

INTUITIONISTIC FUZZY G_δ - e -LOCALLY CLOSED SETS

G. SARAVANAKUMAR¹, S. TAMILSELVAN² AND A. VADIVEL³

¹Research Scholar, Department of Mathematics,
Annamalai University, Annamalainagar, Tamil Nadu-608 002, India.

²Mathematics Section (FEAT),
Annamalai University, Annamalainagar, Tamil Nadu-608 002, India.

³(Deputed) Department of Mathematics,
Government Arts College (Autonomous), Karur, Tamil Nadu-639 005, India.

E-mail: ¹saravananguru2612@gmail.com, ²tamil_au@yahoo.com and ³avmaths@gmail.com

ABSTRACT

This paper is devoted to the study of new class of sets called an intuitionistic fuzzy e -locally closed sets, intuitionistic fuzzy eG_δ -sets, intuitionistic fuzzy eG_δ -locally closed sets and intuitionistic fuzzy G_δ - e -locally closed sets are introduced and studied. Also the concepts of an intuitionistic fuzzy G_δ - e -locally closed set intuitionistic fuzzy subspace, intuitionistic fuzzy G_δ - e -local- δ -semi (resp., δ -pre and β) spaces are introduced and interesting properties are established. In this connection, interrelations are discussed. Examples are provided where necessary.

Keywords and phrases: *Intuitionistic fuzzy e -locally closed sets, intuitionistic fuzzy eG_δ -sets, intuitionistic fuzzy eG_δ -locally closed set, intuitionistic fuzzy G_δ - e -locally closed set, intuitionistic fuzzy G_δ - e -local- $T_{1/2}$ -spaces and intuitionistic fuzzy G_δ - e - δ -semi (resp., δ -pre and β) spaces.*

AMS (2000) subject classification: *54A40, 54A99, 03E72, 03E99.*

1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [14] and Atanassov [1] introduced and studied intuitionistic fuzzy sets. On the other hand, Coker [4] introduced the notions of an intuitionistic fuzzy topological spaces, intuitionistic fuzzy continuity and some other related concepts. The concept of an intuitionistic fuzzy α -closed set was introduced by Biljana Krsteshka and Erdal Ekici [9]. The first step of locally closedness was done by Bourbaki [3]. Ganster and Rely used locally closed sets in [8] to define LC-continuity and LC-irresoluteness. Roja, Uma and Balasubramanian [2] discussed fuzzy G_δ continuous functions. The initiations of e -open sets, e -continuity and e -compactness in topological spaces are due to Ekici [5,6,7]. In fuzzy topology, e -open sets were introduced by Seenivasan in 2014 [11]. Sobana et.al [12] were introduced the concept of fuzzy e -open sets, fuzzy e -continuity and fuzzy e -compactness in intuitionistic fuzzy topological spaces (briefly, IFTS's). In this paper we introduce the concepts of an intuitionistic fuzzy e -locally closed sets, intuitionistic fuzzy eG_δ -sets, intuitionistic fuzzy eG_δ -locally closed sets and intuitionistic fuzzy G_δ - e -locally closed sets in IFTS's. Also the concepts of an intuitionistic fuzzy G_δ - e -locally closed intuitionistic fuzzy subspace, intuitionistic fuzzy G_δ - e -local $T_{1/2} T_{\frac{1}{2}}$ space, intuitionistic fuzzy G_δ - e -local δ -semi (resp., δ -pre and β) spaces are introduced and studied. Some interesting properties and interrelations among sets and spaces are discussed with necessary examples.

