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THE EXTENDED FUZZY LINEAR COMPLEMENTARITY PROBLEM

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ABSTRACT

In this paper the Extended Fuzzy Linear Complementarity Problem (EFLCP),an extension of the well-known Fuzzy Linear Complementarity Problem(FLCP) is defined. The general solution set of an EFLCP is discussed and an algorithm to find the solution is suggested. Finally the applications of EFLCP in the max algebra are discussed with a numerical example by using a triangular fuzzy number.

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1. INTRODUCTION

Fuzzy sets have been introduced by A.Zadeh[16].Fuzzy set theory permits the gradual assessments of the membership of elements in a set which is described in the interval [0,1]. It can be used in a wide range of domains where information is incomplete and imprecise. Dubois and Prade [10] has defined any of the fuzzy numbers as a fuzzy subset of the real line[11].A fuzzy number is a quantity whose values are imprecise, rather than exact is the case with single-valued numbers. Triangular fuzzy numbers and triangular fuzzy matrices are frequently used in application.

The Extended Linear Complementarity Problem (ELCP), an extension of the Linear Complementarity Problem, which is one of the fundamental problems in mathematical programming. In this paper, we shall present an algorithm to find the solution of an EFLCP. The core of this algorithm is formed by an adaption of Motzkin's double description method for solving sets of linear inequalities [13].

The formulation of the EFLCP arose from our work in the study of discrete event systems. We shall briefly indicate how the ELCP can be used to solve a system of multivariate polynomial fuzzy equalities and inequalities in the max algebra, the framework that we use to model a class of discrete event systems. This allows us to solve many other problems in the max algebra such as matrix decompositions, state space transformations, minimal state space realization of single input single output discrete event systems and so on [7,8].

This paper is organized as follows: Firstly in section 2, we recall the definition of triangular fuzzy number, triangular fuzzy matrix and some operations on these two. Section 3 provides the Extended Fuzzy Linear Complementarity problem (EFLCP).Section 4 provides an algorithm for determining the solution of the EFLCP. Section 5 provides a link between the max algebra and the EFLCP. Section 6 provides a numerical example for determining the solution of EFLCP.

2. PRELIMINARIES

2.1 Fuzzy set

A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0, 1]\}$.

In the pair $(x, \mu_A(x))$, the first element x belong to the classical set A, the second element $\mu_A(x)$, belong to the interval [0, 1], called Membership function.

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2.2 Fuzzy number

The notion of fuzzy numbers was introduced by Dubois D. and Prade H [10].A fuzzy subset $\tilde{A}$ of the real line R with membership function$\mu_{\tilde{A}}$:R→[0,1] is called a fuzzy number if

i) A fuzzy set $\tilde{A}$ is normal.
ii) $\tilde{A}$ is fuzzy convex,
(i.e.) $\mu_{\tilde{A}}[\lambda x_1+(1-\lambda)x_2]\geq\mu_{\tilde{A}}(x_1)\wedge\mu_{\tilde{A}}(x_2), x_1, x_2\in R, \forall\lambda\in[0,1]$.
iii) $\mu_{\tilde{A}}$ is upper continuous,and
iv) Supp$\tilde{A}$ is bounded, where supp $\tilde{A}=\{x\in R:\mu_{\tilde{A}}(x)>0\}$.

2.3 Triangular Fuzzy Number

It is a fuzzy number represented with three points as follows: $\tilde{A}=(a_1, a_2, a_3)$. This representation in interpreted as membership functions

$$\mu_{\tilde{A}}(x)=\begin{cases}0 & for x< a_1\\ \dfrac{x-a_1}{a_2-a_1} & for a_1\leq x\leq a_2\\ \dfrac{a_3-x}{a_3-a_2} & for a_2\leq x\leq a_3\\ 0 & for x> a_3\end{cases}$$

