TC_s	Savings (%)
Independent	-	0.500596	2573.757	353.5534	866.0254	3793.336	
VMI &CS	3	0.557673	2133.867	169.9769	413.3239	2717.167	28.36
Ben-Daya VMI&CS[1]	3	0.560605	2122.707	198.045	519.2000	2839.952	25.11

These results illustrate the benefit of co-ordination between the vendor and buyers will some minor advantage to these VMI&CS.

4. CONCLUSION

In this paper, VMI&CS agreement is more beneficial when the vendor has a flexible capacity. It is also more attractive to buyers when they have significant order costs and the vendor's setup cost is not large. The SC cost savings under VMI&CS partnership over the independent case range from 4% to 29% and the buyers can achieve savings of more than 50%. Modified graded mean integration representation method is used for defuzzifying the total cost.

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Source of support: Proceedings of UGC Funded International Conference on Intuitionistic Fuzzy Sets and Systems (ICIFSS-2018), Organized by: Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

