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COMPLETE RANDOM FUZZY METRIC SPACE AND COMMON FIXED POINT THEOREM WITH INTEGRAL TYPE FORM

GUJJETI SANDHYA*

Sidharatha Degree College, Bhupalpalli, Jaya Shakar District, India.

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ABSTRACT

We established some common fixed point theorems satisfying integral type mapping in fuzzy metric space. In fact we proved following fixed point theorems in fuzzy metric spaces

Key words: *fixed point, fixed point theorem. fuzzy set, fuzzy metric space, compatible map of type* (β)*.*

1. INTRODUCTION

Probabilistic functional analysis has emerged as one of the important mathematical disciplines in view of its role in analyzing Probabilistic models in the applied sciences. The study of fixed point of random operator forms a central topic in this area. Random fixed point theorem for contraction mappings in Polish spaces and random Fixed point theorems are of fundamental importance in probabilistic functional analysis. The concept of Fuzzy-random-variable was introduced as an analogous notion to random variable in order to extend statistical analysis to situations when the outcomes of some random experiment are fuzzy sets. But in contrary to the classical statistical methods no unique definition has been established before the work of Volker, Volker [22]. He presented set theoretical concept of fuzzy-random-variables using the method of general topology and drawing on results from topological measure theory and the theory of analytic spaces. No results in fixed point are introduced in fuzzy random spaces. In the present paper we are introducing the fuzzy random spaces and proving a common fixed point theorem.

In this present chapter we prove some common fixed point theorems in fuzzy random metric space by using the concept of β- compatible mapping. First we give some basic and important definitions related to this chapter.

Definition 1.1: Let X be any set. A fuzzy set in X is a function with domain X and values in $[0,1]$.

Definition 1.2: A binary operation $\star : [0,1] \times [0,1] \to [0,1]$ is continuous $t-$norm if $\star$ is satisfying the following conditions:
1.2 (a) $\star$ is commutative and associative,
1.2 (b) $\star$ is continuous,
1.2 (c) $a \star 1 = a$ for all $a \in [0,1]$
1.2 (d) $a \star b \leq c \star d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0,1]$
Examples of $t-$ norm are $a \star b = \min\{a, b\}$ and $a \star b = ab$.

Definition 1.3: A triplet $(X, M, \star)$ is a fuzzy metric space whenever X is an arbitrary set, $\star$ is continuous $t-$norm and M is fuzzy set on $X \times X \times [0, \infty^+)$ satisfying, for every $x, y, z \in X$ and $s, t > 0$, the following condition:
1.3 (a) $M(x, y, t) > 0$
1.3 (b) $M(x, y, 0) = 0$
1.3 (c) $M(x, y, t) = 1$ iff $x = y$
1.3 (d) $M(x, y, t) = M(y, x, t)$
1.3 (e) $M(x, y, t) \star M(y, z, s) \leq M(x, z, t + s)$
1.3 (f) $M(x, y, \cdot)$: $(0, \infty^+) \to [0,1]$ is continuous.

We note that, $M(x, y, t)$ can be realized as the measure of nearness between x and y with respect to t. It is known that $M(x, y, \cdot)$ is non decreasing for all $x, y \in X$. Let $M(x, y, \star)$ be a fuzzy metric space for $t > 0$, the open ball

$$B(x, r, t) = \{y \in X:\ M(x, y, t) > 1 - r\}.$$

Corresponding Author: Gujjeti Sandhya*

Now, the collection $\{B(x,r,t): x \in X, 0 < r < 1, t > 0\}$ is a neighborhood system for a topology τ on X induced by the fuzzy metric M. This topology is Housdroff and first countable.

Example 1.4: Let (X, d) be a metric space. Define $a \star b = \min\{a,b\}$ and $M(x,y,t) = \frac{t}{t+d(x,y)}$ for all $x, y \in X$ and all $t > 0$. Then $(X, M, \star)$ is a fuzzy metric space. It is called the fuzzy metric space induced by d.

Definition 1.5: A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, \star)$ is said to be a converges to x iff for each $\varepsilon > 0$ and each $t > 0$, $n_0 \in N$ such that $M(x_n, x, t) > 1 - \varepsilon$ for all $n \geq n_0$.

Definition 1.6: A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, \star)$ is said to be a Cauchy sequence converges to x iff for each $\varepsilon > 0$ and each $t > 0$, $n_0 \in N$ such that $M(x_m, x_n, t) > 1 - \varepsilon$ for all $m, n \geq n_0$.

A fuzzy metric space $(X, M, \star)$ is said to be complete if every Cauchy sequence in it converges to a point in it.