2. PRELIMINARIES

Let X be a nonempty fixed set and I the closed interval $[0, 1]$. An intuitionistic fuzzy set (IFS) [1] A is an object of the following form $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \}$, where the mapping $\mu_A : X \rightarrow I$ and $\gamma_A : X \rightarrow I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) for each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$. Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form, $A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle : x \in X \}$. For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \gamma_A \rangle$ for the intuitionistic fuzzy set $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \}$. Let A and B be IFSs of the form $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \}$ and $B = \{ \langle x, \mu_B(x), \gamma_B(x) \rangle : x \in X \}$. Then (i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$; (ii) $\bar{A} = \{ \langle x, \gamma_A(x), \mu_A(x) \rangle : x \in X \}$; (iii) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle : x \in X \}$; (iv) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle : x \in X \}$; The IFS's $\underline{0}$ and $\underline{1}$ are defined by, $\underline{0} = \{ \langle x, 0, 1 \rangle : x \in X \}$ and $\underline{1} = \{ \langle x, 1, 0 \rangle : x \in X \}$. An intuitionistic fuzzy topology (IFT) [4] in Coker's sense on a nonempty set X is a family T of intuitionistic fuzzy sets in X satisfying the following axioms: (i) $\underline{0}, \underline{1} \in T$; (ii) $G_1 \cap G_2 \in T$, for any $G_1, G_2 \in T$; (iii) $\cup G_i \in T$ for any arbitrary family $\{G_i : i \in J\} \subseteq T$. In this paper by (X, T) or simply by X we will denote the intuitionistic fuzzy topological space (IFTS). Each IFS which belongs to T is called an intuitionistic fuzzy open set (IFOS) in X . The complement \bar{A} of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X . Let (X, T) be an IFTS and $A = \{ \langle x, \mu_A, \nu_A \rangle : x \in X \}$ be an IFS in X . Then the intuitionistic fuzzy closure and intuitionistic fuzzy interior [4] of A are defined by (i) $cl(A) = \bigcap \{C : C \text{ is an IFCS in } X \text{ and } C \supseteq A\}$; (ii) $int(A) = \bigcup \{D : D \text{ is an IFOS in } X \text{ and } D \subseteq A\}$; It can be also shown that $cl(A)$ is an IFCS, $int(A)$ is an IFOS in X and A is an IFCS in X if and only if $cl(A) = A$; A is an IFOS in X if and only if $int(A) = A$. Let A be IFS in an IFTS (X, T) . A is called an (i) intuitionistic fuzzy regular open set (briefly *IFROS*) [13] if $A = intcl(A)$, (ii) intuitionistic fuzzy regular closed set (briefly *IFRCS*) [13] if $A = clint(A)$. Let (X, T) be a fuzzy topological space and λ be a fuzzy set in X . λ is called G_δ set [2] if $\lambda = \bigwedge_{i=1}^{\infty} \lambda_i$ where each $\lambda_i \in T$. The complement of fuzzy G_λ is fuzzy F_σ . Let (X, T) be an IFTS and $A = \langle x, \mu_A(x), \nu_A(x) \rangle$ be an IFS in X . Then the fuzzy δ closure [13] of A are denoted and defined by $cl_\delta(A) = \bigcap \{K : K \text{ is an IFRCS in } X \text{ and } A \subseteq K\}$ and $int_\delta(A) = \bigcup \{G : G \text{ is an IFROS in } X \text{ and } G \subseteq A\}$. Let A be an IFS in an IFTS (X, T) . A is called an intuitionistic fuzzy δ -semiopen (resp. δ -preopen, β -open) [12] set (IF δ SO (resp. IF δ PO, IF β O), for short), if $A \leq cl(int_\delta(A))$ (resp. $A \leq int(cl_\delta(A))$, $A \leq cl(int(cl(A)))$). A is called an intuitionistic fuzzy δ -semiclosed (resp. δ -preclosed, β -closed) [12] set (IF δ SC (resp. IF δ PC, IF β C) (for short)) if $A \geq int(cl_\delta(A))$ (resp. $A \geq cl(int_\delta(A))$, $A \geq int(cl(int(A)))$). Let A be an IFS in an IFTS (X, T) . A is called an intuitionistic fuzzy e -open set [12] (IFeOS, for short) in X if $A \subseteq clint_\delta(A) \cup intcl_\delta(A)$. Let (X, T) be an IFTS and Y be any intuitionistic fuzzy subset of X . Then $T_Y = (A/Y \mid A \in T)$ is an intuitionistic fuzzy topology on Y and is called the induced or relative intuitionistic fuzzy topology [10]. The pair (Y, T_Y) is called an intuitionistic fuzzy subspace of (X, T) ; (Y, T_Y) is called an intuitionistic fuzzy open/intuitionistic fuzzy closed subspace if the intuitionistic fuzzy characteristic function of (Y, T_Y) viz χ_Y is intuitionistic fuzzy open/intuitionistic fuzzy closed.