2.4 Operations of Triangular Fuzzy Number using Function Principle

Let $\tilde{A}=(a_1, a_2, a_3)$ and $\tilde{B}=(b_1, b_2, b_3)$ be two triangular fuzzy numbers. Then

i. The addition of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}+\tilde{B}=(a_1+b_1, a_2+b_2, a_3+b_3)$ where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers.
ii. The product of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}$ x $\tilde{B}=(c_1, c_2, c_3)$, where $T=\{a_1b_1, a_2b_2, a_3b_3\}$ where $c_1=\min\{T\}$, $c_2=a_2b_2$, $c_3=\max\{T\}$. If $a_1, a_2, a_3, b_1, b_2, b_3$ are all non- zero positive real numbers, then $\tilde{A}$ x$\tilde{B}=(a_1b_1, a_2b_2, a_3b_3)$
iii. $\tilde{B}=(-b_3, -b_2, -b_1)$ then the subtraction of $\tilde{B}$ from $\tilde{A}$ is $\tilde{A}-\tilde{B}=(a_1-b_3, a_2-b_2, a_3-b_1)$ where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers.
iv. The division of $\tilde{A}$ and $\tilde{B}$ is $\frac{\tilde{A}}{\tilde{B}}=(c_1, c_2, c_3)$, where $T=(\frac{a_1}{b_3},\frac{a_2}{b_2},\frac{a_3}{b_1})$,where $c_1=\min\{T\}$,$c_2=\frac{a_2}{b_2}$, $c_3=\max\{T\}$.
If a_1,a_2,a_3,b_1,b_2,b_3 are all non-zero positive real numbers then $\frac{\tilde{A}}{\tilde{B}}=(\frac{a_1}{b_3},\frac{a_2}{b_2},\frac{a_3}{b_1})$.

2.5 Triangular Fuzzy Matrix

A triangular fuzzy matrix of order mxn is defined as $A=(\tilde{a}_{ij})_{mxn}$, where $a_{ij}=(a_{ij1}, a_{ij2}, a_{ij3})$ is the ij^{th} element of A.

2.6 Operations on Triangular Fuzzy Matrices

Let $A=(\tilde{a}_{ij})$ and $B=(\tilde{b}_{ij})$ be two triangular fuzzy matrices of same order. Then we have the following:

i. $A+B=(\tilde{a}_{ij}+\tilde{b}_{ij})$
ii. $A-B=(\tilde{a}_{ij}-\tilde{b}_{ij})$
iii. For $A=(\tilde{a}_{ij})_{mxn}$ and $B=(\tilde{b}_{ij})_{nxk}$ then $AB=(\tilde{c}_{ij})_{mxk}$ where $\tilde{c}_{ij}=\sum_{p=1}^{n}\tilde{a}_{ip}.\tilde{b}_{pj}$, i=1, 2,…,m and j=1,2,…,k.
iv. $A^T=(\tilde{a}_{ji})$
v. $KA=(K\tilde{a}_{ij})$ where K is scalar.

2.7 Fuzzy Linear Complementarity Problem

Assume that all parameters in Linear Complementarity Problem are fuzzy and are described by fuzzy numbers. Then the following fuzzy linear complementarity problem can be obtained by replacing crisp parameters with fuzzy number.

$$\tilde{w}-\tilde{M}\tilde{z}=\tilde{q} \tag{2.1}$$

$$\tilde{w}_i, \tilde{z}_i\geq 0, \text{ for } i=1, 2, \ldots\ldots, n \text{ and} \tag{2.2}$$

$$\tilde{w}_i\tilde{z}_i=0, \text{ for } i=1, 2, \ldots\ldots, n. \tag{2.3}$$

The pair ($\tilde{w}_i, \tilde{z}_i$) is said to be a pair of fuzzy complementarity variables.

2.8 Fuzzy Extended Linear Complementarity Problem

Given two matrices $\tilde{A}\in R^{pXn}$, $\tilde{B}\in R^{qXn}$, two vectors $\tilde{c}\in R^p$, $\tilde{d}\in R^q$ and m subsets $\emptyset_j$of {1, 2, ..., p}, find a vector $\tilde{x}\in R^n$ such that

$$\sum_{j=1}^{m}\prod_{i\in\emptyset_j}(\tilde{A}\tilde{x}-\tilde{c})_i=\tilde{0}, \tag{2.4}$$

$$\tilde{A}\tilde{x}\geq\tilde{c}\ , \tag{2.5}$$

$$\tilde{B}\tilde{x}=\tilde{d}\ , \tag{2.6}$$

or show that no such vector exists.