Definition 1.7: Self mapping A and S of a fuzzy metric space $(X, M, \star)$ are said to be compatible if and only if $M(ASx_n, SAx_n, t) \to 1$ for all $t > 0$, where $\{x_n\}$ is a sequence in X such that $Sx_n, Ax_n \to p$ for some $p \in X$ as $n \to \infty$.

Definition 1.8: Self map A and S of a fuzzy metric space $(X, M, \star)$ are said to be compatible of type (β) if and only if $M(AAx_n, SSx_n, t) \to 1$ for all $t > 0$, where $\{x_n\}$ is a sequence in X such that $Sx_n, Ax_n \to p$ for some $p \in X$ as $n \to \infty$.

Definition 1.9: Two maps A and B from a fuzzy metric space $(X, M, \star)$ into itself are said to be weakly compatible if they commute at their coincidence points i.e., $Ax = Bx$ implies $ABx = BAx$ for some $x \in X$.

Remark 1.10: The concept of compatible map of type (β) is more general then the concept of compatible map in fuzzy metric space.

Definition 1.11: Let A and S be two self maps of a fuzzy metric space $(X, M, \star)$ then A and S is said to be a weakly commuting if $M(ASx_n, SAx_n, t) \leq M(Sx_n, Ax_n, t)$ for all $x \in X$.

It can be seen that commuting maps ($ASx = SAx \ \forall\, x \in X$) are weakly compatible but converse is not true.

Lemma 1.12: In a fuzzy metric space $(X, M, \star)$ limit of a sequence is unique.

Lemma 1.13: Let $(X, M, \star)$ be a fuzzy metric space. Then for all $x, y \in X$ $M(x, y, .)$ is a non decreasing function.

Lemma 1.14: Let $(X, M, \star)$ be a fuzzy metric space. If there exists $k \in (0,1)$ such that for all $x, y \in X$,

$$M(x,y,kt) \geq M(x,y,t)\ \forall t > 0, then\ x = y.$$

Lemma 1.15: Let $\{x_n\}$ be a sequence in a fuzzy metric space $(X, M, \star)$. If there exists a number $k \in (0,1)$ such that

$$M(x_{n+2}, x_{n+1}, kt) \geq M(x_{n+1}, x_n, t)\ \forall\ t > 0\ and\ n \in N$$

Then $\{x_n\}$ is a Cauchy sequence in X.

Lemma 1.16: The only $t-norm$ $\star$ satisfying $r \star r = r$ for all $r \in [0,1]$ is the minimum $t-norm$ that is

$$a \star b = \min\{a,b\} \text{ for all } a, b \in [0,1].$$

In this chapter, (Ω, Σ) denotes a measurable space. $\omega : \Omega \to X$ is a measurable selector. X is any non empty set.* is continuous t-norm, M is a fuzzy set in $X^2 \times [0,\infty)$, then $(X, \Omega, M, \star)$ is said to be randomized fuzzy metric spaces shortly ad R.F.M.S if followings are true.
for every $\xi x, \xi y, \xi z \in X$ and $s, t > 0$, the following condition:
1.16(a) $M(\omega(x), \omega(y), t) > 0$
1.16 (b) $M(\omega(x), \omega(y), 0) = 0$
1.16 (c) $M(\omega(x), \omega(y), t) = 1$ iff $\omega(x) = \omega(y)$
1.16 (d) $M(\omega(x), \omega(y), t) = M(\omega(y), \omega(x), t)$
1.16 (e) $M(\omega(x), \omega(y), t) \star M(\omega(y), \omega(z), s) \leq M(\omega(x), \omega(z), t+s)$
1.16 (f) $M(\omega(x), \omega(y), \cdot) : (0, \infty^+) \to [0,1]$ is continuous.

Lemma 1.17: Let $(X, M, \Omega, \star)$ be a ramdom fuzzy metric space. Then for all $\omega(x), \omega(y) \in X$ $M(\omega(x), \omega(y), .)$ is a non decreasing function.

Lemma 1.18: Let $(X, M, \Omega, \star)$ be a ramdom fuzzy metric space. If there exists $k \in (0,1)$ such that for all $\omega(x), \omega(y) \in X$,

$$M(\omega(x), \omega(y), kt) \geq M(\omega(x), \omega(y), t)$$

for all $t > 0, t\square en\ \omega(x) = \omega(y)$.

Lemma 1.19: Let $\{x_n\}$ be a sequence in a random fuzzy metric space $(X, M, \Omega, \star)$. If there exists a number $k \in (0,1)$ such that

$$M(\omega(x_{n+2}), \omega(x_{n+1}), kt) \geq M(\omega(x_{n+1}), \omega(x_n), t) \;\; \forall \;\; t > 0 \;\; and \;\; n \in N$$

Then $\{\omega(x_n)\}$ is a Cauchy sequence in X.