3. Intuitionistic fuzzy G_δ - e -locally closed sets in an IFTSs

In this section, the concepts of an intuitionistic fuzzy e -locally closed set, intuitionistic fuzzy eG_δ -set, intuitionistic fuzzy eG_δ -locally closed set and intuitionistic fuzzy G_δ - e -locally closed set are introduced and studied. The concepts of an intuitionistic fuzzy G_δ - e -locally closed intuitionistic fuzzy subspace, intuitionistic fuzzy G_δ - e -local $T_{\frac{1}{2}}$ space, intuitionistic fuzzy G_δ - e - δ -semi space, intuitionistic fuzzy G_δ - e - δ -pre space, intuitionistic fuzzy G_δ - β space are introduced and studied. In this connection, interrelation among sets and spaces are discussed with suitable examples.

Definition 3.11: Let (X, T) be an IFTS. Let $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ be an IFS on an IFTS (X, T) . Then A is said to be intuitionistic fuzzy e -locally closed set (in short, IF- e -lcs) if $A = C \cap D$, where $C = \{\langle x, \mu_C(x), \gamma_C(x) \rangle : x \in X\}$ is an IFeOS and $D = \{\langle x, \mu_D(x), \gamma_D(x) \rangle : x \in X\}$ is an IFeCS in (X, T) .

Definition 3.22: Let (X, T) be an IFTS. Let $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ be an intuitionistic fuzzy set on an IFTS X . Then A is said to be an intuitionistic fuzzy eG_δ -set (in short IF- eG_δ -set) if $A = \bigcap_{i=1}^{\infty} A_i$, where $A_i = \{\langle x, \mu_{A_i}(x), \gamma_{A_i}(x) \rangle : x \in X\}$ is an IFeOS in an IFTS (X, T) .

Definition 3.33: Let (X, T) be an IFTS. Let $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ be an IFS on an IFTS (X, T) . Then A is said to be an intuitionistic fuzzy eG_δ -locally closed set (in short, IF- eG_δ -lcs) if $A = C \cap D$, where $C = \{\langle x, \mu_C(x), \gamma_C(x) \rangle : x \in X\}$ is an IF- eG_δ -set and $D = \{\langle x, \mu_D(x), \gamma_D(x) \rangle : x \in X\}$ is an IFeCS in (X, T) .

Definition 3.44: Let (X, T) be an IFTS. Let $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ be an IFS on an IFTS (X, T) . Then A is said to be an intuitionistic fuzzy G_δ - e -locally closed set (in short, IF- G_δ - e -lcs) if $A = B \cap C$, where $B = \{\langle x, \mu_B(x), \gamma_B(x) \rangle : x \in X\}$ is an IF- G_δ -set and $C = \{\langle x, \mu_C(x), \gamma_C(x) \rangle : x \in X\}$ is an IFeCS (X, T) .

The complement of an intuitionistic fuzzy G_δ - e -lcs is said to be an intuitionistic fuzzy G_δ - e -locally open set (in short, IF- G_δ - e -los).

Definition 3.55: Let (X, T) be an IFTS. Let $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ be an IFS on an IFTS (X, T) . The intuitionistic fuzzy G_δ - e -locally closure of A is denoted and defined by IFG_δ - e - $lcl(A) = \bigcap \{B : B = \{\langle x, \mu_B(x), \gamma_B(x) \rangle : x \in X\}$ is an IF- G_δ - e -lcs in X and $A \subseteq B$.

Proposition 3.16: Let (X, T) be an IFTS. For any two IFSs $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x) \rangle : x \in X\}$ of an IFTS (X, T) then the following statements are true. (i) IFG_δ - e - $lcl(\emptyset) = \emptyset$ (ii) $A \subseteq B \Rightarrow IFG_\delta$ - e - $lcl(A) \subseteq IFG_\delta$ - e - $lcl(B)$ (iii) IFG_δ - e - $lcl(IFG_\delta$ - e - $lcl(A)) = IFG_\delta$ - e - $lcl(A)$ (iv) IFG_δ - e - $lcl(A \cup B) = (IFG_\delta$ - e - $lcl(A)) \cup (IFG_\delta$ - e - $lcl(B))$

Definition 3.67: Let (X, T) be an IFTS. Let $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ be an IFS on an IFTS (X, T) . The IF- G_δ - e -locally interior of A is denoted and defined by IFG_δ - e -

$$lint(A) = \bigcup \{B : B = \{\langle x, \mu_B(x), \gamma_B(x) \rangle : x \in X\} \text{ is an IF- } G_\delta\text{-}e\text{-los in } X \text{ and } B \subseteq A\}.$$