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2.9 The homogeneous EFLCP

In order to solve the EFLCP we have to homogenize it: we introduce a scalar $\alpha \geq 0$ and define$\tilde{u} = \begin{bmatrix} \tilde{x} \\ \alpha \end{bmatrix}$,

$$\tilde{P} = \begin{bmatrix} \tilde{A} & -\tilde{c} \\ \tilde{O}_{n,1} & \tilde{1} \end{bmatrix} \text{ and } \tilde{P} = [\tilde{B} \quad -\tilde{d}].$$

Then we get a homogeneous EFLCP of the following form:

Given two matrices $\tilde{P} \in R^{pXn}$, $\tilde{P} \in R^{qXn}$ and m subsets $\emptyset_j$ of {1, 2, ..., p}, find a non-trivial vector u$\in R^n$ such that

$$\sum_{j=1}^{m} \prod_{i \in \emptyset_j} (\tilde{\boldsymbol{P}}\tilde{\boldsymbol{u}})_i = \tilde{\mathbf{0}}, \tag{2.7}$$

$$\tilde{\boldsymbol{P}}\tilde{u} \geq \tilde{\mathbf{0}}\,, \tag{2.8}$$

$$\tilde{Q}\tilde{u} = \tilde{\mathbf{0}}\,, \tag{2.9}$$

or show that no such vector exists.

3. THE EXTENDED FUZZY LINEAR COMPLEMENTARITY PROBLEM ALGORITHM

When we consider the homogeneous EFLCP we see that we have a system of homogeneous fuzzy linear equalities and inequalities subject to a complementarity. The solution set of the system of homogeneous fuzzy linear inequalities and equalities (2.8) – (2.9) is a polyhedral cone P and can be described using two sets of rays: a set of central rays $\tilde{C}$ and a set of extreme rays $\tilde{\mathcal{E}}$. A set of central rays can be considered as a basis for the largest linear subspace of the polyhedral cone P. If $\tilde{c} \in \tilde{C}$ then $\tilde{P}\tilde{c} = \tilde{\mathbf{0}}$, and if $\tilde{e} \in \tilde{\mathcal{E}}$ then $\tilde{P}\tilde{c} \neq \tilde{\mathbf{0}}$.

A vector $\tilde{u}$ is a solution of (2.8) – (2.9) if and only if it can be written as

$$\tilde{u} = \sum_{\tilde{c}_i \in \tilde{C}} \tilde{\alpha}_i \tilde{c}_i + \sum_{\tilde{e}_i \in \tilde{\mathcal{E}}} \tilde{\beta}_i \tilde{e}_i, \tag{3.1}$$

with $\tilde{\alpha}_i \in$ R and $\tilde{\beta}_i \geq 0$.To calculate $\tilde{C}$ and $\tilde{\mathcal{E}}$ we use an iterative algorithm that is an adaptation and extension of the double description method of Motzkin [13]. During the iteration we already remove rays that do not satisfy the partial complementarity condition since such rays cannot yield solutions of the EFLCP. In the kth step the partial complementarity condition is defined as follows:

$$\prod_{i \in \varphi_j} (\tilde{\boldsymbol{P}}\tilde{\boldsymbol{u}})_i = \tilde{\mathbf{0}}, \qquad \forall j \in \{1,2, \ldots, m\} \text{ such that } \varphi_j \subset \{1,2,\ldots,k\}. \tag{3.2}$$

For k≥p the partial complementarity condition (3.2) coincides with the full complementarity condition (2.7). This leads to the following algorithm:

Algorithm: Calculation of the central and extreme rays Initialization:

$\tilde{C}_0 = \{\tilde{c}_i \mid \tilde{c}_i = (\tilde{I}_n)_{.i}$, for i = 1,2,...,n$\}$

$\tilde{\mathcal{E}}_0 = \varphi$

Iteration: For k = 1, 2,...,p+q,

Calculate the intersection of the current polyhedral cone (described by $\tilde{C}_{k-1}$ and $\tilde{\mathcal{E}}_{k-1}$) with the half-space or hyperplane determined by the kth fuzzy inequality or equality. This yields a new polyhedral cone described by $\tilde{C}_k$ and $\tilde{\mathcal{E}}_k$. Remove the rays that do not satisfy the partial complementarity condition.