2. MAIN THEOREM

Theorem 2.1: Let $(X, M, \Omega, \star)$ be a complete random fuzzy metric space and let A, B, S, T, P and Q be mappings from X into itself such that the following conditions are satisfied:
2.1(a) $P(X) \subset ST(X)$ and $Q(X) \subset AB(X)$,
2.1 (b) $AB = BA, ST = TS, PB = BP, QT = TQ$,
21 (c) either P or AB is continuous,
2.1 (d) (P, AB) is compatible of type (β) and (Q, ST) is weak compatible,
2.1 (e) there exists $k \in (0,1)$ such that for every $\omega(x), \omega(y) \in X$ and $t > 0$

$$\int_0^{M^2(P\omega(x), Q\omega(y), kt)} \xi(v)\, dv \geq \int_0^{W(\omega(x), \omega(y), t)} \xi(v)\, dv$$

$$W(\omega(x), \omega(y), t) = \min \begin{Bmatrix} M^2(AB\omega(x), ST\omega(y), t), \\ M^2(P\omega(x), AB\omega(x), t), \\ M^2(Q\omega(y), ST\omega(y), t) , \end{Bmatrix}$$

Where $\xi : [0, +\infty] \to [0, +\infty]$ is a lebgesgue integrable mapping which is summable on each compact subset of $[0, +\infty]$, non negative, and such that, $\forall \; \varepsilon > 0$, $\int_0^{\varepsilon} \xi(v)\, dv > 0$. Then A, B, S, T, P and Q have a unique common fixed point in X.

Proof: Let $x_0 \in X$, then from 2.1(a) we have $x_1, x_2 \in X$ such that $P\omega(x_0) = ST\omega(x_1)$ and $Q\omega(x_1) = AB\omega(x_2)$
Inductively, we construct sequences $\{\omega(x_n)\}$ and $\{\omega(y_n)\}$ in X such that for $n \in N$

$$P\omega(x_{2n-2}) = ST\omega(x_{2n-1}) = \omega(y_{2n-1}) \text{ and } Q\omega(x_{2n-1}) = AB\omega(x_{2n}) = \omega(y_{2n})$$

Step-1: Put $\omega(x) = \omega(x_{2n})$ and $\omega(y) = \omega(x_{2n+1})$ in 2.1 (e) then we have

$$\int_0^{M^2(P\omega(x_{2n}), Q\omega(x_{2n+1}), kt)} \xi(v)\, dv \geq \int_0^{W(\omega(x_{2n}), \omega(x_{2n+1}), t)} \xi(v)\, dv$$

$$W(\omega(x_{2n}), \omega(x_{2n+1}), t) = \min \begin{Bmatrix} M^2(AB\omega(x_{2n}), ST\omega(x_{2n+1}), t), \\ M^2(P\omega(x_{2n}), AB\omega(x_{2n}), t), \\ M^2(Q\omega(x_{2n+1}), ST\omega(x_{2n+1}), t), \end{Bmatrix}$$

$$\int_0^{M^2(\omega(y_{2n+1}), \omega(y_{2n+2}), kt)} \xi(v)\, dv \geq \int_0^{W(\omega(y_{2n+1}), \omega(y_{2n+2}), t)} \xi(v)\, dv$$

$$W(\omega(y_{2n+1}), \omega(y_{2n+2}), t) = \min \begin{Bmatrix} M^2(\omega(y_{2n}), \omega(y_{2n+1}), t), \\ M^2(\omega(y_{2n+1}), \omega(y_{2n}), t), \\ M^2(\omega(y_{2n+2}), \omega(y_{2n+1}), t) , \end{Bmatrix}$$

$$\int_0^{M^2(\omega(y_{2n+1}), \omega(y_{2n+2}), kt)} \xi(v)\, dv \geq \int_0^{\min\begin{Bmatrix} M^2(\omega(y_{2n}), \omega(y_{2n+1}), t), \\ M^2(\omega(y_{2n+2}), \omega(y_{2n+1}), t) \end{Bmatrix}} \xi(v)\, dv$$

From Lemma .1.13 and 1.14 we have

$$\int_0^{M^2(\omega(y_{2n+1}), \omega(y_{2n+2}), kt)} \xi(v)\, dv \geq \int_0^{M^2(\omega(y_{2n}), \omega(y_{2n+1}), t)} \xi(v)\, dv$$

Since $\xi(v)$ is Lesbesgue integrable function so that

$$M^2(\omega(y_{2n+1}), \omega(y_{2n+2}), kt) \geq M^2(\omega(y_{2n}), \omega(y_{2n+1}), t)$$