Proposition 3.28: Let (X, T) be an IFTS. Let $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ and

$B = \{\langle x, \mu_B(x), \gamma_B(x) \rangle : x \in X\}$ are IFS in an IFTS (X, T) . Then the following statements are true. (i) IFG_δ - e - $lcl(A)$ is the largest IF G_δ - e -los contained in A (ii) If A is an IF G_δ - e -los then $A = IFG_\delta$ - e - $lint(A)$ (iii) If A is an IF- G_δ - e -los then IFG_δ - e - $lint(IFG_\delta$ - e - $lint(A)) = IFG_\delta$ - e - $lint(A)$ (iv) IFG_δ - e - $lint(A) = IFG_\delta$ - e - $lcl(\bar{A})$ (v) IFG_δ - e - $lcl(A) = IFG_\delta$ - e - $lint(\bar{A})$ (vi) If $A \subseteq B$ then IFG_δ - e - $lint(A) \subseteq IFG_\delta$ - e - $lint(B)$ (vii) $(IFG_\delta$ - e - $lint(A)) \cap (IFG_\delta$ - e - $lint(B)) \supseteq IFG_\delta$ - e - $lint(A \cap B)$.

Remark 3.1:9 (i) IFG_δ - e - $lcl(A) = A$ if and only if A is an IF- G_δ - e -lcs (ii) IFG_δ - e - $lint(A) \subseteq A \subseteq IFG_\delta$ - e - $lcl(A)$ (iii) IFG_δ - e - $lint(\underline{1}) = \underline{1}$ (iv) IFG_δ - e - $lint(\underline{0}) = \underline{0}$ (v) IFG_δ - e - $lcl(\underline{1}) = \underline{1}$

Proposition 3.310: Every IF- e -lcs is an IF eG_δ -lcs.

Remark 3.211: The converse of the Proposition (3.3) need not be true as show in Example (3.1).

Example 3.112: Let $X = \{a, b\}$ be a nonempty set. Let $A = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.3}\right), \left(\frac{a}{0.6}, \frac{b}{0.6}\right) \right\rangle$,

$$B = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.1}\right), \left(\frac{a}{0.5}, \frac{b}{0.4}\right) \right\rangle, A \vee B = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.3}\right), \left(\frac{a}{0.5}, \frac{b}{0.4}\right) \right\rangle \text{ and}$$

$A \wedge B = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.1}\right), \left(\frac{a}{0.6}, \frac{b}{0.6}\right) \right\rangle$ be IFS of X . Then the family $T = \{0, 1, A, B, A \vee B, A \wedge B\}$ is an IFT

on X . Now $C = \left\langle x, \left(\frac{a}{0}, \frac{b}{0.1}\right), \left(\frac{a}{0.9}, \frac{b}{0.9}\right) \right\rangle$ be IF- eG_δ -set let $D = \left\langle x, \left(\frac{a}{0}, \frac{b}{0.1}\right), \left(\frac{a}{1}, \frac{b}{1}\right) \right\rangle$ be an IF- e -closed

set. Hence $E = C \cap D = \left\langle x, \left(\frac{a}{0}, \frac{b}{0.1}\right), \left(\frac{a}{1}, \frac{b}{1}\right) \right\rangle$ is IF- eG_δ -lcs. But, E is not an IF- e -lcs. Hence, IF- eG_δ -set need not be an IF- e -lcs.

Proposition 3.413: Every IF G_δ -lcs is an IF- G_δ - e -lcs.

Remark 3.314: The converse of the Proposition (3.4) need not be true as shown in Example (3.2).

Example 3.2:15 In Example (3.1), $A \wedge B$ is an IF- G_δ -set. Let $F = \left\langle x, \left(\frac{a}{0}, \frac{b}{0.1}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}\right) \right\rangle$ be an IF- e -closed

set. Hence $E = (A \wedge B) \cap F = \left\langle x, \left(\frac{a}{0}, \frac{b}{0.1}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}\right) \right\rangle$ is IF- G_δ - e -lcs. But, E is not an IF- G_δ -lcs. Hence,

IF- G_δ - e -lcs need not be an IF- G_δ -lcs

Remark 3.416: IF- e -lcs and IF- G_δ - e -lcs are independent of each other as shown by the following Example (3.3).

