Volume 9, No. 1, Jan. - 2018 (Special Issue)
International Journal of Mathematical Archive-9(1), 2018, 54-57
Available online through www.ijma.info ISSN 2229 – 5046

DOMINATION DOT-CRITICAL FUZZY GRAPH

[1]R. JAHIR HUSSAIN & [2]S. AMEENA BANU

[1,2]PG and Research Department of Mathematics,
Jamal Mohamed College (Autonomous), Tiruchirappalli–620020, India.

E-mail: [1]hssn_jhr@yahoo.com & [2]banuameena3@gmail.com

ABSTRACT

In general $\gamma_f(G)$ can be made to increase or decrease by the removal of nodes from G. In this paper we discussed the edge contraction (or identifying any two strong adjacent vertices) and domination dot critical fuzzy graph. A fuzzy graph is said to be domination dot critical if contracting any strong edge or identifying any two strong adjacent vertices decreases the fuzzy domination number.

Key words: *Fuzzy dominating set, fuzzy domination number, Edge contraction and Domination dot- critical fuzzy graph.*

AMS Mathematics Subject Classification (2010): *03E72, 05C72.*

I. INTRODUCTION

Rosenfeld[7] introduced the notion of fuzzy graph and several fuzzy analogous of graph theoretic concepts such as paths, cycles, connectedness and etc. Somasundaram. A and Somasundaram. S [8] discussed domination in fuzzy graphs. Burton. T and Sumner. D [1] discussed domination dot critical in graphs. In this paper we investigate the changes in the fuzzy cardinality of fuzzy dominating set when we identifying any two strong adjacent nodes.

II. PRELIMINARIES

A *fuzzy graph* G=(σ,μ) is a non-empty set V together with a pair of functions $\sigma: V \rightarrow [0,1]$ and μ:VxV→[0,1] such that μ(u,v) $\leq \sigma(u) \wedge \sigma(v)$ for all u,v $\in V$, where $\sigma(u) \wedge \sigma(v)$ is the minimum of $\sigma(u)$ *and* $\sigma(v)$.The *underlying crisp graph* of the fuzzy graph G=(σ,μ) is denoted as $G^*=(\sigma^*, \mu^*)$ where σ^*={u∈ $V/\ \sigma(u) > 0$} and μ^*= {(u,v)∈VxV / μ(u,v)>0}. Let G =(σ,μ) be a fuzzy graph and τ be any fuzzy subset of σ,i.e $\tau(u) \leq \sigma(u)$for all u,. Then the fuzzy subgraph of G = (σ,μ) induced by τ is the maximal fuzzy sub graph of G = (σ,μ) that has fuzzy node set τ. Evidently this is just the fuzzy graph (τ,ρ), where ρ(u,v) = $\tau(u) \wedge \tau(v)$for all u, v ∈ V.The complement of a fuzzy graph G=(σ,μ) is a fuzzy graph $\bar{G}$ =$(\bar{\sigma},\bar{\mu})$ where $\bar{\sigma}$= σ and $\bar{\mu}$(u,v) = σ(u)∧ σ(v)- μ(u,v) for all u,v in V.

Two nodes u and v are said to be *neighbours* if μ(u, v) > 0.The *strong neighbourhood* of u is $N_S(u)$ = {v ∈ V: (u, v) is a strong arc}. $N_S[u]$ = $N_S(u)$ ∪ {u} is the *closed strong neighbourhood* of u. Let G=(σ,μ) be a fuzzy graph. Two nodes u and v of G are *strong adjacent* if (u, v) is strong arc. The *strong degree* of a node v is the minimum number of nodes that are strong adjacent to v. It is denoted by $d_S(v)$. The minimum cardinality of strong neighbourhood $\delta_S(G)$ = min{$|N_S(u)|$:u∈ $V(G)$} and the maximum cardinality of strong neighbourhood Δ_S (G) = max {$|N_S(u)|$: u∈ $V(G)$}. Let G be a fuzzy graph. Let S be a set of vertices in G. Let u∈S then the *private neighbourhood* of u is pn(u, S) = {v: $N_S(v)$∩S = {u}}. The *external private neighbourhood* epn(v, S) = pn(u, S) \ S. A node u is called a *fuzzy end node* of G = (σ,μ) if it has atmost one strong neighbour in G = (σ,μ).