Result: $\tilde{C} = \tilde{C}_{p+q}$ and $\tilde{\mathcal{E}} = \tilde{\mathcal{E}}_{p+q}$

Not every combination of the form (3.1) satisfies the complementarity condition. Although every linear combination of the central rays satisfies the complementarity condition, not every positive combination of the extreme rays satisfies the complementarity condition. Therefore we introduce the concept of cross-complementarity:

Definition 2.1 (Cross-complementarity): Let $\tilde{\mathcal{E}}$ be a set of extreme rays of an homogeneous EFLCP .A subset $\tilde{\mathcal{E}}_s$ of $\tilde{\mathcal{E}}$ is cross-complementary if every combination of the form $\tilde{u} = \sum_{\tilde{e}_i \in \tilde{\mathcal{E}}_s} \tilde{\beta}_i \tilde{e}_i$, with $\tilde{\beta}_i \geq \tilde{0}$, satisfies the complementarity condition.In [6] we have proven that in order to check whether a set $\mathcal{E}_s$ is cross-complementary it suffices to test only one strictly positive combination of the rays in $\mathcal{E}_s$, e.g., the combination with ∀i : β_i=1.

Theorem 4.1: The general solution set of an EFLCP consists of the union of faces of a polyhedron.

Theorem 4.2: The general EFLCP is an NP hard problem.

Proof: The decision problem that corresponds to the EFLCP belongs to NP: a non-deterministic algorithm can guess a vector $\tilde{x}$ and then check in polynomial time whether $\tilde{x}$ satisfies the complementarity condition and the system of linear equalities and inequalities. Chung [3] has proven that the decision problem that corresponds to the FLCP is in general

an NP – complete problem. The FLCP is a special case of the EFLCP and therefore the decision problem that corresponds to the EFLCP is also NP-complete. This means that in general the EFLCP is NP hard.
So the EFLCP can probably not be solved in polynomial time (unless the class P would coincide with the class NP).

4. A LINK BETWEEN THE MAX ALGEBRA AND THE EFLCP

The formulation of the EFLCP arose in this research on discrete event systems, examples of which are flexible manufacturing systems, traffic networks and telecommunications networks. Normally the behaviour of discrete event systems is highly nonlinear. However, when the order of the events is known or fixed some of these systems can be described by a linear description in the max algebra [1].

The basic operations of the max algebra are the maximum (represented by$\oplus$) and the addition (represented by$\otimes$):

$$\tilde{x} \oplus \tilde{y} = \max(\tilde{x}, \tilde{y}) \,, \tag{4.1}$$

$$\tilde{x} \otimes \tilde{y} = \tilde{x}+\tilde{y} \tag{4.2}$$

The reason for choosing these symbols is that many results from linear algebra can be translated to the max algebra simply by replacing + by$\oplus$ and X by$\otimes$. The max-algebraic power is defined as follows:

$$\tilde{x}^{\otimes^{a}} = a.\tilde{x} \tag{4.3}$$

Many important problems in the max algebra can be reformulated as a set of max-algebraic polynomial fuzzy equalities and inequalities in the max algebra:

Given a set of integers $\{\tilde{m}_k\}$ and three sets of real numbers $\{\tilde{a}_{ki}\}$, $\{\tilde{b}_k\}$ and $\{\tilde{c}_{kij}\}$ with i ∈{1,2,...,$\tilde{m}_k$}, j∈{1,2,...,n} and k∈{1,2,...p_1, p_1+1,...,p_1+p_2}, find a vector $\tilde{x} \in R^n$ that satisfies

$\oplus_{i=1}^{m_k} \tilde{a}_{ki} \otimes \otimes_{j=1}^{n} \tilde{x}_j^{\otimes^{\tilde{c}_{kij}}} = \tilde{b}_k$ for $k = 1,2, \ldots, p_1$,

$\oplus_{i=1}^{m_k} \tilde{a}_{ki} \otimes \otimes_{j=1}^{n} \tilde{x}_j^{\otimes^{\tilde{c}_{kij}}} \leq \tilde{b}_k$ for$k = p_1 +1, p_1 + 2, \ldots, p_1 + p_2$,

or show that no such vector $\tilde{x}$ exists.

Theorem 4.1: A set of multivariate polynomial fuzzy equalities and inequalities in the max algebra is equivalent to an extended fuzzy linear complementarity problem.