Example 3.317: In Example (3.1), $A \wedge B$ is an IF- G_δ -set. Let $G = \left\langle x, \left(\frac{a}{0.1}, \frac{b}{0.2}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}\right) \right\rangle$ be an IF- e -closed set. Hence $E = (A \wedge B) \cap G = \left\langle x, \left(\frac{a}{0.1}, \frac{b}{0.1}\right), \left(\frac{a}{0.6}, \frac{b}{0.6}\right) \right\rangle$ is IF- G_δ - e -lcs. But, E is not an IF- e -lcs and $H = \left\langle x, \left(\frac{a}{0}, \frac{b}{0.1}\right), \left(\frac{a}{0.5}, \frac{b}{0.4}\right) \right\rangle$ be an IF- e -open set Hence $E = G \cap H = \left\langle x, \left(\frac{a}{0}, \frac{b}{0.1}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}\right) \right\rangle$ is IF- e -lcs. But, E is not IF- G_δ - e -lcs.

Proposition 3.518: Every IF- G_δ - e -lcs is an IF- eG_δ -lcs.

Remark 3.519: The converse of the Propositionn (3.5) need not be true as shown in Example (3.4).

Example 3.420: In Example (3.1), $C = \left\langle x, \left(\frac{a}{0}, \frac{b}{0.1}\right), \left(\frac{a}{0.9}, \frac{b}{0.9}\right) \right\rangle$ be IF- eG_δ -set and $G = \left\langle x, \left(\frac{a}{0.1}, \frac{b}{0.2}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}\right) \right\rangle$ be an IF- e -closed set. Hence $E = C \cap G = \left\langle x, \left(\frac{a}{0}, \frac{b}{0.1}\right), \left(\frac{a}{0.9}, \frac{b}{0.9}\right) \right\rangle$ is IF- eG_δ -lcs. But, E is not an IF- G_δ - e -lcs. Hence, IF- eG_δ -lcs need not be an IF- G_δ - e -lcs.

Remark 3.621: IF- G_δ -lcs and IF- eG_δ - e -lcs are independent of each other as shown by the following Example (3.5).

Example 3.522: In Example (3.1), $C = \left\langle x, \left(\frac{a}{0}, \frac{b}{0.1}\right), \left(\frac{a}{0.9}, \frac{b}{0.9}\right) \right\rangle$ be IF- eG_δ -set and $G = \left\langle x, \left(\frac{a}{0.1}, \frac{b}{0.2}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}\right) \right\rangle$ be an IF- e -closed set. Hence $E = C \cap G = \left\langle x, \left(\frac{a}{0}, \frac{b}{0.1}\right), \left(\frac{a}{0.9}, \frac{b}{0.9}\right) \right\rangle$ is IF- eG_δ -lcs. But, E is not an IF- G_δ -lcs and $A \wedge B$ is an IF- G_δ -set let $I = \left\langle x, \left(\frac{a}{0.6}, \frac{b}{0.6}\right), \left(\frac{a}{0.3}, \frac{b}{0.1}\right) \right\rangle$ be an intuitionistic fuzzy closed set. Hence $E = (A \wedge B) \cap I = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.1}\right), \left(\frac{a}{0.6}, \frac{b}{0.6}\right) \right\rangle$ is IF- G_δ -lcs. But, E is not an IF- eG_δ -lcs.

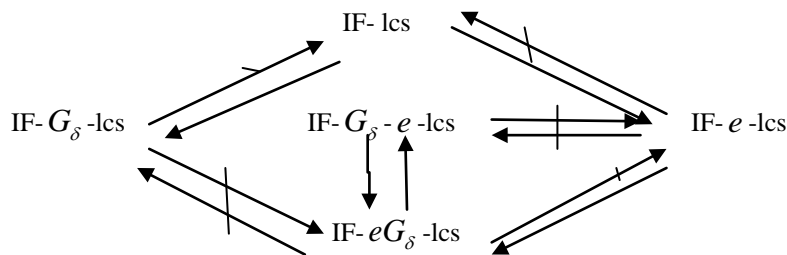
Proposition 3.623: Every IF-locally closed set is an IF- e -lcs.