A *path* ρ in a fuzzy graph is a sequence of distinct nodes $u_0,u_1,u_2,...u_n$ such that $\mu(u_{i-1},u_i)>0$; $1\leq i \leq n$ here n≥ 0 is called the *length* of the path ρ .The consecutive pairs (u_{i-1},u_i) are called the *arcs* of the path. A path ρ is called a *cycle* if $u_o=u_n$ and n≥3. An arc (u,v) is said to be a *strong arc* if μ(u,v) ≥ μ^∞ (u,v) and the node v is said to be a *strong neighbour* of u. If μ(u,v) = 0 for every v∈V then u is called *isolated node*. Two nodes that are joined by a path are said to be *connected*. The relation connected is a reflexive, symmetric and transitive. The equivalence classes of nodes

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018 on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

under this relation are the *connected components* of the given fuzzy graph. Let G= (σ,μ) be a fuzzy graph and u be a node in G then there exist a node v such that (u, v) is a strong arc then we say that *u dominates v*. Let G=(σ,μ) be a fuzzy graph. A set D of V is said to be *fuzzy dominating set* of G if every v ∈ V-D there exist u ∈ D such that u dominates v. A fuzzy dominating set D of G is called a *minimal fuzzy dominating set* of G if no proper subset of D is a fuzzy dominating set. The *fuzzy domination number* γ_f(G) of the fuzzy graph G is the smallest number of nodes in any fuzzy dominating set of G. A fuzzy dominating set D of a fuzzy graph G such that $|D| = \gamma_f$(G) is called minimum fuzzy dominating set.

III. FUZZY DOMINATING CRITICAL NODES

Definition 3.1: Let G = (σ,μ) be a fuzzy graph. A node v of G is said to be *fuzzy dominating critical node* if its removal either increases (or) decreases the fuzzy domination number.

We partition the nodes of G into three disjoint sets according to how their removal affects γ_f (G).

Let V= $V_f^0 \cup V_f^+ \cup V_f^-$ for

V_f^0= {v∈V: γ_f (G-v) = γ_f (G)}

V_f^+ = {v∈V: γ_f (G-v) $>$ γ_f (G)}

V_f^- = {v∈V: γ_f (G-v) $<$ γ_f(G)}.

We denote the set of critical nodes of G by G'.

Definition 3.2: A vertex v of G is γ_f- *fixed* if v belongs to every γ_f-set of G. If every vertex of G is γ_f- *fixed* then G is γ_f- fixed.

A vertex v of G is γ_f-*free* if v belongs to some γ_f-set but not all γ_f-sets. If every vertex of G is γ_f- *free* then G is γ_f- free.

Definition 3.3: A fuzzy graph G is said to be *critically dominated* if its vertices forms a dominating set.

Definition 3.4: Let G = (σ,μ) be a fuzzy graph whose underlying crisp graph is G^*= (σ^*, μ^*). Let uv be an edge in G, then the *edge contraction or identifying any two strong adjacent vertices* of fuzzy graph G with respect to the edge uv is denoted by G.uv = ($\sigma_{G.uv}$, $\mu_{G.uv}$) where $\sigma_{G.uv}$ is a fuzzy set on V'= V/{u, v}∪{w} and is defined as $\sigma_{G.uv}$(x) = σ_G(x) ∀ x∈V and

$\sigma_{G.uv}$(w) = min {$\sigma_{G.uv}$(u), $\sigma_{G.uv}$(v)

$\mu_{G.uv}$ is defined as if two vertices x and y are adjacent in G.uv if any one of the following conditions hold.

(i) If x,y ∈ V and x, y are adjacent in G then $\mu_{G.uv}$(xy) = μ_G(xy).
(ii) If x∈ V and y=w then x and y are adjacent in G.uv if either x is adjacent to u or v in G and $\mu_{G.uv}$(xy) = min {$\sigma_{G.uv}$(x), $\sigma_{G.uv}$(w)
(iii) If x∈ V and y=w then x and y are adjacent in G.uv if x is adjacent to both u and v in G and $\mu_{G.uv}$(xy) = min {$\mu_{G.uv}$(xu), $\mu_{G.uv}$(xv).