That is

$$M(\omega(y_{2n+1}), \omega(y_{2n+2}), kt) \geq M(\omega(y_{2n}), \omega(y_{2n+1}), t)$$

Similarly we have

$$M(\omega(y_{2n+2}), \omega(y_{2n+3}), kt) \geq M(\omega(y_{2n+1}), \omega(y_{2n+2}), t)$$

Thus we have

$$M(\omega(y_{n+1}), \omega(y_{n+2}), kt) \geq M(\omega(y_n), \omega(y_{n+1}), t)$$
$$M(\omega(y_{n+1}), \omega(y_{n+2}), t) \geq M\left(\omega(y_n), \omega(y_{n+1}), \frac{t}{k}\right)$$
$$M(\omega(y_n), \omega(y_{n+1}), t) \geq M\left(\omega(y_0), \omega(y_1), \frac{t}{k^n}\right) \to 1 \text{ as } n \to \infty,$$

and hence $M(\omega(y_n), \omega(y_{n+1}), t) \to 1$ as $n \to \infty$ for all $t > 0$.

For each $\epsilon > 0$ *and* $t > 0$, we can choose $n_0 \in N$ such that

$$M(\omega(y_n), \omega(y_{n+1}), t) > 1 - \epsilon \text{ for all } n > n_0.$$

For any $m, n \in N$ we suppose that $n_0 \geq n$. Then we have

$$M(\omega(y_n), \omega(y_m), t) \geq M\left(\omega(y_n), \omega(y_{n+1}), \frac{t}{m-n}\right) \star M\left(\omega(y_{n+1}), \omega(y_{n+2}), \frac{t}{m-n}\right) \star \dots \star \left(\omega(y_{m-1}), \omega(y_m), \frac{t}{m-n}\right)$$
$$M(\omega(y_n), \omega(y_m), t) \geq (1-\epsilon) \star (1-\epsilon) \star \dots \dots \star (1-\epsilon)(m-n)\text{times}$$
$$M(\omega(y_n), \omega(y_m), t) \geq (1-\epsilon)$$

And hance $\{\omega(y_n)\}$ is a Cauchy sequence in X.

Since $(X, M, \Omega, \star)$ is complete, $\{\omega(y_n)\}$ converges to some point $\omega(z) \in X$. Also its subsequences converges to the same point $\omega(z) \in X$.
That is

$$\{P\omega(x_{2n+2})\} \to \omega(z) \text{ and } \{ST\omega(x_{2n+1}\} \to \omega(z) \qquad 2.1(i)$$

$$\{Q\omega(x_{2n+1})\} \to \omega(z) \text{ and } \{AB\omega(x_{2n})\} \to \omega(y_{2n}) \qquad 2.1(ii)$$

Case-1: Suppose AB is continuous

Since AB is continuous, we have

$$(AB)^2\omega(x_{2n}) \to AB\omega(z) \text{ and } ABP\omega(x_{2n}) \to AB\omega(z)$$

As (P, AB) is compatible pair of type (β), we have

$$M(PP\omega(x_{2n}), (AB)(AB)\omega(x_{2n}), t) = 1, \quad \text{for all } t > 0$$

Or $\qquad M(PP\omega(x_{2n}), AB\omega(z), t) = 1$

Therefore, $\qquad PP\omega(x_{2n}) \to AB\omega(z)$.

Step-2: Put $\omega(x) = (AB)\omega(x_{2n})$ and $\omega(y) = \omega(x_{2n+1})$ in 6.3.1(e) we have

$$\int_0^{M^2(P(AB)\omega(x_{2n},Q\omega(y,kt)}\xi(v)\,dv \geq \int_0^{W(P(AB)\omega(x_{2n},Q\omega(y,kt)}\xi(v)\,dv$$
$$W(P(AB)\omega(x_{2n}), Q\omega(y), t) = \min\left\{\begin{matrix} M^2(AB(AB)\omega(x_{2n}), ST\omega(x_{2n+1}), t), \\ M^2(P(AB)\omega(x_{2n}), AB(AB)\omega(x_{2n}), t), \\ M^2(Q\omega(x_{2n+1}), ST\omega(x_{2n+1}), t), \end{matrix}\right\}$$

Taking $n \to \infty$ we get

$$\int_0^{M^2(P(AB)\omega(x_{2n}),Q\omega(y),kt)}\xi(v)\,dv \geq \int_0^{W(P(AB)\omega(x_{2n}),Q\omega(y),t)}\xi(v)\,dv$$
$$M^2\big((AB)\omega(z), \omega(z), kt\big) \geq \min\left\{\begin{matrix} M^2\big((AB)\omega(z), \omega(z), t\big), \\ M^2\big((AB)\omega(z), (AB)\omega(z), t\big), \\ M^2\big((AB)\omega(z), \omega(z), t\big), \end{matrix}\right\}$$

$$\int_0^{M^2((AB)\omega(z),\omega(z),kt)}\xi(v)\,dv \geq \int_0^{\min\left\{\begin{matrix} M^2((AB)\omega(z),\omega(z),t), \\ M^2((AB)\omega(z),\omega(z),t)\end{matrix}\right\}}\xi(v)\,dv$$