Remark 3.724: The converse of the Propositionn (3.6) need not be true as shown in Example (3.6)

Example 3.6: In Example (3.1), Let $J = \left\langle x, \left(\frac{a}{0}, \frac{b}{0.1}\right), \left(\frac{a}{0.5}, \frac{b}{0.4}\right) \right\rangle$ be IFeOS and $H = \left\langle x, \left(\frac{a}{0.1}, \frac{b}{0.2}\right), \left(\frac{a}{0.4}, \frac{b}{0.4}\right) \right\rangle$ be an IFeCS. Hence $E = J \cap H = \left\langle x, \left(\frac{a}{0}, \frac{b}{0.1}\right), \left(\frac{a}{0.5}, \frac{b}{0.4}\right) \right\rangle$ is IF- e -lcs. But, E is not an IF-locally closed set.

Remark 3.826: Every IF-locally closed set is an IF- G_δ -lcs but the converse need not be true as shown in [10].

Remark 3.927: Clearly the following diagram holds.



Definition 3.728: Let (X, T) be an IFTS and Y be any intuitionistic fuzzy subset of X . Then $T_Y = \{A/Y \mid A \in T\}$ is an IFT on Y and is called the induced or relative intuitionistic fuzzy topology. The pair (Y, T_Y) is called an intuitionistic fuzzy subspace of (X, T) : (Y, T_Y) is called an IF- G_δ - e -locally closed intuitionistic fuzzy subspace if the intuitionistic fuzzy characteristic function of (Y, T_Y) viz χ_Y is IF- G_δ - e -lcs.

Proposition 3.729: Let (X, T) be an IFTS. Suppose $Z \subseteq Y \subseteq X$ and (Y, T_Y) is an IF- G_δ - e -locally closed intuitionistic fuzzy subspace of an IFTS (X, T) . If (Z, T_Z) is an IF- G_δ - e -locally closed intuitionistic fuzzy subspace in an IFTS $(X, T) \Leftrightarrow (Z, T_Z)$ is an intuitionistic fuzzy G_δ - e -locally closed intuitionistic fuzzy subspace in an IFTS (Y, T_Y) .

Definition 3.830: An IFTS (X, T) is said to be an intuitionistic fuzzy G_δ - e -local- $T_{\frac{1}{2}}$ space if for every intuitionistic fuzzy G_δ - e -locally closed set is an intuitionistic fuzzy closed set in an IFTS (X, T) .

Definition 3.931: An IFTS (X, T) is said to be an intuitionistic fuzzy G_δ - e -local- δ -semi (resp., δ -pre and β) space if for every IF- G_δ - e -lcs is an intuitionistic fuzzy δ -semi (resp., δ -pre and β) closed set in an IFTS (X, T) .

Proposition 3.832: Every intuitionistic fuzzy G_δ - e -local- $T_{\frac{1}{2}}$ space is an intuitionistic fuzzy G_δ - e -local- δ -semi (resp., G_δ - e -local- δ -pre and G_δ - e -local- β) space.

Remark 3.1033: The converse of the Proposition (3.8) need not be true as shown in Examples (3.7) and (3.8).

Example 3.7:34 Let $X = \{a, b\}$ be a nonempty set. Let $A = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.3}\right), \left(\frac{a}{0.6}, \frac{b}{0.6}\right) \right\rangle$, $B = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.1}\right), \left(\frac{a}{0.5}, \frac{b}{0.4}\right) \right\rangle$, $A \vee B = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.3}\right), \left(\frac{a}{0.5}, \frac{b}{0.4}\right) \right\rangle$ and $A \wedge B = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.1}\right), \left(\frac{a}{0.6}, \frac{b}{0.6}\right) \right\rangle$ be IFS of X . Then the family $T = \{0, 1, A, B, A \vee B, A \wedge B\}$ is an IFT on X . Now, $A \wedge B$ is an IF- G_δ -set. Let $C = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.1}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}\right) \right\rangle$ be an intuitionistic fuzzy e -closed set. Hence $E = (A \wedge B) \cap C = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.1}\right), \left(\frac{a}{0.6}, \frac{b}{0.6}\right) \right\rangle$ is δ -semi-closed. Hence (X, T) is an intuitionistic fuzzy G_δ - e -local- δ -semi space. But, E is not an intuitionistic fuzzy closed set. Thus, (X, T) is not an intuitionistic fuzzy G_δ - e -local $T_{1/2}$ space. Hence, intuitionistic fuzzy G_δ - e -local- δ -semi space need not be an intuitionistic fuzzy G_δ - e -local $T_{1/2}$.