Theorem 3.5[4]: For any fuzzy graph G = (σ,μ) if V_f^- = {v} then A*(V) = ∅. Here A*(V) = {u:u∉D and N_S(u)∩D = {v}}

Theorem 3.6[6]: Let G = (σ,μ) be a fuzzy graph then (i) Every node v is in V_f^- belongs to a γ_f-set of G (ii) Every node is in V_f^+ is γ_f- *fixed.*

Theorem 3.7 [6]: Let G be a fuzzy graph. A node v is inV_f^+ if and only if

(i) v is not an isolate in G and v is γ_f- *fixed.*
(ii) There is no dominating set for G-N_S[v] with domination number $\gamma_f(G)$ which also dominates N_S(v).

Theorem 3.8[6]: Let G be a fuzzy graph. A node v is in V_f^+ then for every γ_f-set of D of G, v∈D and pn[v,D] contains atleast one node.

Theorem 3.9[6]: Let G= (σ,μ) be a fuzzy graph. A node x is in V_f^+ and y is in V_f^- then (x,y) is not a strong arc.

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

IV. DOMINATION DOT- CRITICAL FUZZY GRAPH

Definition 4.1: A fuzzy graph G is said to be *domination dot – critical* if contracting any strong edge or identifying any two strong adjacent vertices decreases the fuzzy domination number.*(i.e)* γ_f (G.uv) $<$ γ_f(G) for any two strong adjacent vertices.

Definition 4.2: A fuzzy graph G is said to be *total domination dot – critical* if identifying any two strong vertices decreases the fuzzy domination number.(i.e) γ_f (G.uv) $<$ γ_f(G) for any two strong vertices.

Definition 4.3: A fuzzy graph G is *point-distinguishing* if every two distinct vertices have distinct strong closed neighbourhoods.

Theorem 4.4: Let u,v $\in V$ for a fuzzy graph G. Then γ_f (G.uv) $<$ γ_f(G) if and only if either there exists a γ_f-set D of G such that u,v $\in D$ or at least one of u or v is critical in G.

Proof: Let u,v $\in V$ such that γ_f (G.uv) $<$ γ_f(G). Let D be γ_f-set of G.uv. If (uv) $\in D$, then D* = D / {uv}∪{u,v} is a γ_f-set of G containing u and v. If (uv) $\notin D$, then there exists a t $\in D$ such that t strong adjacent to (uv).

If t ∈ N_S(u)∩ N_S(v), then D dominates G which contradicts that γ_f (G.uv) $<$ γ_f(G). Thus t is strong adjacent to exactly one of u or v in G say t strong adjacent to u. Then D dominates G-V which implies that v∈G'.

Conversely if u,v belong to a common γ_f-set of G, then D* = D/{u,v}∪{uv} is a dominating set of G.uv of fuzzy cardinality γ_f(G) – 1. On the other hand, if u∈G', then any γ_f-set of G-u is a dominating set of G. uv of fuzzy cardinality γ_f(G) – 1. Hence γ_f (G.uv) $<$ γ_f(G).

Lemma 4.5: If G is any fuzzy graph with γ_f(G) ≥ 2, then G is domination dot-critical if and only if every two strong adjacent non-critical vertices belong to a common γ_f-set.

Theorem 4.6: Let G be any fuzzy graph and v∈G'. Then all of N_S[v] is γ_f- *free.*

Proof: Let D be a γ_f-set of G-v and u∈ N_S(v). Then D ∪{v} and D ∪{u}are both γ_f-set of G.

Theorem 4.7: For any fuzzy graph G,
(i) If G is domination dot-critical , then G is γ_f- *free.*(ii) If G is critically dominated , then G is γ_f- *free.*

Proof:

(i) Let v∈ V and u any strong neighbour of v.Then if v or u is critical, then v is γ_f- *fixed* by theorem 4.4. Otherwise there is a γ_f-set that contains both v and u and again v is γ_f- *fixed.*
(ii) This is immediate from theorem 4.4.