That is form the property of $\xi(v)$ we have

$$M\big((AB)\omega(z), \omega(z), kt\big) \geq M\big((AB)\omega(z), \omega(z), t\big)$$

Therefore by lemma 1.14 we have

$$AB\omega(z) = \omega(z). \qquad 2.1(iii)$$

Step-3: Put $\omega(x) = \omega(z)$ and $\omega(y) = \omega(x_{2n+1})$ in 2.1(e) we have

$$\int_0^{M^2(P\omega(z),Q\,\omega(x_{2n+1}),kt)}\xi(v)\,dv \geq \int_0^{W(P\omega(z),Q\,\omega(x_{2n+1}),t)}\xi(v)\,dv$$
$$W(P\omega(z), Q\,\omega(x_{2n+1}), t) = \min\left\{\begin{matrix} M^2(AB\omega(z), ST\,\omega(x_{2n+1}), t), \\ M^2(P\omega(z), AB\omega(z), t), \\ M^2(Q\omega(x_{2n+1}), ST\omega(x_{2n+1}), t), \end{matrix}\right\}$$

Taking $n \to \infty$ and using equation .2.1 (i) we have

$$\int_0^{M^2(P\omega(z),\omega(z),kt)}\xi(v)\,dv \geq \int_0^{W(P\omega(z),\omega(z),t)}\xi(v)\,dv$$
$$W(P\omega(z), \omega(z), t) \geq \min\left\{\begin{matrix} M^2(AB\omega(z), \omega(z), t), \\ M^2(P\omega(z), AB\omega(z), t), \\ M^2(\omega(z), \omega(z), t), \end{matrix}\right\}$$

So that $\quad M^2(P\omega(z), \omega(z), kt) \geq M^2(P\omega(z), \omega(z), t)$

And hance $M(P\omega(z), \omega(z), kt) \geq M(P\omega(z), \omega(z), t)$

Therefore by using lemma 1.14, we get $P\omega(z) = \omega(z)$

So we have $AB\omega(z) = P\omega(z) = \omega(z)$.

Step-4: Putting $\omega(x) = B\omega(z)$ and $\omega(y) = \omega(x_{2n+1})$ in 2.1(e), we get

$$\int_0^{M^2(PB\omega(z),Q\omega(x_{2n+1}),kt)} \xi(v)\ dv \geq \int_0^{W(PB\omega(z),Q\omega(x_{2n+1}),t)} \xi(v)\ dv$$

$$W(PB\omega(z), Q\omega(x_{2n+1}), t) = \min\begin{Bmatrix} M^2(ABB\omega(z), ST\omega(x_{2n+1}), t), \\ M^2(Q\omega(x_{2n+1}), ST\omega(x_{2n+1}), t), \\ M^2(PB\omega(z), ABB\omega(z), t), \end{Bmatrix}$$

As $BP = PB$ and $AB = BA$, so we have
$P(B\omega(z)) = B(P\omega(z)) = B\omega(z)$ and $(AB)(B\omega(z)) = (BA)(B\omega(z)) = B(AB\omega(z)) = B\omega(z))$.

Taking $n \to \infty$ and using 6.3.1(i) we get

$$\int_0^{M^2(PB\omega(z),Q\omega(x_{2n+1}),kt)} \xi(v)\ dv \geq \int_0^{W(PB\omega(z),Q\omega(x_{2n+1}),t)} \xi(v)\ dv$$

$$\int_0^{M^2(B\omega(z),\omega(z),kt)} \xi(v)\ dv \geq \int_0^{W(B\omega(z),\omega(z),t)} \xi(v)\ dv$$

$$W(B\omega(z), \omega(z), t) = \min\begin{Bmatrix} M^2(B\omega(z), \omega(z), t), \\ M^2(B\omega(z), B\omega(z), t), \\ M^2(\omega(z), \omega(z), t), \end{Bmatrix}$$