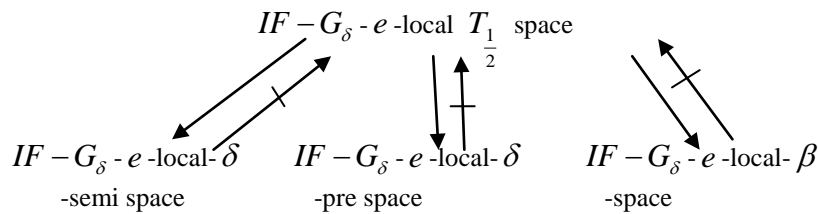
Example 3.8:35 In Example (3.7), Let $D = \left\langle x, \left(\frac{a}{0}, \frac{b}{0}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}\right) \right\rangle$ be an IFeCS. Hence

$E = (A \wedge B) \cap D = \left\langle x, \left(\frac{a}{0}, \frac{b}{0}\right), \left(\frac{a}{0.6}, \frac{b}{0.6}\right) \right\rangle$ is δ -pre-closed (resp., β -closed). Hence (X, T) is an

intuitionistic fuzzy G_δ - e -local- δ -pre (G_δ - e -local- β -space). But, E is not an intuitionistic fuzzy closed set.

Thus, (X, T) is not an intuitionistic fuzzy G_δ - e -local $T_{\frac{1}{2}}$ space. Hence, intuitionistic fuzzy G_δ - e -local- δ -pre (G_δ - e -local- β)space need not be an intuitionistic fuzzy G_δ - e -local $T_{\frac{1}{2}}$.

Remark 3.1136: Clearly the following diagram holds.



REFERENCES

1. Atanassov K. T., *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, 20, (1986),87-96.
2. Balasubramanian G., *Maximal fuzzy topologies*, Kybernetika, 31, (1995),459-465.
3. Bourbaki N., *General topology*, Part 1, Addison- Wesley, Reading, Mass (1966).
4. Coker D., *An introduction to Intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems, 88,(1997),81-89.
5. Ekici E., *On e -open sets, DP^* -sets and $DP\mathcal{E}^*$ -sets and decompositions of continuity*, Arabian Journal for Science and Engineering, 33 (2A), (2008), 269-282.
6. Ekici E., *Some generalizations of almost contra-super-continuity*, Filomat, 21 (2) (2007), 31-44.
7. Ekici E., *New forms of contra-continuity*, Carpathian Journal of Mathematics, bf 24(1), (2008), 37-45.
8. Ganster M., and Relly I. L., *Locally closed sets and and LC-continous functions*, Intr., J.Math and Math. Sci., 12(3), (1989), 417-424.
9. Krsteska B., and Ekici E., *Intuitionistic fuzzy contra strong precontinuity*, Faculty of Sciences and Mathematics University of Nis, Serbia, Filomat., 21(2), (2007), 273-284.
10. Narmada Devi R., Roja E., and Uma M. K., *On Some Applications of Intuitionistic Fuzzy G_δ - α -locally Closed Sets*, The Journal of Fuzzy Mathematics, 21(1),(2013),85-98.
11. Seenivasan V., and Kamala K., *Fuzzy e -continuity and fuzzy e -open sets*, Annals of Fuzzy Mathematics and Informatics, 8, (1) (2014), 141-148.
12. Sobana D., Chandrasekar V., and Vadivel A., *On Fuzzy e -open Sets, Fuzzy e -continuity and Fuzzy e -compactness in IFTSs*, Accepted in Sahand Communications in Mathematical Analysis.
13. Thakur S.S., and Singh S., *On fuzzy semi-pre open sets and fuzzy semi-pre continuity*, Fuzzy Sets and Systems, (1998), 383-391.
14. Zadeh L. A., *Fuzzy Sets*, Information and Control, 8, (1965), 338-353.

Source of support: Proceedings of UGC Funded International Conference on Intuitionistic Fuzzy Sets and Systems (ICIFSS-2018), Organized by: Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.