We shall illustrate this by an example:

Consider the following set of multivariate polynomial fuzzy equalities and inequalities:

$$(2.75,3,3.25)\otimes(0.75,1,1.25)\tilde{x}_1^{\otimes^3} \otimes (0.75,1,1.25)\, \tilde{x}_2^{\otimes^{-1}} \otimes (0.75,1,1.25)\tilde{x}_4^{\otimes^{-2}} \oplus (0.75,1,1.25)\tilde{x}_1^{\otimes^{-1}} \otimes (0.75,1,1.25)\, \tilde{x}_3^{\otimes^3} \oplus (6.75,7,7.25) \otimes (0.75,1,1.25)\tilde{x}_2 \otimes (0.75,1,1.25)\, \tilde{x}_3^{\otimes^2} = (2.75,3,3.25) \tag{4.4}$$

$$(1.75,2,2.25)\otimes(0.75,1,1.25)\tilde{x}_1 \otimes (0.75,1,1.25)\tilde{x}_3^{\otimes^2} \oplus (-1.25,-1,-0.75) \otimes (0.75,1,1.25)\tilde{x}_1 \otimes (0.75,1,1.25)\, \tilde{x}_4^{\otimes^{-2}} = (3.75,4,4.25) \tag{4.5}$$

$$(3.75,4,4.25) \otimes (0.75,1,1.25)\tilde{x}_1 \otimes (0.75,1,1.25)\tilde{x}_3^{\otimes^{-4}} \leq (5.75,6,6.25) \tag{4.6}$$

Let us first consider the first term of (4.4). Using definitions (4.2) and (4.3)we find that this term is equivalent to (2.75,3,3.25)+3(0.75,1,1.25)$\tilde{x}_1$+(-1)(0.75,1,1.25)$\tilde{x}_2$+(-2)(0.75,1,1.25)$\tilde{x}_4$= (2.75,3,3.25)
i.e., (-3.75,-3,-2.25)$\tilde{x}_1$+(0.75,1,1.25)$\tilde{x}_2$+(1.5,2,2.5)$\tilde{x}_4$ = (-1,0,1)

The other terms of (4.3) can be transformed to linear algebra in a similar way. Each term has to be smaller than $\tilde{3}$ and atleast one of them has to be equal to$\tilde{3}$. So we get a group of three inequalities in which atleast one inequality should hold with equality. If we also include expressions (4.4) and (4.5) we get the following EFLCP:

$$\text{Given } \tilde{A} = \begin{pmatrix} (-3.75,-3,-2.25) & (0.75,1,1.25) & (0,0,0) & (1.5,2,2.5) \\ (0.75,1,1.25) & (0,0,0) & (-3.75,-3,-2.25) & (0,0,0) \\ (0,0,0) & (-1.25,-1,-0.75) & (-2.5,-2,-1.5) & (0,0,0) \\ (-1.25,-1,-0.75) & (0,0,0) & (-2.5,-2,-1.5) & (0,0,0) \\ (-1.25,-1,-0.75) & (0,0,0) & (0,0,0) & (1.5,2,2.5) \\ (-1.25,-1,-0.75) & (0,0,0) & (3,4,5) & (0,0,0) \end{pmatrix}$$

and $\tilde{c} = \begin{pmatrix} (-1,0,1) \\ (-3.25,-3,-2.75) \\ (3.5,4,4.5) \\ (-2.5,-2,-1.5) \\ (-5.5,-5,-4.5) \\ (-2.5,-2,-1.5) \end{pmatrix}$

find a column vector $\tilde{x} \in R^4$ such that

$(\tilde{A}\tilde{x} - \tilde{c})_1(\tilde{A}\tilde{x} - \tilde{c})_2(\tilde{A}\tilde{x} - \tilde{c})_3 + (\tilde{A}\tilde{x} - \tilde{c})_4(\tilde{A}\tilde{x} - \tilde{c})_5 = \tilde{0}$ and $\tilde{A}\tilde{x} \geq \tilde{c}$

Many problems in the max algebra such as matrix decompositions, transformation of state space models, construction of matrices with a given characteristic polynomial, minimal state space realization and so on, can be reformulated as a set of multivariate max-algebraic polynomial fuzzy equalities and inequalities. These problems are equivalent to an EFLCP and can thus be solved using the EFLCP algorithm. In general their solution set consists of the union of faces of a polyhedran.