So we have $M^2(B\omega(z), \omega(z), kt) \geq M^2(B\omega(z), \omega(z), t)$

That is $M(B\omega(z), \omega(z), kt) \geq M(B\omega(z), \omega(z), t)$

Therefore by Lemma 1.14 we have $B\omega(z) = \omega(z)$

And also we have $AB\omega(z) = \omega(z)$ implies $A\omega(z) = \omega(z)$

Therefore

$$A\omega(z) = B\omega(z) = P\omega(z) = \omega(z). \qquad 2.1\ (iv)$$

Step–5: As $P(X) \subset ST(X)$ there exists $u \in X$ such that

$$\omega(z) = P\omega(z) = ST\omega(u)$$

Putting $\omega(x) = \omega(x_{2n})$ and $\omega(y) = \omega(u)$ in 6.3.1(e) we get

$$\int_0^{M^2(P\omega(x_{2n}),Q\omega(u,)kt)} \xi(v)\ dv \geq \int_0^{W(P\omega(x_{2n}),Q\omega(u),t)} \xi(v)\ dv$$

$$W(P\omega(x_{2n}), Q\omega(u), t) = \min\begin{Bmatrix} M^2(AB\omega(x_{2n}), ST\omega(u), t), \\ M^2(P\omega(x_{2n}), AB\omega(x_{2n}), t), \\ M^2(Q\omega(u), ST\omega(u), t), \end{Bmatrix}$$

Taking $n \to \infty$ and using 6.3.1(i) and 6.3.1(ii) we get

$$\int_0^{M^2(\omega(z),Q\omega(u),kt)} \xi(v)\ dv \geq \int_0^{W(\omega(z),Q\omega(u),t)} \xi(v)\ dv$$

$$W(\omega(z), Q\omega(u), t) = \min\begin{Bmatrix} M^2(\omega(z), ST\omega(u), t), \\ M^2(\omega(z), \omega(z), t), \\ M^2(Q\omega(u), ST\omega(u), t), \end{Bmatrix}$$

So we have $M^2(\omega(z), Q\omega(u), kt) \geq M^2(\omega(z), Q\omega(u), t)$

That is $M(\omega(z), Q\omega(u), kt) \geq M(\omega(z), Q\omega(u), t)$

Therefore by using Lemma 1.13 we have $Q\omega(u) = \omega(z)$

Hence $ST\omega(u) = \omega(z) = Q\omega(u)$.

Hence (Q, ST) is weak compatible, therefore, we have

$$QST\omega(u) = STQ\omega(u)$$

Thus $Q\omega(z) = ST\omega(z)$.

Step–6: Putting $\omega(x) = \omega(x_{2n})$ and $\omega(y) = \omega(z)$ in 6.3.1(e) we get

$$\int_0^{M^2(P\omega(x_{2n}),Q\omega(z),kt)} \xi(v)\ dv \geq \int_0^{W(P\omega(x_{2n}),Q\omega(z),t)} \xi(v)\ dv$$

$$W(P\omega(x_{2n}),Q\omega(z),t) = \min\begin{Bmatrix} M^2(AB\omega(x_{2n}),ST\omega(z),t), \\ M^2(P\omega(x_{2n}),AB\omega(x_{2n}),t), \\ M^2(Q\omega(z),ST\omega(z),t), \end{Bmatrix}$$

Taking $n \to \infty$ and using 6.3.1(ii) and step 5 we get

$$\int_0^{M^2(\omega(z),Q\omega(z),kt)} \xi(v)\ dv \geq \int_0^{W(\omega(z),Q\omega(z),t)} \xi(v)\ dv$$

$$W(\omega(z),Q\omega(z),t) = \min\begin{Bmatrix} M^2(\omega(z),ST\omega(z),t), \\ M^2(\omega(z),\omega(z),t), \\ M^2(Q\omega(z),ST\omega(z),t), \end{Bmatrix}$$

That is $M^2(\omega(z),Q\omega(z),kt) \geq M^2(\omega(z),Q\omega(z),t)$

And hence $M(\omega(z),Q\omega(z),kt) \geq M(\omega(z),Q\omega(z),t)$

Therefore by using Lemma 1.13 we get $Q\omega(z) = \omega(z)$.

Step–7: Putting $\omega(x) = \omega(x_{2n})$ and $\omega(y) = T\omega(z)$ in 3.1(e) we get

$$\int_0^{M^2(P\omega(x_{2n}),QT\omega(z),kt)} \xi(v)\ dv \geq \int_0^{W(P\omega(x_{2n}),QT\omega(z),t)} \xi(v)\ dv$$

$$W(P\omega(x_{2n}),QT\omega(z),t) = \min\begin{Bmatrix} M^2(AB\omega(x_{2n}),STT\omega(z),t), \\ M^2(P\omega(x_{2n}),AB\omega(x_{2n}),t), \\ M^2(QT\omega(z),STT\omega(z),t), \end{Bmatrix}$$

As $QT = TQ$ and $ST = TS$ we have $QTz = TQz = Tz$

And $ST(T\omega(z)) = T(ST\omega(z)) = TQ\omega(z) = T\omega(z)$.