Lemma 4.8: G is dot-critical (resp. totally dot-critical) if and only if each of its components is dot-critical (resp. totally dot-critical)

Proof: This lemma is clear for dot-critical fuzzy graphs, and it is also apparent that every component of a totally dot-critical fuzzy graph is also totally dot-critical. So suppose that each component of G is totally dot-critical. We argue that G itself is totally dot-critical.

Clearly, identifying any two strong vertices in the same component of G decreases γ_f(G).So suppose that x, y belong to separate components of G. Then by theorem 4.7 we may select an γ_f-set S_1 of the component containing x so that x∈ S_1 and an γ_f-set S_2 of the component containing y so that y∈ S_2. Let S be an γ_f-set for the remaining components, if any. Then S* = S_1∪ S_2∪S is an γ_f-set that contains both x and y.

Theorem 4.9: If u, v $\in V$ for a fuzzy graph G such that N_S[v] = N_S[u], then γ_f (G.uv) $=$ γ_f(G).

Proof: Suppose that γ_f (G.uv) $<$ γ_f(G). Then by lemma 4.3 either one of u,v ∈G' or there is a γ_f-set D of G containing u and v. Suppose that v∈G', and let D_v be a γ_f-set of D-v of fuzzy cardinality γ_f(G) – 1.

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

Then any vertex of D_v that is strong adjacent to u in G is also strong adjacent to v in G. This implies that D_v dominates G. Thus, neither v nor u is critical in G. if D is a dominating set of G with v, u both in D, then D-u is also dominating set of G.
Hence no γ_f-set of G may contain both v and u and so γ_f (G.uv) = γ_f(G).

Lemma 4.10: For any fuzzy graph G, v∈G'if and only if some γ_f-set S of G in which v is needed only to dominate itself.

Theorem 4.11: Let G = (σ,μ) be a fuzzy graph with γ_f(G) = 2. Then the critical vertices of G are precisely those, which are strong adjacent to a fuzzy end node in $\bar{G}$.

Proof: Let x∈V such that x is a fuzzy end node in $\bar{G}$. If x is adjacent to y in $\bar{G}$, then x dominates G-y, and hence y∈G'.On the other hand, suppose that y∈G' and that x dominates G-y, Then in $\bar{G}$, x is a fuzzy end node strong adjacent to y.

Lemma 4.12: If G is a dot-critical fuzzy graph and $N_S[v] \subseteq N_S[u]$, then v∈G'.

Proof: In this case, G. uv≅ G-v, and so γ_f (G-v) = γ_f (G.uv) = γ_f(G)-1.

Corollary 4.13: Every fuzzy end node of a dot-critical graph is a critical node.

REFERENCES

1. Burton.T and Sumner.D, Domination dot-critical graphs, Discrete Math. 306, (2206), 11-18
2. Harary.F, Bauer.D, Nieminen.J and Suffel.C.L, Domination alteration sets in graphs, Discrete Math.47, (1983), 153-161.
3. Haynes.T, Hedetniemi.T, Slater.P, Fundamentals of Domination in Graphs, Marcel Dekker, Newyork, 1998.
4. Kumar NV and Ramani GG, Different Types of Dominating Critical in Fuzzy graphs, Fuzzy systems, 3(9), (2011), 277-279.
5. Nagoor Gani.A and Chandrasekaran.V.T, A First look at Fuzzy Graph Theory, Allied publishers (p) Ltd, 2010.
6. Nagoor Gani.A and Vijayalakshmi.P, Domination critical nodes in fuzzy graph, International. Journal of Math. Sci & Engg. App, vol 5 No.1 (January 2011), pp -295-301.
7. Rosenfeld A (1975) Fuzzy graphs: In: Zadeh LA, FU KS,Shimura M (eds) Fuzzy sets and their applications Academic press, New York.
8. Somasundaram.A and Somasundaram.S, Domination in fuzzy graphs, Pattern Recognition Letters, 19(9), (1998), 787-791.
9. Yamuna. M and Karthika. K, Domination Subdivision Stable graphs, International Journal of Mathematical Archeive, 3(4), 2012, 1467-1471.

Source of support: Proceedings of UGC Funded International Conference on Intuitionistic Fuzzy Sets and Systems (ICIFSS-2018), Organized by: Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.