5. NUMERICAL EXAMPLE

Solve the following EFLCP:

$$\tilde{P} = \begin{pmatrix} (0.75,1,1.25) & (0,0,0) & (0.75,1,1.25) & (0,0,0) \\ (0.75,1,1.25) & (0.75,1,1.25) & (0.75,1,1.25) & (-2.25,-2,-1.75) \\ (0.75,1,1.25) & (-1.25,-1,-0.75) & (0,0,0) & (0.75,1,1.25) \end{pmatrix}$$

$and\ \tilde{Q} = [(0,0,0)\quad (0,0,0)\quad (0.75,1,1.25)\quad (0.75,1,1.25)], find\ \tilde{u} \in R^4\ such\ that$

$$(\tilde{P}\tilde{u})_1(\tilde{P}\tilde{u})_2 + (\tilde{P}\tilde{u})_2(\tilde{P}\tilde{u})_3 = \tilde{0} \tag{5.1}$$

$Subject\ to\ \tilde{P}\tilde{u} \geq \tilde{0}$

$\tilde{Q}\tilde{u} = \tilde{0}.$

Solution: Since all inequalities of $\tilde{P}\tilde{u} \geq \tilde{0}$ appear in the complementarity condition we do not have to split $\tilde{P}\tilde{u} \geq \tilde{0}$.

First we process the inequalities of $\tilde{P}\tilde{u} \geq \tilde{0}$:

k= 0:

Initialization:

$\tilde{c}_{0,1} = \begin{bmatrix} (0.75,1,1.225) \\ (0,0,0) \\ (0,0,0) \\ (0,0,0) \end{bmatrix}$, $\tilde{c}_{0,2} = \begin{bmatrix} (0,0,0) \\ (0.75,1,1.25) \\ (0,0,0) \\ (0,0,0) \end{bmatrix}$, $\tilde{c}_{0,3} = \begin{bmatrix} (0,0,0) \\ (0,0,0) \\ (0.75,1,1.25) \\ (0,0,0) \end{bmatrix}$, $\tilde{c}_{0,4} = \begin{bmatrix} (0,0,0) \\ (0,0,0) \\ (0,0,0) \\ (0.75,1,1.25) \end{bmatrix}$

k= 1:

First we calculate the residues:

res $(\tilde{c}_{0,1})$ = (0.75,1,1.25), res$(\tilde{c}_{0,2})$ = (0,0,0), res$(\tilde{c}_{0,3})$ = (0.75,1,1.25), res$(\tilde{c}_{0,4})$ = (0,0,0).

Hence $\tilde{C}^+ = \{\tilde{c}_{0,1}, \tilde{c}_{0,3}\}$ and $\tilde{C}^0 = \{\tilde{c}_{0,2}, \tilde{c}_{0,4}\}$

Since $\tilde{C}^+$ is not empty, therefore we put the elements of $\tilde{C}^0$ in $\tilde{C}$: $\tilde{c}_{1,1} = \tilde{c}_{0,2}$ and $\tilde{c}_{1,2} = \tilde{c}_{0,4}$.

Since $\tilde{C}^-$ is empty we do not have to transfer rays from $\tilde{C}^-$ to $\tilde{C}^+$. We set $\tilde{c} = \tilde{c}_{0,1}$ and put it in $\tilde{}$:$\tilde{e}_{1,1} = \tilde{c}_{0,1}$. Because no group of inequalities has been processed entirely yet, condition (3.2) is void.So $\tilde{c}$ satisfies the partial complementarity condition.