Taking $n \to \infty$ we get

$$\int_0^{M^2(\omega(z),T\omega(z),kt)} \xi(v)\ dv \geq \int_0^{W(\omega(z),T\omega(z),t)} \xi(v)\ dv$$

$$W(\omega(z),T\omega(z),t) = \min\begin{Bmatrix} M^2(\omega(z),T\omega(z),t), \\ M^2(\omega(z),\omega(z),t), \\ M^2(T\omega(z),T\omega(z),t), \end{Bmatrix}$$

And hence $M^2(\omega(z),T\omega(z),kt) \geq M^2(\omega(z),T\omega(z),t)$

Therefore $M(\omega(z),T\omega(z),kt) \geq M(\omega(z),T\omega(z),t)$

Therefore by Lemma 1.13 we have $T\omega(z) = \omega(z)$

Now $ST\omega(z) = T\omega(z) = \omega(z)$ implies $S\omega(z) = \omega(z)$.

Hence

$$S\omega(z) = T\omega(z) = Q\omega(z) = \omega(z) \qquad 2.1(v)$$

Combining 2.1(iv) and 2.1(v) we have

$$A\omega(z) = B\omega(z) = P\omega(z) = S\omega(z) = T\omega(z) = Q\omega(z) = \omega(z)$$

Hence z is the common fixed point of A, B, S, T, P and Q.

Case-II: suppose P is continuous

As P is continuous

$$P^2\omega(x_{2n}) \to P\omega(z) \text{ and } P(AB)\omega(x_{2n}) \to P\omega(z)$$

As (P,AB) is compatible pair of type (β),

$$M(PP\omega(x_{2n}),(AB)(AB)\omega(x_{2n}),t) = 1 \text{ forall } t > 0$$

Or $\quad M(P\omega(z),(AB)(AB)\omega(x_{2n}),t) = 1$

Therefore $(AB)^2 x_{2n} \to Pz$.

Step-8: Putting $\omega(x) = P\omega(x_{2n})$ and $\omega(y) = \omega(x_{2n+1})$ in 2.3.2(e) then we get

$$\int_0^{M^2(PP\omega(x_{2n}),Q\omega(x_{2n+1}),kt)} \xi(v)\ dv \ge \int_0^{W(PP\omega(x_{2n}),Q\omega(x_{2n+1}),t)} \xi(v)\ dv$$

$$W(PP\omega(x_{2n}),Q\omega(x_{2n+1}),t) = \min\begin{Bmatrix} M^2(ABP\omega(x_{2n}),ST\omega(x_{2n+1}),t), \\ M^2(PP\omega(x_{2n}),ABP\omega(x_{2n}),t), \\ M^2(Q\omega(x_{2n+1}),ST\omega(x_{2n+1}),t), \end{Bmatrix}$$

Taking $n \to \infty$, we get

$$\int_0^{M^2(P\omega(z),\omega(z),kt)} \xi(v)\ dv \ge \int_0^{W(P\omega(z),\omega(z),t)} \xi(v)\ dv$$

$$W(P\omega(z),\omega(z),t) = \min\begin{Bmatrix} M^2(P\omega(z),\omega(z),t), \\ M^2(P\omega(z),P\omega(z),t), \\ M^2(\omega(z),\omega(z),t), \end{Bmatrix}$$

$$\int_0^{M^2(P\omega(z),\omega(z),kt)} \xi(v)\ dv \ge \int_0^{M^2(P\omega(z),\omega(z),t)} \xi(v)\ dv$$

Therefore we have

$$M^2(P\omega(z),\omega(z),kt) \ge M^2(P\omega(z),\omega(z),t)$$

Hence $\quad M(P\omega(z),\omega(z),kt) \ge M(P\omega(z),\omega(z),t)$

Therefore by Lemma 1.13 we get $P\omega(z) = \omega(z)$

Step-9: Put $\omega(x) = AB\omega(x_{2n})$ and $\omega(y) = \omega(x_{2n+1})$ in 2.2(e) then we get

$$\int_0^{M^2(P\,AB\omega(x_{2n}),Q\omega(x_{2n+1}),kt)} \xi(v)\ dv \ge \int_0^{W(P\,AB\omega(x_{2n}),Q\omega(x_{2n+1}),t)} \xi(v)\ dv$$

$$W(P\,AB\omega(x_{2n}),Q\omega(x_{2n+1}),t) = \min\begin{Bmatrix} M^2(AB\,AB\omega(x_{2n}),ST\omega(x_{2n+1}),t), \\ M^2(P\,AB\omega(x_{2n}),AB\,AB\omega(x_{2n}),t), \\ M^2(Q\omega(x_{2n+1}),ST\omega(x_{2n+1}),t), \end{Bmatrix}$$