So we find $\tilde{c}_{1,1} = \begin{bmatrix} (0,0,0) \\ (0.75,1,1.25) \\ (0,0,0) \\ (0,0,0) \end{bmatrix}$, $\tilde{c}_{1,2} = \begin{bmatrix} (0,0,0) \\ (0,0,0) \\ (0,0,0) \\ (0.75,1,1.25) \end{bmatrix}$, $\tilde{c}_{1,3} = \begin{bmatrix} (0.56,1,1.56) \\ (0,0,0) \\ (-1.56,-1,-0.56) \\ (0,0,0) \end{bmatrix}$, $\tilde{e}_{1,1} = \begin{bmatrix} (0.75,1,1.25) \\ (0,0,0) \\ (0,0,0) \\ (0,0,0) \end{bmatrix}$

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Using the same procedure as in the previous step we find that the following:

k= 2: $\tilde{c}_{2,1}=\begin{bmatrix}(056,1,1.56)\\(0,0,0)\\(-1.56,-1,-0.56)\\(0,0,0)\end{bmatrix}$, $\tilde{c}_{2,2}=\begin{bmatrix}(0,0,0)\\(1.27,2,2.74)\\(0,0,0)\\(0.42,1,1.95)\end{bmatrix}$, $\tilde{e}_{2,1}=\begin{bmatrix}(0,0,0)\\(0.75,1,1.25)\\(0,0,0)\\(0,0,0)\end{bmatrix}$, $\tilde{e}_{2,2}=\begin{bmatrix}(0.75,1,1.25)\\(-1.25,-1,-0.75)\\(0,0,0)\\(0,0,0)\end{bmatrix}$

k = 3: $\tilde{c}_{3,1}=\begin{bmatrix}(-1.33,1,1)\\(0.53,2,5.34)\\(-2.71,-1,0.48)\\(0.18,1,3.8)\end{bmatrix}$, $\tilde{e}_{3,1}=\begin{bmatrix}(0.56,1,1.56)\\(0,0,0)\\(-1.56,-1,-0.56)\\(0,0,0)\end{bmatrix}$, $\tilde{e}_{3,2}=\begin{bmatrix}(0.56,1,1.47)\\(032,1,2.44)\\(-1.56,-1,-0.53)\\(0,0,0)\end{bmatrix}$,

$\tilde{e}_{3,3}=\begin{bmatrix}(-4.55,-1,1.81)\\(-1.46,-1,-0.52)\\(1.75,2,2)\\(0,0,0)\end{bmatrix}$

We do not have to reject any extreme rays since they all satisfy complementarity condition (6.1).

Since we did not encounter any redundant inequalities, we have that $\tilde{P}_{nec}=\tilde{P}$. No we take the equality $\tilde{Q}\tilde{u}=\tilde{0}$ into account:

k= 1: $\tilde{c}_{4,1}=\tilde{c}_{3,1}=\begin{bmatrix}(-1.33,1,1)\\(0.53,2,5.34)\\(-2.71,-1,0.48)\\(0.18,1,3.8)\end{bmatrix}$

Thus $\tilde{e}_{4,1}=\begin{bmatrix}(-4.59,1,5.17)\\(-1.71,-1,-0.36)\\(-0.64,0,0.65)\\(0,0,0)\end{bmatrix}$

Since the combination of the adjacent rays $\tilde{e}_{3,3}$ and $\tilde{e}_{3,2}$does not satisfy the complementarity condition, we have to reject it.

Now we have $\tilde{C}=\begin{bmatrix}(-1.33,1,1)\\(0.53,2,5.34)\\(-2.71,-1,0.48)\\(0.18,1,3.8)\end{bmatrix}$ and $\tilde{}=\begin{bmatrix}(-4.59,1,5.17)\\(-1.71,-1,-0.36)\\(-0.64,0,0.65)\\(0,0,0)\end{bmatrix}$

Hence every combination of the form

$\tilde{C}=\lambda\begin{bmatrix}(-1.33,1,1)\\(0.53,2,5.34)\\(-2.71,-1,0.48)\\(0.18,1,3.8)\end{bmatrix}+\kappa\begin{bmatrix}(-4.59,1,5.17)\\(-1.71,-1,-0.36)\\(-0.64,0,0.65)\\(0,0,0)\end{bmatrix}$ with $\lambda\in R$ and $\kappa\geq 0$

is a solution of the EFLCP.

CONCLUSION

In this paper we have introduced the Extended Fuzzy Linear Complementarity Problem (EFLCP) and sketched an algorithm to find the solution of it. We have also demonstrated that the EFLCP can be used to solve a set of max-algebraic polynomial fuzzy equalities and inequalities and related problems. Therefore the EFLCP is a powerful mathematical tool for solving max-algebraic problems.

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