Taking $n \to \infty$ we get

$$\int_0^{M^2(AB\omega(z),\omega(z),kt)} \xi(v)\ dv \ge \int_0^{W(AB\omega(z),\omega(z),t)} \xi(v)\ dv$$

$$W(AB\omega(z),\omega(z),t) = \min\begin{Bmatrix} M^2(AB\,\omega(z),\omega(z),t), \\ M^2(AB\omega(z),\omega(z),t), \\ M^2(\omega(z),\omega(z),t), \end{Bmatrix}$$

$$\int_0^{M^2(AB\omega(z),\omega(z),kt)} \xi(v)\ dv \ge \int_0^{M^2(AB\omega(z),\omega(z),t)} \xi(v)$$

Therefore $M^2(AB\omega(z),\omega(z),kt) \ge M^2(AB\omega(z),\omega(z),t)$

And hence $M(AB\omega(z),\omega(z),kt) \ge M(AB\omega(z),\omega(z),t)$

By lemma .1.13 we get $AB\omega(z) = \omega(z)$

By applying step 4, 5, 6, 7, 8 we get

$$A\omega(z) = B\omega(z) = S\omega(z) = T\omega(z) = P\omega(z) = Q\omega(z) = \omega(z).$$

That is $\omega(z)$ is a common fixed point of A, B, S, T, P, Q in X.

Uniqueness: Let u be another common fixed point of A, B, S, T, P and Q. Then

$$A\omega(u) = B\omega(u) = S\omega(u) = T\omega(u) = P\omega(u) = Q\omega(u) = \omega(u)$$

Putting $\omega(x) = \omega(u)$ and $\omega(y) = \omega(z)$ in 2.1(e) then we get

$$\int_0^{M^2(P\omega(u),Q\omega(z),kt)} \xi(v)\ dv \ge \int_0^{W(P\omega(u),Q\omega(z),t)} \xi(v)\ dv$$

$$W(P\omega(u),Q\omega(z),t) = \min\begin{Bmatrix} M^2(AB\omega(u),ST\omega(z),t), \\ M^2(P\omega(u),AB\omega(u),t), \\ M^2(Q\omega(z),ST\omega(z),t), \end{Bmatrix}$$

Taking limit both side then we get

$$\int_0^{M^2(\omega(u),\omega(z),kt)} \xi(v)\ dv \geq \int_0^{W(\omega(u),\omega(z),t)} \xi(v)\ dv$$

$$W(\omega(u),\omega(z),t) = \min \begin{Bmatrix} M^2(\omega(u),\omega(z),t), \\ M^2(\omega(u),\omega(u),t), \\ M^2(\omega(z),\omega(z),t), \end{Bmatrix}$$

That is $M^2(\omega(u),\omega(z),kt) \geq M^2(\omega(u),\omega(z),t)$

And hence $M(\omega(u),\omega(z),kt) \geq M(\omega(u),\omega(z),t)$

By lemma 1.13 we get $\omega(z) = \omega(u)$.
That is z is a unique common fixed point of A, B, S, T, P and Q in X.

Remark 3.3: If we take $B = T = I$ identity map on X in Theorem 2.2.3.2 then condition 6.3.1(b) is satisfy trivially and we get following Corollary

Corollary 2.4: Let $(X, M, \Omega, \star)$ be a complete random fuzzy metric space and let A, S, P and Q be mappings from X into itself such that the following conditions are satisfied:
2.4(a) $P(X) \subset S(X)$ and $Q(X) \subset A(X)$,
2.4 (b) either P or AB is continuous,
2.4 (c) (P, AB) is compatible of type (β) and (Q, ST) is weak compatible,
2.4 (d) there exists $k \in (0,1)$ such that for every $x, y \in X$ and $t > 0$

$$\int_0^{M^2(P\omega(x),Q\omega(y),kt)} \xi(v)\ dv \geq \int_0^{W(\omega(x),\omega(y),t)} \xi(v)\ dv$$

$$W(\omega(x),\omega(y),t) = \min \begin{Bmatrix} M^2(A\omega(x),S\omega(y),t), \\ M^2(P\omega(x),A\omega(x),t), \\ M^2(Q\omega(y),S\omega(y),t), \end{Bmatrix}$$

Where $\xi : [0,+\infty] \to [0,+\infty]$ is a lebgesgue integrable mapping which is summable on each compact subset of $[0,+\infty]$, non negative, and such that, $\forall\ \varepsilon > 0$, $\int_0^{\varepsilon} \xi(v)\ dv > 0$. Then A, B, S, T, P and Q have a unique common fixed point in X.

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